Functional limit theorems for weakly dependent heavy tailed time series

Philippe Soulier
Université Paris Ouest Nanterre

Joint work with Bojan Basrak and Hrvoje Planinić
Zagreb University

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II. Convergence to a stable process

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I. Regularly varying time series

We are interested in stationary time series $\{X_t, t \in \mathbb{Z}\}$ with heavy tails or regularly varying tail.

A random variable X is said to have a regularly varying right tail with tail index α (denoted $RV(\alpha)$) if

$$\lim_{t\to\infty} \frac{\mathbb{P}(X > tx)}{\mathbb{P}(X > t)} = x^{-\alpha} \; .$$

The tail balance condition is said to hold if $|X|$ is regularly varying at infinity and

$$\lim_{x\to\infty} \frac{\mathbb{P}(X > x)}{\mathbb{P}(|X| > x)} = 1 - \lim_{x\to\infty} \frac{\mathbb{P}(X < -x)}{\mathbb{P}(|X| > x)} = p \in [0,1] \; .$$

Let $\{X_i, i \in \mathbb{N}\}$ is a sequence of i.i.d. random variables with the same distribution as X. Set $\mathbb{P}(X > a_n) \sim 1/n$ and

$$b_n = \begin{cases} 0 & \text{if } \alpha < 1 , \\ a_n^{-1}\mathbb{E}[X_1 \mathbb{1}_{\{|X_1| \leq a_n\}}] & \text{if } \alpha = 1 , \\ a_n^{-1}\mathbb{E}[X_1] & \text{if } \alpha > 1 , \end{cases}$$

Then

$$\{a_n^{-1} \sum_{n=1}^{[nt]} (X_k - b_n), t \geq 0\} \Rightarrow \Lambda$$

where Λ is an α-stable Lévy process with skewness $\beta = 2p - 1$ and the convergence is with respect to Skorohod's J_1 topology on $\mathcal{D}([0,1])$.

[Feller, 1971] [Resnick, 1987]

Multivariate regular variation

We use the following non standard but convenient definition.

A vector $(X_0, \ldots, X_d)$ is multivariate regularly varying if there exists a vector $(Y_0, \ldots, Y_d)$ with $Y_0 \not\equiv 0$ such that

$$\mathcal{L}\left(\frac{X_0}{x}, \frac{X_1}{x} \ldots, \frac{X_d}{x} \mid |X_0| > x\right) \to (Y_0, \ldots, Y_d) \,.$$

- This convergence implies that X_0 is multivariate regularly varying and Y_0 is a two sided Pareto random variable.
- If for some $j \in \{1, \ldots, d\}$, $Y_j \equiv 0$, then X_j is said to be extremally independent from X_0.

Regularly varying time series

A strictly stationary real valued times series $\{X_t, t \in \mathbb{Z}\}$ is said to be regularly varying if all its finite dimensional distributions are regularly varying, i.e. for each $n \geq m \in \mathbb{Z}$, the vector $(X_m, \ldots, X_n)$ is regularly varying.

A sequence of i.i.d. random variables with regularly marginal distribution is regularly varying.

The tail process

It is convenient to characterize the extremal behaviour of the time series by its tail process

$$\{Y_t, t \in \mathbb{Z}\} = \mathrm{fidi} - \lim_{x\to\infty} \{x^{-1} X_t, t \in \mathbb{Z} \mid |X_0| > x\} \, .$$

[Basrak and Segers, 2009]

- The only inconvenient is that it gives an arbitrary role to the time origin 0.
- Y_0 has a two sided Pareto distribution with skewness p:

$$\mathbb{P}(Y_0 > x) = p\mathbb{P}(|Y_0| > x) = px^{-\alpha} \, .$$

Examples

- Linear processes with heavy tailed innovation;
- Stochastic recurrence equations (ARCH, GARCH);
- Stochastic volatility models with heavy tailed innovation or heavy tailed volatility;
- MCMC algorithms with heavy tailed proposal distribution.

Clusters

- For iid observations, extremes appear in isolation.
- For dependent time series, extremes may appear in cluster.

Precise definition of a cluster? related to the blocking method: the original series is split into $[n/r_n]$ blocks of size r_n.

