A Test for Local White Noise Components (and the Absence of Aliasing) in Locally Stationary Wavelet Time Series

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(joint work with I. Eckley, Lancaster)

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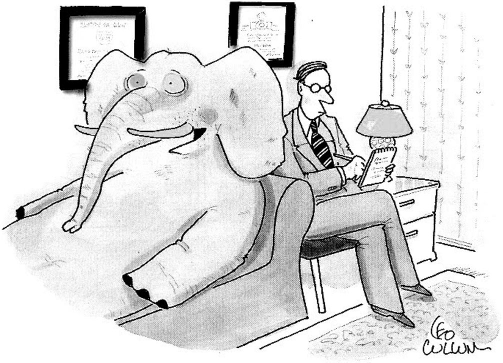
Outline

Elephant in the room: what sample rate is adequate?



"I'm right there in the room, and no one even acknowledges me."

Model Setup: LSW Processes (NvSK00)

Let X_t be time series of interest.

Suppose X_t modeled by a locally stationary wavelet process

with evolutionary wavelet spectrum $\{S_j(z)\}_{j=1}^{\infty}$, $z \in (0, 1)$.

That is:

$$X_t = \sum_{j=1}^{\infty} \sum_{k=-\infty}^{\infty} w_{j,k} \, \psi_{j,k-t} \, \xi_{j,k}, \quad (1)$$

where $\{\xi_{j,k}\}$ is set of uncorrelated random variables with mean zero and variance one and ...

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Model Setup 2 (NvSK2000, JRSSB)

$\{\psi_{j,k}\}$ are nondecimated discrete wavelets & amplitudes $w_{j,k}$.

Process 'controlled' by time-scale spectrum $\{S_j(z)\}_{j=1}^{\infty}$, where $z \in (0,1)$ is rescaled time (i.e. $z = t/T$).

Have evolutionary wavelet spectrum (EWS): $S_j(k/T) \approx w_{j,k}^2$.

Smoothness of $S_j(z)$ as fn of z, controls nonstationarity.

$S_j(z) = S_j \implies$ stationarity.

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Oscillatory vectors. E.g. Haar

$$\psi_1 = 2^{-1/2}(1, -1),$$

$$\psi_2 = 2^{-1}(1, 1, -1, -1),$$

$$\psi_3 = 2^{-3/2}(1, 1, 1, 1, -1, -1, -1, -1),$$

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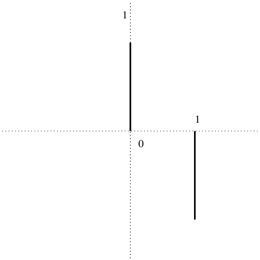
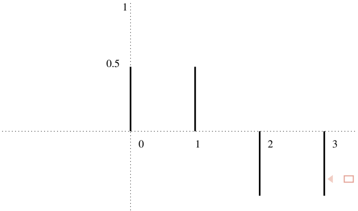
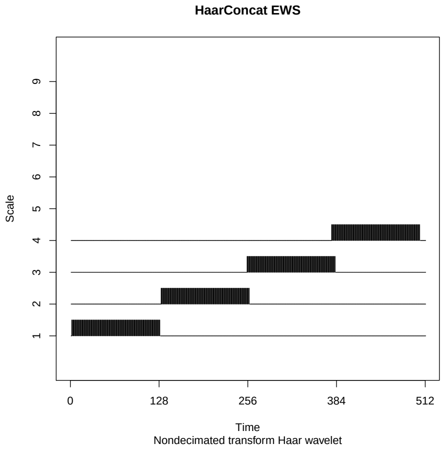
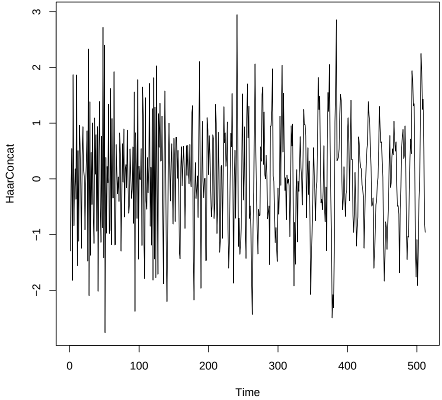




Example: EWS for concatenated Haar (NvSK00)



Example: Concatenated Haar realization (NvSK00)



Definitions (NvSK00)

Autocorrelation wavelet:

$$\Psi_j(\tau) = \sum_k \psi_{j,k}\psi_{j,k-\tau},$$

for $j \in \mathbb{N}, \tau \in \mathbb{Z}$.

see, e.g. Saito & Beylkin, 92, Berkner & Wells 98, NvSK00, E&N 05.

