# Algebraic Geometry and Complex Geometry
# 23 - 27 juanuary 2017
# **Mini-Courses**

Alena Pirutka (New York University)

**Title:** Rationalité stable

**Abstract:** Soit X une variété algébrique complexe, projective et lisse. On dispose de plusieurs notions pour déterminer si X est "proche" à un espace projectif : la variété X est rationnelle si un ouvert de X est isomorphe à un ouvert d'un espace projectif, on dit que X est stablement rationnelle si cette propriété vaut en remplaçant X par un produit avec un espace projectif, enfin X est unirationnelle si X est rationnellement dominée par un espace projectif. Dans le problème classique de Lüroth on s'intéresse à trouver des exemples de variétés unirationnelles non rationnelles. Ce problème fut ouvert jusqu'à les années 1970, où trois séries d'exemples ont étés construites : les solides cubiques (Clemens et Griffiths), certaines solides quartiques (Iskovskikh et Manin), ainsi qu'une fibration en coniques (Artin et Mumford). Dans ce dernier exemple il s'agit d'une variété qui n'est même pas stablement rationnelle. Pour d'autres exemples la question de la stabilité rationnelle était ouverte.

Dans un travail récent C. Voisin montre qu'un solide double ramifié le long d'une quartique très générale n'est pas stablement rationnel. Inspirés par son travail, on montre que "beaucoup" de solides quartiques ne sont pas stablement rationnelles (travail en commun avec J.-L. Colliot-Thélène). B. Totaro a ensuite établi qu'une hypersurface très générale de degré d n'est pas stablement rationnelle, si d/2 est au moins le plus petit entier supérieur à (n+2)/3. Les mêmes méthodes ont permis d'établir que la rationalité n'est pas stable par déformation (travail en commun avec B. Hassett et Y. Tschinkel).

Dans ce mini-cours, on va présenter des méthodes pour obtenir les résultats ci-dessus : l'étude des propriétés universelles du groupe de Chow des zéro-cycles, la décomposition diagonale, ainsi que des méthodes de spécialisation.

References:

- https://webusers.imj-prg.fr/~claire.voisin/Articlesweb/jdgvoisin.pdf;
- http://math1.unice.fr/~beauvill/conf/Cime.pdf;
- http://cims.nyu.edu/~pirutka/survey.pdf;

Jérémy Blanc (University of Basel)

**Title:** Dynamical degrees of birational transformations of surfaces

**Abstract:** The dynamical degree $\lambda(f)$ of a birational transformation $f$ of a surface measures the exponential growth of the formula that define the iterates of $f$. It allows to study the complexity of the dynamic of $f$. For instance, over the field of complex number, the number $\log(\lambda(f))$ gives a upper bound for the topological entropy. The number $\lambda(f)$ is an algebraic integer of a special kind: a Pisot or Salem number. Its value gives informations on the transformation, for instance on the fact that it is conjugate or not to an automorphism of a projective surface. More generally, I will try to explain in this mini-course how a precise information on the dynamical degree allows to understand the geometry of the transformation, for instance its conjugacy class or its centraliser in the group of all birational transformations.

References:


- J. Diller and C. Favre, Dynamics of bimeromorphic maps of surfaces, Amer. J. Math. 123 (2001), no. 6, 1135–1169
- J. Blanc and S. Cantat, Dynamical degrees of birational transformations of projective surfaces. J. Amer. Math. Soc. 29 (2016), no. 2, 415–471.



Mark Gross (University of Cambridge)


**Title:** Log Calabi-Yau varieties, degenerations of Calabi-Yau varieties, and mirror symmetry


**Abstract:** I will try to provide some context and background for recent breakthroughs by Siebert and myself involving constructions of mirrors to various types of varieties. In particular, I will give a gentle introduction to logarithmic Gromov-Witten theory and explain how log GW invariants can be used to generalize constructions of Gross-Siebert and Gross-Hacking-Keel to give a very general mirror symmetry construction.


References: Suggested background reading is our survey/announcement paper https://arxiv.org/abs/1609.00624, and background on log geometry that should be useful for the talks can be found in Chapter 3 of my book, ”Tropical geometry and mirror symmetry”.


Radu Laza (Stony Brook University)


**Title:** Birational geometry of moduli spaces


**Abstract:** Overview: The ideal situation in moduli theory is to have a geometric/modular compactification for a given moduli space, and to have a good understanding of its structure so that one can compute various invariants (e.g. Betti numbers). Unfortunately, this is rarely the case (e.g. for varieties of general type, there is a geometric compactification, the so called KSBA compactification, but we have little understanding of the boundary objects, and even less is known about the global structure of the KSBA compactification). In recent years, starting with the VGIT theory of Thaddeus and Dolgachev-Hu, a different perspective appeared: It might be better to study the birational geometry of the moduli space itself. Namely, in certain cases, there might be two or more natural compactifications for a moduli space (one might be of interest as a geometric compactification, while another one might have a simple description). Then the interest is to understand (explicitly) the birational geometry relating these various models. More precisely, the goal is to decompose the birational maps relating various models of a moduli space into simple birational maps (simple flips and divisorial contractions), and then compute the invariants of interest via “wall-crossing” formulas.

Plan of the lectures: I will start by reviewing the basic story of moduli spaces and methods of constructing them (see [K] and [L2]). I will then briefly discuss the VGIT theory (see [L2]), and the variation of log canonical models for the moduli space of curves (aka Hassett-Keel program, see [HH]). I will then focus (probably half of my lecture time) on the emerging story of the variation models for moduli spaces of polarized K3 surfaces (aka Hassett-Keel-Looijenga program, see [LOG1, LOG2]).

References:

General Moduli Theory:


- [K] Kollár "Moduli of varieties of general type", Hanbook of moduli, vol II
- [L1] Laza "GIT and moduli with a twist", Handbook of moduli, vol II
- [L2] Laza "Perspectives on the construction and compactification of moduli spaces", in Compactifying moduii spaces, (Birkhauser, 2016)


Hassett-Keel Program:


- [HH] Hassett-Hyeon "Log canonical models for the moduli space of curves: the first divisorial contraction". Trans. Amer. Math. Soc. 361 (2009)
- [FS] Fedorchuck-Smyth "Alternate compactifications of moduli spaces of curves", Handbook of Moduli, vol I


Hassett-Keel-Looijenga Program:


- [Lo] Looijenga "Compactifications defined by arrangements II", Duke Math. J. 119 (2003)
- [LOG1] Laza-O'Grady "Birational geometry of the moduli space of quartic K3 surfaces", arXiv:1607.01324 (2016)
- [LOG2] Laza-O'Grady "GIT versus Baily-Borel compactification for K3's which are quartic surfaces or double covers of quadrics", arXiv:1612.07432 (2016)



Johannes Nicaise (KU Leuven)


**Title:** The non-archimedean SYZ fibration


**Abstract:** The SYZ conjecture gives a geometric description of the relation between mirror pairs of Calabi-Yau varieties. It was a fundamental insight of Kontsevich and Soibelman that the structures predicted by the SYZ conjecture can be found in the world of non-archimedean geometry (Berkovich spaces). I will explain some of the main ideas, as well as the connections with the minimal model program in birational geometry. If time permits, I will also discuss how these results have led to a proof of Veys's conjecture on poles of maximal order of p-adic zeta functions. These talks are based on joint work with Mircea Mustata and Chenyang Xu.


References:


- "Berkovich skeleta and birational geometry" in: M. Baker and S. Payne (eds.), Nonarchimedean and Tropical Geometry, Simons Symposia, pages 179-200 (2016), arXiv:1409.5229