The extremal index

The extremal index is the inverse mean cluster size. Let u_n and r_n be sequences such that $r_n \to \infty$, $u_n \to \infty$ and $r_n \mathbb{P}(X_1 > u_n) \to 0$.

$$\begin{aligned} \frac{1}{\theta} &= \lim_{n\to\infty} \mathbb{E}\left[\sum_{i=0}^{r_n} \mathbb{1}_{\{X_i > u_n\}} \mid X_0 > u_n \right] \\ &= \lim_{n\to\infty} \sum_{i=0}^{r_n} \mathbb{P}(X_i > u_n \mid X_0 > u_n) \,. \end{aligned}$$

This limit does not always exist and may be infinite. If the extremal index exists then $\theta \in [0, 1]$. For i.i.d. observations or extremally independent times series $\theta = 1$.

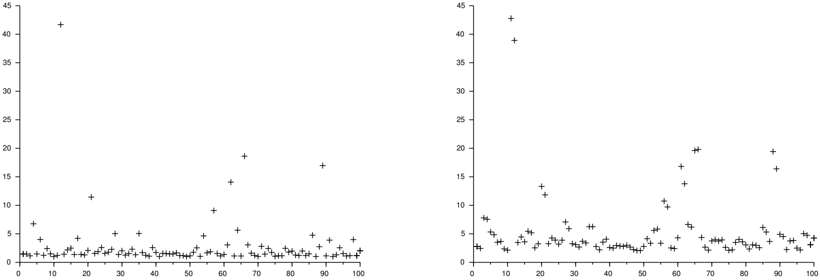

Figure : Pareto 3/2 distribution: iid and MA(1) with $\rho = .9$

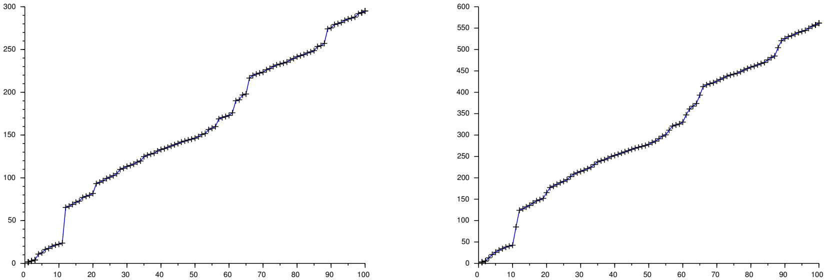

Figure : Partial sum process; Pareto 3 distribution: iid and MA(1) with $\rho = .9$

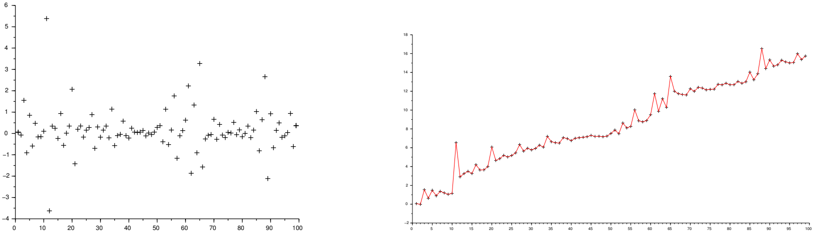

Figure : Partial sum process; Pareto 3/2 distribution: MA(1) with $\rho = -.7$

II. Convergence to a stable process

We want to investigate the convergence of the partial sum process

$$S_n(t) = a_n^{-1} \sum_{k=1}^{[nt]} (X_k - b_n)$$

to a Lévy stable process, both fidi convergence and functional convergence.

Very old story: [Davis, 1983] $\rightarrow$ [Basrak and Krizmanić, 2014]

The main ingredients are a weak dependence condition and an anticlustering condition.

Temporal weak dependence condition

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A convenient assumption is that the series is absolutely regular (β-mixing) with fast rate. This allows to replace the original sequence $X_1, \ldots, X_n$ by a sample $X_1^*, \ldots, X_n^*$ which consists of n/r_n independent blocks of length $r_n = o(n)$ and each block has the same distribution as the corresponding original block.

Under geometric decay of the coefficients, the choice block size r_n is largely irrelevant (logarithmic).

Ex: functions of geometrically ergodic irreducible Markov chains.

