CARMA (ALGEBRAIC COMBINATORICS, RESURGENCE, MOULDS AND APPLICATIONS)

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Henrik Bachmann: *A simultaneous q-analogue of finite and symmetrized multiple zeta values*
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Karen Yeats: *Connected chord diagrams, bridgeless maps, and perturbative quantum field theory*

Abstracts

Ines Aniceto
Beyond asymptotics in gauge and string theories

Abstract: In order to study the weakly coupled regime of some given quantum theory we often make use of perturbative expansions of the physical quantities of interest. But such expansions are often divergent, and defined only as asymptotic series. This divergence is connected to the existence of nonperturbative contributions, i.e. instanton effects not captured by a perturbative analysis. The theory of resurgence perfectly captures this connection and its consequences. Moreover, it allows us to construct a full non-perturbative solution from perturbative data.

In this talk, I will analyse essential role of resurgence theory in a variety of problems appearing in gauge and string theories. I will focus in how to effectively use the resurgent properties to go beyond the perturbative results and obtain (analytically and numerically) nonperturbative data. I will focus on examples of the cusp anomalous dimension in $\mathcal{N} = 4$ SYM, where resummation techniques allow us to probe the single parameter space, then turning to the related Bremsstrahung function to exemplify the issues of having multi-parameter dependence. The next example is a toy example of hydrodynamic theories mimicking the existence of black brane quasinormal modes appearing on the gravity dual description of a $\mathcal{N} = 4$ SYM plasma. I will finalise with the mention of novel results in the large-N resurgent dynamics of certain matrix models.

Henrik Bachmann
A simultaneous q-analogue of finite and symmetrized multiple zeta values

Abstract: In this talk we will introduce a new object, which can be seen as a q-analogue of finite multiple zeta values and symmetrized multiple zeta values (modulo $\zeta(2)$) simultaneously. For a fixed positive number n, this object will be an element in the cyclotomic field $\mathbf{Q}(q)$, with q being a primitive n-th root of unity. We will discuss its algebraic structure and prove a variant of the double shuffle relations for these objects. After giving results on their special values and various families of linear relations, we will explain their connections to finite and symmetrized multiple zeta values. Our results will lead to another interpretation of a conjecture of Kaneko and Zagier for finite multiple zeta values. This talk is based on a current joint work with Y. Takeyama and K. Tasaka.

Michael Borinsky
Flag decompositions of graphs and their Hopf algebraic structure

Abstract: The period integral of a primitive Feynman diagram is bounded by the so called Hepp bound. This bound can be formulated as a sum over flags of the graph. The notion of flag is a small generalization of the ear decomposition of a graph.

I will explain how these flags can be put into the framework of a Hopf algebra and how this structure can be exploited. It can be used to sum the bound over all Feynman diagrams of a specific theory. The generating function of this sum fulfills a nonlinear differential equation.

Using the same techniques the generating function of all flag decompositions of graphs with prescribed degree distributions can be expressed as the solution of a differential equation. The generating function of all ear decompositions fulfills a simpler linear differential equation.

David Broadhurst

Explosive combinatorics from Feynman integrals

Abstract: Very recently, David Roberts and I have discovered wonderful conditions imposed on Feynman integrals by Betti and de Rham homology. In decoding the corresponding matrices, we encounter asymptotic expansions of a refined nature. In making sense of these, we appear to have some refuge in resurgence.

Emily Burgunder

Generalised algebras and freeness

Abstract: For commutative algebras, there exists some tools to compute an algebraic basis of a given algebra. For associative algebras these tools have been adapted. When considering generalised algebras little or no tool exist. A first step is try to determine its freeness. We will present some tools to study the freeness of certain type of algebras and we will give some combinatorial example.

Ovidiu Costin

Resurgence analysis and effective resummation of divergent series

Abstract: Perturbative expansions arising in difficult mathematical or physical problems have, more often than not, zero radius of convergence. In Dyson's argument, convergence is obstructed by qualitatively different behaviors for different complex phases of the coupling constants. Mathematically this translates into nontrivial Stokes phenomena. I will describe a set of methods designed to overcome these difficulties and restore convergence.

Work in collaboration with Rodica D Costin and Gerald Dunne.

Gerald Dunne

Quantum Geometry and Resurgent Perturbative/Non-perturbative Relations

Abstract: Certain quantum spectral problems have the remarkable property that the formal perturbative series for the energy spectrum can be used to generate all other terms in the entire trans-series, in a completely constructive manner. I explain a geometric all-orders WKB approach to these perturbative/non-perturbative relations, which reveals surprising connections to number theory and modular forms.

