VORTICITY, ROTATION AND SYMMETRY (IV) - COMPLEX FLUIDS AND THE ISSUE OF REGULARITY

CIRM, LUMINY, 8-12 MAY, 2017

Speakers:

Helmut Abels
Sharp Interface Limit for a Stokes/Allen-Cahn System

Abstract: We consider the sharp interface limit of a coupled Stokes/Allen-Cahn system, when a parameter $\varepsilon > 0$ that is proportional to the thickness of the diffuse interface tends to zero, in a two dimensional bounded domain. For sufficiently small times we prove convergence of the solutions of the Stokes/Allen-Cahn system to solutions of a sharp interface model, where the interface evolution is given by the mean curvature equation with an additional convection term coupled to a two-phase Stokes system with an additional contribution to the stress tensor, which describes the capillary stress. In the talk we will first discuss the corresponding result for the Allen-Cahn equation and its proof. To this end a suitable approximation of the solution of the Allen-Cahn system is constructed with the aid of finitely many terms from formally matched asymptotic calculations. Then a spectral estime for the linearized Allen-Cahn operator is used to estimate the difference of exact and approximate solution. Afterwards we will explain how this strategy can be adapted to the case of the coupled Stokes/Allen-Cahn system. In particular a suitable refinement of the estimates for the linearized operator and a careful treatment of the coupling terms is needed.

References

[1] H. Abels and Y. Liu. Sharp interface limit for a Stokes/Allen-Cahn system. *Preprint arXiv:1611.04422*, 2016.
[2] X. Chen. Spectrum for the Allen-Cahn, Cahn-Hilliard, and phase-field equations for generic interfaces. *Comm. Partial Differential Equations*, 19(7-8):1371–1395, 1994.
[3] X. Chen, D. Hilhorst, and E. Logak. Mass conserving Allen-Cahn equation and volume preserving mean curvature flow. *Interfaces Free Bound.*, 12(4):527–549, 2010.
[4] P. De Mottoni and M. Schatzman. Geometrical evolution of devoloped interfaces. *Trans. Amer. Math. Soc.*, 347(5):1533–1589, 1995.

Chérif Amrouche

New Regularity Results for Elliptic Problems

Abstract: We are interested here in questions concerning the regularity of solutions of elliptic problems

$$\operatorname{div}(A \operatorname{grad} u) = f \quad \text{in} \quad \Omega$$

with Dirichlet or Neumann boundary condition and where A is a matrix or a function. As particular case, we study the case of the Laplacian ($A = -\Delta$) in Lipschitz domains or in domains more regular. We will also revisit the work of Lions-Magenes [3], concerning the method of transposition, which permits to get so-called very weak solutions, when the data are less regular. Thanks to the interpolation theory, this will allow us to extend the classes of solutions and then to obtain new results of regularity.

References

[1] C. Amrouche, M. Moussaoui, H.H. Nguyen. Laplace equation with non regular data in bounded domains. Work in Progress.

[2] D. Jerison, C.E. Kenig. The Inhomogeneous Dirichlet Problem in Lipschitz Domains. *J. Funct. Anal.*, **130**, (1995), 161–219.

[3] J.L. Lions, E. Magenes. *Problèmes aux limites non-homogènes et applications.* Vol. 1, Dunod, Paris, (1969).

[4] J. Necas. *Direct methods in the theory of elliptic equations.* Springer Monographs in Mathematics. Springer, Heidelberg, (2012).

Corentin Audiard

Global solutions of the Euler-Korteweg equations

Abstract: The Euler-Korteweg equations are a modification of the usual compressible Euler equations that take into account capillary effects. Without vacuum they read

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0, \\ \partial_t u + u \cdot \nabla u + \nabla g(\rho) = \nabla\left(K(\rho)\Delta\rho + \frac{1}{2}K'(\rho)|\nabla\rho|^2\right), \end{cases} \quad (x,t) \in \mathbb{R}^d \times \mathbb{R}^+.$$

The third order derivative of ρ is a dispersive term, and the equation can be reformulated in appropriate variables as a quasilinear Schrödinger equation. A general expectation for dispersive equations is that in dimension large enough and for initial data smooth and small enough, the solution should be global, and converge to a solution of a linearized system. We prove here that this principle is true up to $d = 3$.

This result is sharp in the sense that in dimension 2 an opposite dynamic appears : there exists smooth traveling waves -a purely nonlinear phenomenon- arbitrarily small.

The existence of global solutions is based on a bootstrap argument involving various dispersive estimates in the spirit of [2,3], and is a work in collaboration with B. Haspot (Univ. Paris Dauphine). The existence of traveling waves relies on a variational argument similar to the one developed in the framework of the Gross-Pitaevskii equation [1].