Temporal weak dependence condition

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Under geometric decay of the coefficients, the choice block size r_n is largely irrelevant (logarithmic).

Ex: functions of geometrically ergodic irreducible Markov chains.

- Linear processes can be studied by ad-hoc techniques (truncation).
- Certain forms of long memory processes can also be dealt with.

The anticlustering condition

Let u_n and r_n be sequences such that $r_n \to \infty$, $u_n \to \infty$ and $r_n \mathbb{P}(X_1 > u_n) \to 0$.

$$\lim_{m\to\infty} \limsup_{n\to\infty} \mathbb{P}\left(\max_{m \le i \le r_n} X_i > u_n \mid X_0 > u_n \right) = 0 \qquad (AC)$$

Obviously holds for m-dependent sequences; hard to check in general.

Condition AC implies that the extremal index exists and is positive. The mean cluster size is finite.

[Smith, 1992], [Davis and Hsing, 1995].

The point process of exceedences

The point process of exceedence is defined by

$$N_n = \sum_{k=1}^{n} \delta_{\frac{k}{n} \frac{X_k}{a_n}} \ .$$

For $t \in [0,1]$ and $u > 0$,

$$N_n([0,t] \times [u,\infty]) = \#\{k \leq n \mid k/n \leq t \ , \ X_k > a_n u\} \ ,$$

$$\mathbb{P}\left(N_n([0,t] \times [u,\infty]) = 0\right) = \mathbb{P}\left(\max_{1 \leq k \leq nt} X_k \leq a_n u\right) \ .$$

Convergence of the point process of exceedences

Under the regular variation condition and conditions *AC* and *WD*, $N_n \Rightarrow N$ with

$$N = \sum_{i=1}^{\infty} \sum_{j \in \mathbb{Z}} \delta_{T_i, P_i Q_{i,j}}$$

- $N_0 = \sum_{\imath=1}^{\infty} \delta_{T_i, P_i}$ is a PRM on $[0,1] \times ([-\infty, 0) \cup (0, \infty])$ with mean measure $\mathrm{d}t\{(1-p)\mathbb{1}_{\{x<0\}} + p\mathbb{1}_{\{x>0\}}\}\alpha|x|^{-\alpha-1}\mathrm{d}x$.
- the sequences $\{Q_{i,j}, j \in \mathbb{Z}\}$ are i.i.d. and independent of N_0; their common distribution is related to that of the tail process conditioned to have a record at time 0..

[Davis and Hsing, 1995], [Basrak et al., 2012], [Basrak and Tafro, 2015].

The summation functional

Let $\mathcal{M}_p$ be the set of finite point measures on $[0,1] \times \mathbb{E}^0 = [0,1] \times \bar{\mathbb{R}} \setminus \{0\}$. The summation functional

$$\begin{aligned} \mathcal{M}_p &\to \mathbb{R} \\ N &\mapsto \int_0^1 \int_{\mathbb{E}^0} x N(\mathrm{d}t\mathrm{d}x) \end{aligned}$$

is continuous[1] on $\mathcal{M}_p$ endowed with the topology of vague convergence.

[1] nearly

For simplicity of notation, we assume that the distribution of X_1 is symmetric, so we forget the centering. Fix $\epsilon > 0$.

$$S_n(t) = a_n^{-1} \sum_{k=1}^{[nt]} X_k \mathbb{1}_{\{|X_k| > \epsilon a_n\}} + a_n^{-1} \sum_{k=1}^{[nt]} X_k \mathbb{1}_{\{|X_k| \le \epsilon a_n\}} = S_{1,n} + S_{2,n} \, .$$

Asymptotic negligibility of small jumps: for every $\eta > 0$ and $t \in (0,1]$,

$$\lim_{\epsilon \to 0} \limsup_{n \to \infty} \mathbb{P}(S_{2,n}(t) > \eta) = 0 \, . \qquad (AN)$$

True for $\alpha < 1$; must be assumed for $\alpha \in (1,2)$.

By a continuous mapping argument, for fixed t,

$$S_{2,n}(t) = a_n^{-1} \sum_{k=1}^{[nt]} X_k \mathbb{1}_{\{|X_k| > \epsilon a_n\}} = \int_0^t \int_{\epsilon < |x| \leq \infty} x N_n(\mathrm{d}t \mathrm{d}x)$$

$$\Rightarrow_{n\to\infty} \int_0^t \int_{\epsilon < |x| \leq \infty} x N(\mathrm{d}t \mathrm{d}x) \Rightarrow_{\epsilon \to 0} \Lambda(t) \, .$$

where Λ is an α-stable Lévy process.