Kurusch Ebrahimi–Fard

Monotone, free, and boolean cumulants from a shuffle algebra viewpoint

Abstract: The monotone, free, and boolean moment-cumulant relations are expressed in terms of non-crossing set partitions. In this talk we propose a shuffle algebra approach to those relations. Cumulants are considered as infinitesimal characters over a particular combinatorial Hopf algebra, which is neither commutative nor cocommutative. As a result the moment-cumulant relations can be encoded in terms of shuffle and half-shuffle exponentials, which in return permits to express monotone, free, and boolean cumulants in terms of each other using the pre-Lie Magnus expansion together with shuffle and half-shuffle logarithms. Our shuffle algebra approach is further illustrated by studying additive convolution in monotone, free and boolean probability. Based on joint work with F. Patras (CNRS).

Jean Ecalle

Taming the coloured multizetas

Abstract: 1. We shall briefly describe the ARI-GARI structure; recall its double origin in Analysis and mould theory; explain what makes it so well-suited to the study of multizetas; and review the most salient results it led to, beginning with the exchanger *adari*(*pal*$^\bullet$) of double symmetries ($\underline{al}/\underline{il}$) $\leftrightarrow$ ($\underline{al}/\underline{al}$), and culminating in the explicit decomposition of multizetas into a remarkable system of *irreducibles*, positioned exactly half-way between the two classical multizeta encodings, symmetral resp. symmetrel.

2. Although the coloured, esp. two-coloured, multizetas are in many ways more regular and better-behaved than the plain sort, their sheer numbers soon make them computationally intractable as the total weight $\sum s_i$ increases. But help is at hand: we shall show a conceptual way round this difficulty; make explicit its algebraic implementation; and sketch some of the consequences.

Michael E. Hoffman

Quasi-shuffle algebras

Abstract: In recent decades quantities like multiple zeta values, multiple polylogarithms, and multiple harmonic sums have often been encountered in quantum field theory. Quasi-shuffle algebras have been a useful tool in understanding them: the idea is that we have an algebraic object, the quasi-shuffle algebra, which has as homomorphic images the various numbers that appear in calculations. I will talk about the Hopf algebra, infinitesimal Hopf algebra, and other algebraic structures carried by a quasi-shuffle algebra, and how they can be put to use.

Carlos Mafra

Superstring scattering amplitudes and combinatorics

Abstract: The recent discovery of the pure spinor formalism for the superstring has led to important advances in the computation of scattering amplitudes. For instance, a general solution for tree-level amplitudes has been found using the fact that the amplitudes live in the cohomology of a certain operator (the BRST charge).

In this talk I will show some surprising relations between the above tree-level solution and some combinatoric structures such as planar binary trees.

More precisely, the cohomology analysis of the amplitude benefits from certain superfields that obey the symmetries of an alternal mould. These objects are constructed from planar binary trees dressed with "propagators" (objects whose labels are totally symmetric) and they are invertible when their numerators satisfy generalized Jacobi symmetries. I will present more examples like this, involving shuffle products, the rho-map (from Reutenauer's book) and Lie series of superfields etc.

If time allows, I will mention how some of the above concepts such as Lie series and alternal moulds have played a central role in the recursive solution (in the number of punctures) for the alpha' expansion of disk integrals using techniques from scattering amplitudes. These expansions give rise to multiple zeta values and it is surprising that they admit such a representation.

If for some miracle I still have time left after presenting the above, I will mention some generalizations of the above combinatoric structures to one-loop (ie genus-one surface). They lead to the definition of elliptic functions constructed out of certain functions that appear in the expansion of the Eisenstein-Kronecker series recently discussed by Brown and Levin.

Dominique Manchon

Free post-Lie algebras, the Hopf algebra of Lie group integrators and planar arborification

Abstract: The Hopf algebra of Lie group integrators has been introduced by H. Munthe-Kaas and W. Wright as a tool to handle Runge-Kutta numerical methods on homogeneous spaces. It is spanned by planar rooted forests, possibly decorated. We will describe a canonical surjective Hopf algebra morphism onto the shuffle Hopf algebra which deserves to be called planar arborification. The space of primitive elements is a free post-Lie algebra, which in turn will permit us to describe the corresponding co-arborification process.

Joint work with Charles Curry (NTNU Trondheim), Kurusch Ebrahimi-Fard (NTNU) and Hans Z. Munthe-Kaas (Univ. Bergen).