References

[1] C. Audiard, B. Haspot. Global well-posedness of the Euler-Korteweg system for small irrotational data. *Comm. Math. Phys.*, to appear 2017, doi 10.1007/s00220-017-2843-8

[2] F. Béthuel, P. Gravejat, J.C. Saut. Travelling waves for the Gross-Pitaevskii equation. II. *Comm. Math. Phys.* 285 (2009), no. 2, 567?651.

[3] P. Germain, N. Masmoudi, J. Shatah. Global Solutions for the Gravity Water-Waves Equation in Dimension 3 *Ann. of Math.* (2) 175 (2012), no. 2, 691?754.

[4] S. Gustafson, K. Nakanishi, T.P. Tsai. Scattering Theory for the Gross-Pitaevskii equation in three Dimensions. *Comm. Contemp. Math.*, 11 no. 4 (2009) 657-707.

Didier Bresch

Mathematical results around Euler-Korteweg and Navier-Stokes-Korteweg systems

Abstract: In this talk, we revisit relative entropy estimates for the Euler-Korteweg system that has been recently derived in [2] and [3]. We generalize to the Navier-Stokes-Korteweg system with density dependent viscosities (satisfying the BD relation) the relative entropy proposed in [1] for a linear density dependent shear viscosity and a zero bulk viscosity.

This helps to provide existence of dissipative solutions for Euler-Korteweg systems as the limit of global weak solution of the Navier-Stokes-Korteweg systems. This is a joint work with M. Gisclon and I. Lacroix-Violet.

References

[1] D. Bresch, P Noble, J.-P. Vila. Relative entropy for compressible Navier-Stokes equations with density dependent viscosities and various applications. *To appear in ESAIM Proc (2017).*

[2] D. Donatelli, E. Feireisl, P. Marcati. Well/ill posedness for the Euler-Korteweg-Poisson system and related problems. *Comm. Partial Differential Equations, 40 1314-1335, (2015).*

[3] J. Giesselmann, C. Lattanzio, A.-E. Tzavaras. Relative energy for the Korteweg theory and related Hamiltonian flows in gas dynamics. *Arch. Rational Mech. Analysis, 223, 1427-1484, (2017).*

Dongho Chae

Liouville type theorems for the Euler and related equations

Abstract: In this talk we discuss the discretely self-smilar blow-up scenarios for the 3D incompressible Euler equations in $\mathbb{R}^3$. This corresponds to time periodic solution of the time-dependent self-similar Euler equations, the solution of which is the profile of the discretely self-similar blow-up. In this talk we focus on the energy conserving scale, which was not considered in the previous results. For such energy conserving case we prove Liouville type theorem under the extra assumption of type I condition. Next, we would like to discuss the Liouville type properties of the Beltrami flow, which is a stationary solutions of the 3D Euler equations. For such flows it is known previously by Nadirashvili that decay condition $|v(x)| = o(1/|x|)$ at infinity, or $v \in L^q(R^3)$ with $q \in [2, 3]$ implies the triviality, $v = 0$ on $\mathbb{R}^3$. This shows that the existence result of a special Beltrami flows by Enciso and Peralta-Salas is sharp. Here we first present an elementary and simple proof of similar result, which was obtained jointly with Constantin.

Then, we study recent improvements of those results, deduced jointly with Wolf, which shows that if we assume similar decay rate, but only on the tangential component of the velocity, then the solution becomes trivial.

Hi Jun Choe

On Maximum Modulus Estimates of the Navier-Stolkes Equations with Non-zero Boundary Data

Abstract: We consider discontinuous influx for the Navier-Stokes flow and construct an unbounded solution near discontinuous point for any dimension bigger or equal to two. This is an extension of the previous result that a blow-up solution exists with a bounded and discontinuous boundary data for the Stokes flow. It turns out that Dini continuity in space or log-Dini continuity in time of the normal component of boundary data is crucial for maximum modulus estimates.

Camillo De Lellis

Dissipative Euler flows with Hölder exponents below 1/3

Abstract: In the fifties John Nash astonished the geometers with his celebrated isometric embedding theorems. A folkloristic explanation of his first theorem is that you should be able to put any piece of paper in your pocket without crumpling or folding it, no matter how large it is.

Ten years ago La'szlo' Sze'kelyhidi and I discovered unexpected similarities with the behavior of some classical equations in fluid dynamics. Our remark sparked a series of discoveries and works which have gone in several directions. Among them the most notable is the recent proof of Phil Isett of a long-standing conjecture of Lars Onsager in the theory of turbulent flows. In a joint work with La'szlo', Tristan Buckmaster and Vlad Vicol we improve Isett's theorem to show the existence of dissipative solutions of the incompressible Euler equations below the Onsager's threshold.