The asymptotic negligibility of the small jumps ensures that $S_n(t) \Rightarrow \Lambda(t)$.

Fidi convergence is proved similarly.

In the case $\alpha < 1$, the points of the limit of the PPE are summable and the limiting process Λ has an explicit form:

$$\Lambda(t) = \sum_{i:T_i \le t} \sum_{k \in \mathbb{Z}} P_i Q_{i,j} \ .$$

The order of the points is lost.

What more do you want?

Tightness, functional convergence, convergence of functionals. Do the following convergence hold?

$$\sup_{0\le t\le 1} S_n(t) \Rightarrow \sup_{0\le t\le 1} \Lambda(t)\,,$$

$$\int_0^1 S_n(t)\mathrm{d}t \Rightarrow \int_0^1 \Lambda(t)\mathrm{d}t\,.$$

We will only discuss convergence of the supremum.

Example

Consider the MA(1) process $X_t = Z_t + \vartheta Z_{t-1}$ where $\{Z_t, t \in \mathbb{Z}\}$ is an i.i.d. positive regularly varying sequence with tail index $\alpha \in (0, 1)$. Let a_n be the $1 - 1/n$ quantile of Z_1.

$$S_n(t) = a_n^{-1} \vartheta Z_0 + a_n^{-1}(1 + \vartheta) \sum_{k=1}^{[nt]-1} Z_k + a_n^{-1} Z_{[nt]} \, .$$

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$$S_n(t) = a_n^{-1}\vartheta Z_0 + a_n^{-1}(1+\vartheta) \sum_{k=1}^{[nt]-1} Z_k + a_n^{-1} Z_{[nt]} \; .$$

If $\vartheta > 0$ then

$$\sup_{0 \le t \le 1} S_n(t) \sim (1+\vartheta) \sup_{0 \le t \le 1} a_n^{-1} \sum_{k=1}^{[nt]} Z_k \; .$$

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If $\vartheta > 0$ then

$$\sup_{0\le t\le 1} S_n(t) \sim (1+\vartheta) \sup_{0\le t\le 1} a_n^{-1} \sum_{k=1}^{[nt]} Z_k \ .$$

If $\vartheta < 0$ then this is not true. For $\vartheta = -1$,

$$S_n(t) = a_n^{-1}\vartheta Z_0 + a_n^{-1} Z_{[nt]} \Rightarrow 0 \ ,$$
$$\sup_{0\le t\le 1} S_n(t) \sim a_n{-1} \max_{1\le k\le n} Z_k \to \Phi_\alpha$$

the Fréchet distribution.

The summation functional

We must now consider the summation functional as a functional onto $\mathcal{D}([0,1])$. Set $\mathbb{E}^0 = \bar{\mathbb{R}} \setminus \{0\}$.

$$\begin{aligned} &\mathcal{M}_p \to \mathcal{D}([0,1]) \\ &\quad N \to \left(t \to \int_0^t \int_{\mathbb{E}^0} x N(\mathrm{d}t\mathrm{d}x) \right) \ . \end{aligned}$$

We must endow $\mathcal{D}([0,1])$ with a topology.

III. Skorohod's topologies

The J_1 topology is finer than the M_1 topology which is finer than the M_2 topology.

The supremum functional is continuous wrt to the J_1, M_1 and M_2 topologies.

The J_1 topology can handle discontinuities but not unmatched discontinuities nor multiple jumps.

The M_1 topology can handle unmatched discontinuities and multiple jumps of the same sign.

The M_1 topology cannot handle multiple jumps of different signs.

The M_2 topology can handle some cases of multiple jumps of different signs.