Jean-Christophe Novelli

Hopf Algebras on m-permutations and $(m+1)$-ary trees

Abstract: The m-Tamari lattice of F. Bergeron is an analogue of the clasical Tamari order defined on objects counted by Fuss-Catalan numbers, such as $(m+1)$-ary trees. On another hand, the Tamari order is related to the product in the Loday-Ronco Hopf algebra of planar binary trees. We introduce new combinatorial Hopf algebras based on $(m+1)$-ary trees, whose structure is described by the m-Tamari lattices.

In the same way as planar binary trees can be interpreted as sylvester classes of permutations, we obtain $(m+1)$-ary trees as sylvester classes of what we call m-permutations. These objects are no longer in bijection with decreasing $(m+1)$-ary trees, and a finer congruence, called metasylvester, allows us to build Hopf algebras based on these decreasing trees. At the opposite, a coarser congruence, called hyposylvester, leads to Hopf algebras of graded dimensions $(m+1)^{n-1}$, generalizing noncommutative symmetric functions and quasi-symmetric functions in a natural way.

Frédéric Patras

Pictures!

Abstract: The theory of pictures between posets is known to encode much of the combinatorics of symmetric group representations and related topics such as Young diagrams and tableaux. Many reasons, combinatorial (e.g. since semi-standard tableaux can be viewed as double quasi-posets) and topological (quasi-posets identify with finite topologies) lead to extend the theory to quasi-posets. Joint work with L. Foissy and C. Malvenuto.

Sylvie Paycha

Branched zeta functions and a refined universal property for trees

Abstract: Multizeta functions generalise the well-known ordinary zeta function and branched zeta functions are discrete sums attached to trees that generalise multizeta functions which correspond to ladder trees. We view branched zeta functions as discrete sums of pseudodifferential symbols and accordingly, we decorate the trees with pseudodifferential symbols. We then implement a multivariate regularisation procedure in replacing the symbols in the decoration by holomorphic families of symbols. Using the universal property of trees, we further build the corresponding regularised branched zeta functions and show that they *are multivariate meromorphic functions with linear poles*.

In order to renormalise branched zeta functions at poles, we need a good control of the poles. For this purpose, we use a *refined universal property for trees*, which involves the notion of locality, reminiscent of locality in quantum field theory. In our framework locality is encoded in a binary

symmetric relation, with which we equip sets that we call *L-sets*. We introduce the L-monoid of properly decorated forests, which serves as a model for what we call *partially operated L- monoids*. A refined universal property for partially operated L-monoids provides us with a good knowledge of the pole structure of the branched zeta functions. Branched zeta functions are then renormalised at poles using a multivariate minimal subtraction scheme, which generalises the (univariate) minimal subtraction scheme known to physicists. We show that the renormalised branched zeta values are rational.

This talk is based on joint work with Pierre Clavier, Li Guo and Bin Zhang

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Viviane Pons

Lattice and Hopf algebra of integer relations

Abstract: We present how many well known structures such as the weak order on permutations and the Malvenuto Reutenaurer Hopf algebra can be derived from very simple definitions on the integer relations level. We define both a lattice and a Hopf alegbra on integer relations which gives us in turn both a lattice and Hopf algebra on integer posets. We see that many combinatorial objects, such as permutations and binary trees, can be represented as integer posets which leads us to re-discover some well known algebraic structures on those objects as well as new ones.

David Sauzin

Analytic linearization of multi-dimensional dynamical systems through tree-expansions

Abstract: Ecalle's "arbomoulds" and "co-arbomoulds" provide a concise and elegant representation of the linearizations of non-resonant local analytic dynamical systems (discrete or with continuous time). The arbomould is a family of coefficients depending only on the spectrum of the linear part of the system (assumed to be diagonal and non-resonant); we show how Bruno's condition allows one to bound these coefficients efficiently. The co-arbomould is a family of differential operators depending only on the nonlinear terms, which is related to a pre-Lie structure defined for the infinitesimal characters of the Connes-Kreimer Hopf algebra. Building on the strategy indicated in Ecalle's 1992 article, we show, by a simple majorant series argument, how the previous bounds allow one to prove the convergence of the linearizing transformation in as large a domain as the one recently obtained by Giorgilli-Marmi (in the continous-time case) and Giorgilli-Locatelli-Sansottera (in the discrete case).

Joint work with Frédéric Fauvet and Frédéric Menous.

Karen Yeats

Connected chord diagrams, bridgeless maps, and perturbative quantum field theory

Abstract: Rooted connected chord diagrams can be used to index certain expansions in quantum field theory. There is also a nice bijection between rooted connected chord diagrams and bridgeless maps. I will discuss each of these things as well as how the second sheds light on the first. (Based on work with Nicolas Marie, Markus Hihn, Julien Courtiel, and Noam Zeilberger.)