Inner product operator of $\{\Psi_j(\tau)\}$:

$$A_{j,\ell} = \sum_\tau \Psi_j(\tau)\Psi_\ell(\tau).$$

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NvSK00, Eckley and Nason (2005).

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LSW Process: usual estimation (NvSK00)

Given data, X_t, can compute *raw wavelet periodogram* $I_{j,k} = d_{j,k}^2$ where $d_{j,k}$ is discrete non-decimated wavelet transform of X_t:

$$d_{j,k} = \sum_{t=1}^{T} X_t \, \psi_{j,k-t}.$$

NvSK00 show that ($u \approx v \implies u = v + \mathcal{O}(T^{-1})$):

$$\mathbb{E}(I_{\ell,m}) = \mathbb{E}(d_{\ell,m}^2) \approx \sum_{j=1}^{\infty} A_{j,\ell} S_j(m/T),$$

where A is invertible: get estimator of S from I.

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White Noise

Suppose $X_t \sim \mathrm{WN}(0, \sigma^2)$ (uncorrelated).

Then $\mathbb{E}(d_{j,k}) = 0$ and

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as wavelets have norm one for all scales j.

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Aliasing

Given a time series $\{X_t\}_{t=1}^{T}$ for T some integer.

Integer samples $\implies$ highest (Nyquist) freq is π.

Sample $2\times$ slower, $2t$, then the highest freq halves to $\pi/2$, etc.

Aliasing occurs when $\exists$ power at freqs exceeding Nyquist freq

Wagon wheel effect

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Problem with Aliasing

- Hard to know when it occurs — does anybody test for it?
- Can cause problems for spectrum/covariance estimation.
- Higher freq peaks moved into lower bands.
- Lower freq spectral peaks distorted by higher ones.
- Hence, can have strong influence on scientific understanding, modelling.
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Many sources explain aliasing and some recommend:

low-pass filtering: obvious loss of info

increase sampling rate. Not always possible in, e.g., social sciences, meteorological, climate or finance.

Even when you can increase sample rate, storage waste risk

What about non-stationary series (second-order)?

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- Based on the bispectrum (third-order cumulant estimator).
- Initially, highly controversial, but later clarified by Hinich and Messier (IEEE Trans. Sig. Proc. 1995).
- Permits construction of aliasing hypothesis test for *stationary* series.
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Dyadic subsampling on LSW

We use subsampling to induce aliasing (not the only way)

LSW processes behave nicely under dyadic subsampling . . .

. . . because wavelets behave nicely under dyadic subsampling.

They become increasingly like white noise under subsampling

This can be directly and mathematically seen, as follows

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Impact of subsampling on LSW (Corollary 1)

Let $\{X_t\}$ be LSW with EWS $\{S_j(z)\}_{j=1}^{\infty}$.

If $Y_t = X_{2^r t}$ then Y_t admits the representation

$$Y_t = F_t + L_t,$$

where L_t is LSW with spectrum given in next slide and F_t is process with $\mathbb{E}F_t = 0$ and

$$\mathrm{cov}(F_t, F_{t+\tau}) \approx \delta_{0,\tau} \sum_{j=1}^{r} S_j(2^r t/T).$$

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EW Spectrum of subsampled LSW (Theorem 2)

Let $Y_t = X_{2^r t}$, our new result shows:

$$D^{(r)}_{\ell,m} := \mathbb{E}(d^2_{\ell,m}) \approx \sum_{j=1}^{r} S_j(2^r m/T) + \sum_{j=r+1}^{\infty} A_{j-r,\ell} S_j(2^r m/T).$$

ℓ is relative to the new series, Y_t.

E.g. for $r = 1$ get

$$D^{(1)}_{\ell,m} := \mathbb{E}(d^2_{\ell,m}) \approx S_1(2m/T) + \sum_{j=2}^{\infty} A_{j-1,\ell} S_j(2m/T).$$

Compare to NvSK00 original result:

$$\mathbb{E}(d^2_{\ell,m}) \approx \sum_{j=1}^{\infty} A_{j,\ell} S_j(m/T),$$

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(Subsamping) Aliasing Confounded with White Noise

Would be nice to use result to detect aliasing, BUT . . .