[Whitt, 2002]

For convergence of integrals the S topology is sufficient. [Jakubowski, 1997]

Simple discontinuity

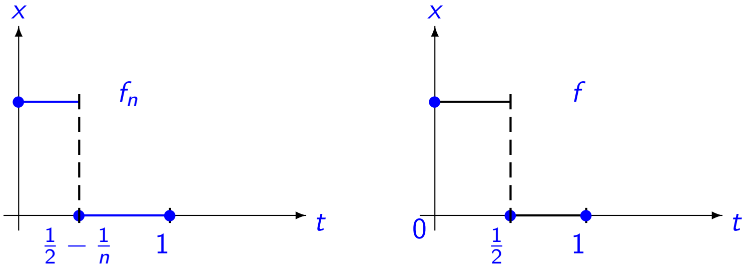


Figure : J_1 convergence

Unmatched jump at the limit

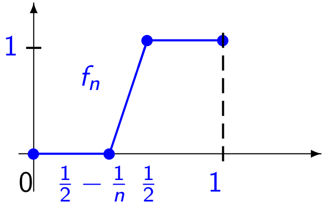
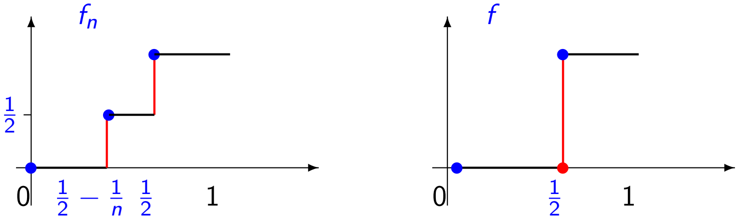


Figure : M_1 but not J_1 convergence.

Multiple jumps of the same sign



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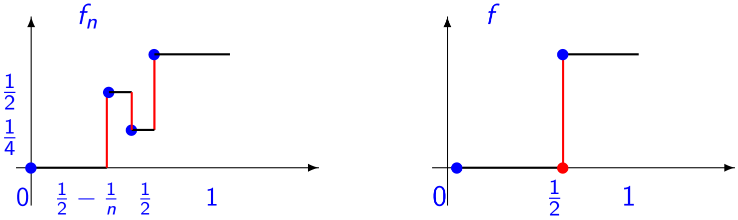

Figure : M_2 but not M_1 convergence.

Multiple jumps of different signs

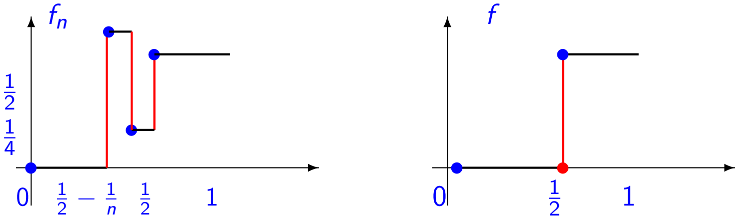

Figure : Not even M_2 convergence!

Convergence of the partial sum process

Let $\{Y_j\}$ be the tail process of the stationary regularly varying time series $\{X_j\}$.

Theorem

Assume that Conditions $AC(a_n, r_n)$ and WD hold. Assume the uniform asymptotic negligibility of small jumps if $\alpha \in [1, 2)$.

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Assume that Conditions $AC(a_n, r_n)$ and WD hold. Assume the uniform asymptotic negligibility of small jumps if $\alpha \in [1, 2)$.

- *If $Y_j = 0$ for all $j \neq 0$ (extremal independence), S_n converges to Λ in the J_1 topology.*

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Theorem

Assume that Conditions $AC(a_n, r_n)$ and WD hold. Assume the uniform asymptotic negligibility of small jumps if $\alpha \in [1, 2)$.

- *If $Y_j = 0$ for all $j \neq 0$ (extremal independence), S_n converges to Λ in the J_1 topology.*
- *If $Y_i Y_j \geq 0$ for all $i, j \in \mathbb{Z}$, then S_n converges to Λ in the M_1 topology.*

[Basrak et al., 2012] (M_1 convergence); [Louhichi and Rio, 2011] (M_1 convergence for associated sequences).