White Noise Gives Same Result

So cannot distinguish between white noise or aliasing

Can detect LACK of white noise/aliasing in this model.

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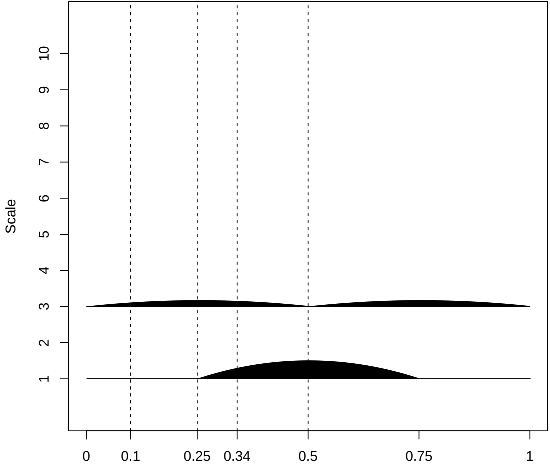
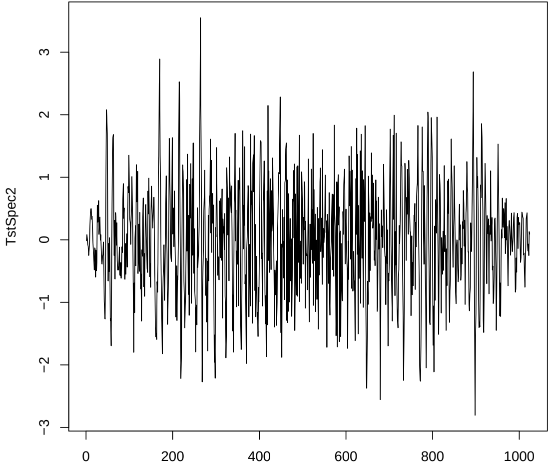
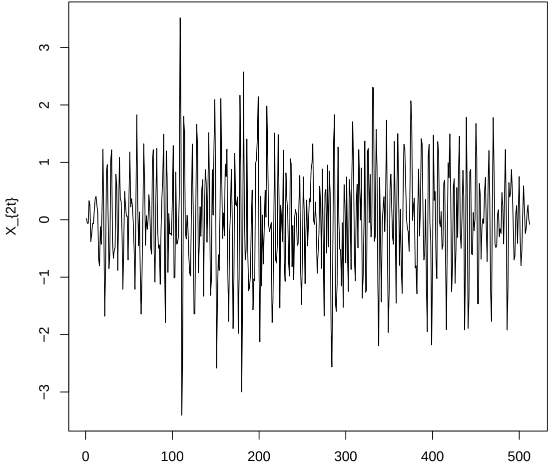
A test LSW spectrum



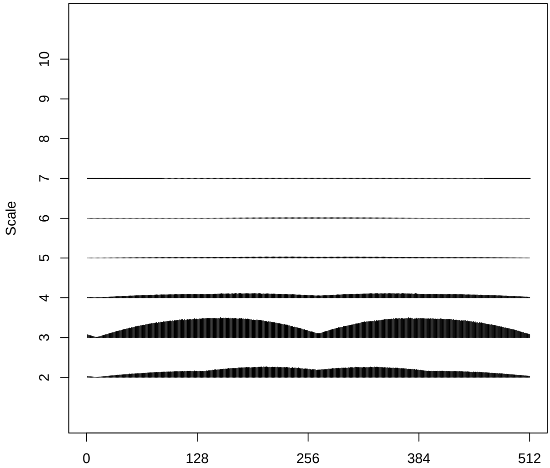
Realization from test spectrum



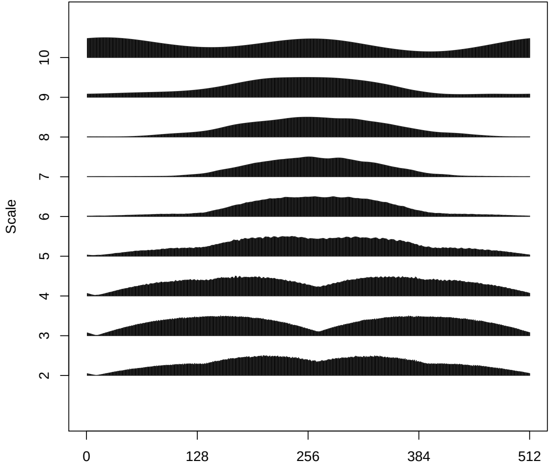
Dyadic subsampled $r = 1$ realization



Dyadic sampled $r = 1$ spectrum



Previous picture: levels enlarged



Detecting White Noise Components

Our LSW “thought experiment” possibilities: what is Y_t?

(a) Y_t is LSW from subsampled LSW X_t.

(b) $Y_t = U_t + \eta_t \epsilon_t$, where

$\{U_t\}$ is LSW with no white noise component $\equiv S_{j^*}^{(U)}(z_0) = 0$ for some $j^* = \{1, \ldots J\}$, $\eta(z)$ some slowly varying function, ϵ_t white noise. Means that Y_t could be any non-subsampled LSW.

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versus $H_A : S_j^{(Y)}(z_0) > 0 \;\; \forall \;\; j \in \{1, \dots, J\}$, is equivalent to

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The Test (outline): Step 1

Define J_{NC} to be the largest integer $< \log_2 \left(\frac{T/2+N_h-2}{N_h-1} \right)$.

Define the set $\mathrm{NC} = \mathrm{NC}(T, N_h) = \{j : 1 \le j \le J_{NC}\}$

NC are the *non-cone* scales.

Wavelet scales not adversely affected by edge effects.

Sanderson *et al.* (2010) show that EWS estimates converge in probability for $J < \log_2(T)$. They choose $J^* = 0.7 \log_2(T)$.

$J_{NC} < \log_2(T)$ for $T > 2(N_h - 2)$.

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The test at $z_0 = k_0/T$ continued.

Step 2: Compute $\mathbf{I}_z = (I_{1,k}, \ldots, I_{J_{NC},k})^T$ raw wavelet pgram, $k = 1, \ldots, T$, $z = k/T$.

Step 3: Apply simple running mean to $I_{j,k}$ over k to obtain estimate $\hat{\mathbf{I}}_{z_0}$ at z_0, using manual or CV bandwidth.

Let $\Lambda = \text{diag}(2, 4, \ldots, 2^{J_{NC}})$. $A_{J_{NC}}$ correction matrix (NvSK00)

Define $\hat{Q}_{z_0} = \Lambda A_{J_{NC}}^{-1} \hat{\mathbf{I}}_{z_0}$.

$\hat{Q}_{z_0}$ is approx. J_{NC}-dim Gaussian: can calculate $\hat{\mu}_Q$, $\hat{\Sigma}_Q$.

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Step 4: construct $100(1-\alpha)\%$ MV conf. region for $Q_{z_0} = \Lambda S_{z_0}$:

$$\Omega_{\alpha,z_0} = \{x : (x - \hat{\mu}_Q)^T \hat{\Sigma}_Q^{-1} (x - \hat{\mu}_Q) \leq c_{J_{NC},\alpha}\}.$$

where $c_{J_{NC},\alpha}$ is appropriate point of $\chi^2_{J_{NC}}$ distribution.

Step 5: $\forall j \in$ NC determine if Ω_{α,z_0} intersects axis j.

Step 6: If any do, then accept H_0, no aliasing nor white noise.

Equivalent to finding bound. box of MV ellipsoid (Barnes, 2014)

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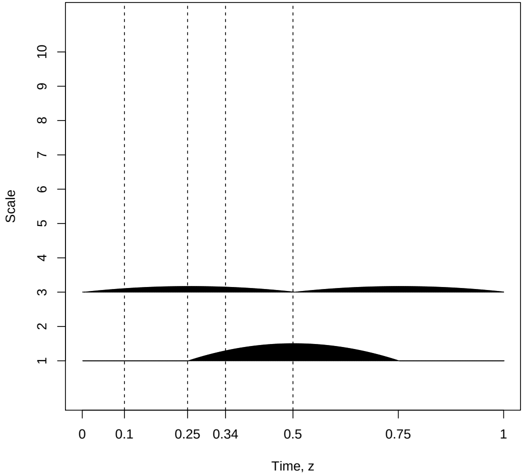
Simulation using earlier test spectrum