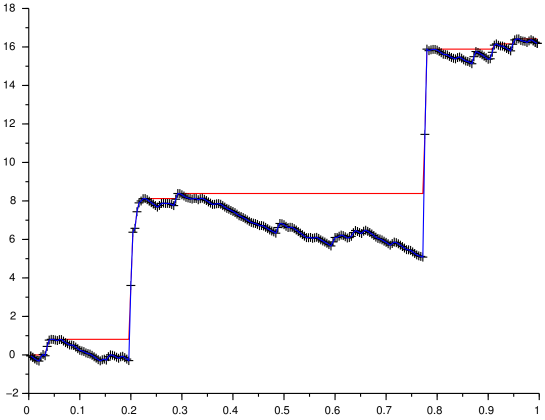

Figure : partial sum proces of MA(1) $\rho = .7$, $\alpha = 3/2$. M_1 convergence.

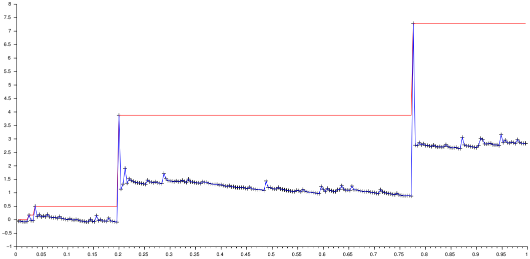

Figure : MA(1) process $\rho = -.7$, $\alpha = 3/2$

Beyond the point process of exceedences and M_2 convergence

The convergence of the point process of exceedences sums points within a cluster so that the order of the exceedences within on cluster is lost. The order determines the nature of the convergence.

- For the AR(1) process with positive coefficent, the exceedences occur in increasing order; M_1 convergence holds.
- For the AR(1) process with negative coefficent, the exceedences may occur in decreasing order; even M_2 convergence fails.

The clusters must be considered as ordered sequences and the order be kept. The point process of exceedences must be replaced by the point process of clusters.

Related to cluster functionals [Drees and Rootzén, 2010], [Mikosch and Wintenberger, 2015]

Under the anticlustering condition, $\lim_{|t|\to\infty} Y_t = 0$.

Let $\ell_0 = \{(u_n)_{n\in\mathbb{Z}}, \lim_{|n|\to\infty} u_n = 0\}$ and let $\tilde{\ell_0}$ be the quotient of ℓ_0 by the shift operator. $\tilde{\ell_0}$ is a CSMS. The theory of PP convergence of Daley and Vere-Jones can be applied.

For a sequence r_n of block size and $m_n = [n/r_n]$, define the cluster

$$C_{n,k} = \{a_n^{-1} X_i, (k-1)r_n + 1 \le i \le kr_n\} ,$$

add zeroes to the left and to the right and consider it as an element of $\tilde{\ell_0}$.

Define the point process of clusters $\tilde{N}$ on $[0,1] \times \tilde{\ell_0}$:

$$\tilde{N} = \sum_{k=1}^{m} \delta_{\frac{k}{n}, C_{n,k}}$$

Convergence of the point process of clusters

Under the regular variation condition and conditions *AC* and *WD*, $\tilde{N}_n \Rightarrow \tilde{N}$ with

$$\tilde{N} = \sum_{i=1}^{\infty} \delta_{T_i, \{P_i Q_{i,j}, j \in \mathbb{Z}\}}$$

where as previously

- $N_0 = \sum_{i=1}^{\infty} \delta_{T_i, P_i}$ is a PRM on $[0,1] \times (0,\infty]$ with mean measure $\mathrm{d}t\mathrm{d}(-x^{-\alpha})$;
- the sequences $\{Q_{i,j}, j \in \mathbb{Z}\}$ are i.i.d. and independent of N_0 and their distribution is that of the tail process conditioned to have a record at time 0.

The supremum of the partial sum process

For $\epsilon > 0$, consider the mappings f_ϵ and g_ϵ on $\tilde{\ell}_0$ by

$$g_\epsilon(\mathbf{x}) = \sum_j x_j \mathbb{1}_{\{|x_j|>\epsilon\}} , \quad f_\epsilon(\mathbf{x}) = \sup_k \sum_{j\le k} x_j \mathbb{1}_{\{|x_j|>\epsilon\}} .$$

We define the mapping

$$T_\epsilon : \mathcal{M}_p([0,1] \times \tilde{\ell}_0) \to \mathcal{D}([0,1])$$

$$\sum_{i\in\mathbb{N}} \delta_{t_i,\mathbf{x}_i} \mapsto \left(t \to \sup_{t_i \le t} \left\{ \sum_{t_j < t_i} g_\epsilon(\mathbf{x}_j) + f_\epsilon(\mathbf{x}_i) \right\} \right) .$$

For every $\epsilon > 0$, the mapping T_ϵ is continuous[2] with respect to the M_1 topology on $\mathcal{D}([0,1])$.