Suppose LSW, X_t, has evolutionary wavelet spectrum

$$S_j(z) = \begin{cases} \frac{3}{2}\max\left\{1 - 4(2z-1)^2, 0\right\} & j = 1, \\ \frac{1}{2}\left[\max\left\{1-(4z-1)^2, 0\right\} + \max\left\{1-(4z-3)^2, 0\right\}\right] & j = 3, \\ 0 & \text{otherwise}, \end{cases} \quad (3)$$

where $z \in (0, 1)$

Simulation: Test Spectrum



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Simulation: Empirical Size/Power

Table : Empirical size/power (%) of the test over 1000 realizations from the test EWS.

	z, D5 wavelet				z, D10 wavelet			
T	0.1	0.25	0.34	0.50	0.1	0.25	0.34	0.50
256	1	81	96	100	0	79	97	99
512	0	30	76	95	0	14	71	92
1024	0	14	79	99	0	11	82	99

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Real data: hi-res wind speed data

Data: hi-res wind speed data at 1Hz

Simple Trend removed by first differences

Evidence of nonstationarity (subjective and objectively)

Is 1Hz sample rate enough?

Real data: hi-res wind speed data

Data: hi-res wind speed data at 1Hz

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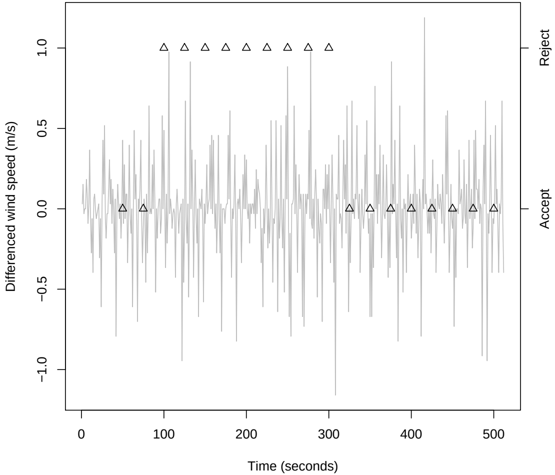
Is 1Hz sample rate enough?

Wind Speed Data + Results of Test



Wind Speed Discussion

Evidence for aliasing/white noise compt. between $t = 100$ & 300.

Working "guess": aliasing before about $t = 300$ (or could be white noise)

Idea: apply regular rolling local periodogram "after" $t = 300$

See what happens to frequency content after that ...

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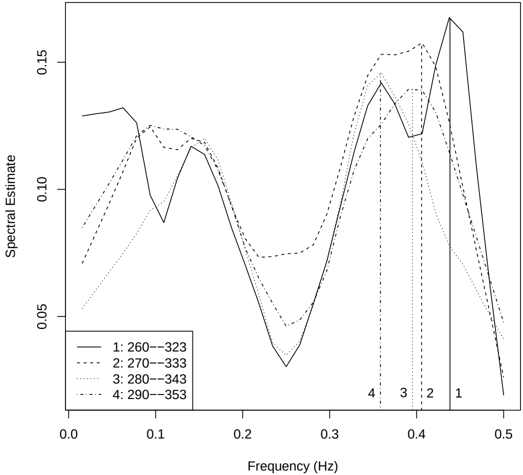
Wind Example: Rolling Spectral Estimates



Peak frequency decreases over time

Let t_c be centre of rolling window.

Let f_c be peak frequency in that time window.

t_c	292	302	312	322
f_c	0.438	0.406	0.395	0.358

Indicator of power redistributing to lower frequencies.

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Robust to mismatch of analysis and synthesis wavelet

X_t LSW process synthesized using wavelets $\psi^{(s)}$.

X_t analysed using $\psi^{(a)}$ wavelets to form periodogram.

Method works irrespective of choice of synthesis and analysis wavelets because it works through white noise components.

Subsampling produces white noise components (irrespective of $\psi^{(s)}$);

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IF your time series is LSW then

- You can identify periods of aliasing or white noise components.
- You can identify periods of *NOT ALIASED*.

Similar ideas do not work for Fourier because ...

...dyadic subsampling folds, but might not generate any white noise component.

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