[2] nearly

The case $\alpha < 1$

In that case the points are summable and it is possible to let $\epsilon \to 0$ in the limit point process T_ϵ. Set $U_n(t) = \sup_{0 \le s \le t} S_n(s)$. Then

$$U_n \Rightarrow U \ , \ \text{wrt the } M_1 \text{ topology,}$$

$$U(t) = \sup_{i : t_i \le t} \left\{ \sum_{j<i} \sum_{k \in \mathbb{Z}} P_j Q_{j,k} + \sup_k \sum_{\ell \le k} P_i Q_{i,\ell} \right\} \ .$$

Questions:

We know that S_n converges fidi to a stable Lévy process Λ.

- Is U the supremum of Λ?
- Under what condition does S_n converge to Λ in the M_2 topology?

A (N)SC for M_2 convergence

If conditions WD and AC hold and

$$\sup_{k\in\mathbb{Z}} \sum_{\ell\leq k} Q_{1,\ell} \leq \left(\sum_{\ell\in\mathbb{Z}} Q_{1,\ell} \right)_+ , \qquad (C^*)$$

then $U = \Lambda^*$.

A (N)SC for M_2 convergence

If conditions WD and AC hold and

$$\sup_{k\in\mathbb{Z}} \sum_{\ell\leq k} Q_{1,\ell} \leq \left(\sum_{\ell\in\mathbb{Z}} Q_{1,\ell} \right)_+ , \qquad (C^*)$$

then $U = \Lambda^*$. If moreover

$$\left(\sum_{\ell\in\mathbb{Z}} Q_{1,\ell} \right)_- \leq \inf_{k\in\mathbb{Z}} \sum_{\ell\leq k} Q_{1,\ell} ,$$

then M_2 converence holds.

Causal moving averages

Consider a causal moving average with regularly varying i.i.d. innovation $\{Z_t, t \in \mathbb{Z}\}$:

$$X_t = \sum_{k=0}^{\infty} a_k Z_{t-k} \ ,$$

with $\sum_{k=0}^{\infty} |a_k| < \infty$ and $\sum_{k=0}^{\infty} a_k > 0$.

Then condition (C^*) holds iff for all $k \geq 0$,

$$0 \leq \sum_{k=0}^{k} a_k \leq \sum_{k=0}^{\infty} a_k \ .$$

Then the partial sum process converges wrt M_2 topology.

Conjectured by [Avram and Taqqu, 1992], proved by [Basrak and Krizmanić, 2014].

Records

Let $m = \sum_{i=1}^{\infty} \delta_{t_i, \mathbf{x}^i} \in \mathcal{M}_p([0,1] \times \tilde{\ell}_0)$. Define, for $i \geq 1$ and $j \in \mathbb{Z}$

$$M_{i,j} = \sup_{j' \leq j} x^i_{j'} \ , \quad M_i = \sup_{j \in \mathbb{Z}} x^i_j \ ,$$
$$M'_i = \sup_{i' < i} M_{i'} \ , \quad M'_{i,j} = M'_i \vee M_{i,j} \ .$$

The counting process of records is defined on $(0, \infty)$ by

$$R_m(a, b] = \sum_{i : a < t_i \leq b} \mathbb{1}_{\{X^i_j > M'_{i,j-1}\}} \ , 0 < a \leq b \ .$$

The application

$$\mathcal{M}_p([0,1] \times \tilde{\ell}_0) \to \mathcal{M}_p((0, \infty))$$
$$m \to R_m$$

is continuous at every m such that for all $i, i' \geq 1$ and $j, j' \in \mathbb{Z}$, $x_{i,j} \neq x_{i',j'}$ if $x_{i,j} > 0$.

Conjecture: the number of significant records is $O(\log(n))$.

Checking the anticlustering condition for Markov chains

For functions of irreducible Markov chains, the anticlustering condition is implied by geometric convergence and nothing less.

[Roberts et al., 2006], [Mikosch and Wintenberger, 2013], [Kulik et al., 2015].

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