Paul Deuring

Decay in time and in space of solutions to the time-dependent Oseen system

Abstract: olutions in L^2-spaces to the time-dependent Oseen system in 3D are represented by a sum of three integrals, each involving a fundamental solution of the Oseen system. Two of these integrals are volume potentials, whereas the third consists of a boundary integration with respect to the space variable, a Volterra integral with respect to the time variable, weighted by a function solving an integral equation. Under suitable assumptions on the data, each of these integrals decays in time and pointwise in space. As main problem in the proof, the L^2-norm in space of the weight function in the boundary potential must be shown to decay with an algebraic rate in time. It remains an open question how to extend this approach to the nonlinear case.

Carlo R. Grisanti

The p-Laplacian singular system in exterior domains: regularity and time behaviors

Abstract: We consider the IBVP in exterior domains for the p-Laplacian parabolic system in the singular case $1 < p < 2$. We obtain L^q regularity up to the boundary for the second derivatives $(q \geqslant 2)$. We prove the existence of a solution for initial data in L^s with $1 < s < 2$ and we study its behavior in time. We find extinction properties for $p \in \left(\frac{2n}{n+2}, \frac{2n}{n+1}\right)$ and exponential decay for $p = \frac{2n}{n+1}$.

Toshiaki Hishida

L^q-L^r *estimates of a generalized Oseen evolution operator, with applications to the Navier-Stokes flow past a rotating obstacle*

Abstract: Consider the motion of a viscous incompressible fluid in a 3D exterior domain D when a rigid body $\mathbb{R}^3 \setminus D$ moves with prescribed time-dependent translational and angular velocities. For the linearized non-autonomous system, L^q-L^r smoothing action near $t = s$ as well as generation of the evolution operator $\{T(t,s)\}_{t \geqslant s \geqslant 0}$ was shown by Hansel and Rhandi [1] under reasonable conditions. In this presentation we develop the L^q-L^r decay estimates of the evolution operator $T(t,s)$ as $(t-s) \to \infty$ and then apply them to the Navier-Stokes initial value problem.

References

[1] T. Hansel and A. Rhandi. The Oseen-Navier-Stokes flow in the exterior of a rotating obstacle: the non-autonomous case. *J. Reine Angew. Math.* **694** (2014), 1–26.

Yoshiyuki Kagei

Bifurcation of the compressible Taylor vortex

Abstract: In this talk I will consider the Taylor problem, a flow between two concentric cylinders, whose inner cylinder is rotating with uniform speed and the outer one is at rest. If the rotating speed is sufficiently small, the Couette flow is stable. When the rotating speed increases, beyond a certain value of the rotating speed, a vortex flow pattern appears. In this talk this problem will be considered for a compressible fluid. We study the spectrum of the linearized operator around the Couette flow and show the bifurcation of the Taylor vortex when the Mach number is sufficiently small. This talk is based on a joint work with Prof. Takaaki Nishida (Kyoto University) and Ms. Yuka Teramoto (Kyushu University).

Kyungkeun Kang

Existence of regular solutions for non-Newtonian Navier-Stokes equations of power-law type

Abstract: We are concerned with non-Newtonian Navier-Stokes equations of a power law type with an exponent q in three dimensions.

$$u_t - \nabla \cdot S + (u \cdot \nabla)u + \nabla p = f,$$

$$\text{div } u = 0$$

in $\mathbb{R}^3 \times (0, T)$. Here the strain tensor $S = (S_{ij})_{i,j=1,2,3}$ is given as

$$S(Du) = \mu_1(1 + |Du|^2)^{\frac{q-2}{2}} Du,$$

where

$$Du = (D_{ij}) := \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big),\ i, j = 1, 2, 3.$$

We establish the local existence of regular solutions for $2 < q$ locally in time, if initial data is sufficiently smooth. Furthermore, if initial data is sufficiently small, the regular solutions are extended globally in time. The method of proof also works for more general type of strain tensors as long as some structure conditions are satisfied.

This is a joint work with H.-K. Kim and J.-M. Kim.

Grzegorz Karch

Blowup phenomena in conservation laws with fractional Laplacian and nonlocal fluxes

Abstract: Recent results on a blowup of solutions to the nonlinear evolution equation $u_t + (-\Delta)^{\alpha/2}u + \nabla \cdot (u\nabla K(u)) = 0$ in $\mathbb{R}^n$ will be presented. This model contains, as a particular example, the celebrated parabolic-elliptic model of chemotaxis. I shall discuss a certain method which allowed us to show that sufficiently well-concentrated initial conditions lead to solutions which blowup in a finite time.

This is a joint work with Piotr Biler and Jacek Zienkiewicz.

Hideo Kozono

Finite energy for the Navier-Stokes equations and Liouville-type theorems in two dimensional domains

Abstract: Introducing a new notion of *generalized suitable weak solutions*, we first prove validity of the energy inequality for such a class of weak solutions to the Navier-Stokes equations in the whole space $\mathbb{R}^n$. Although we need certain growth condition on the pressure, we may treat the class even with infinite energy quantity except for the initial velocity. We next handle the equation for vorticity in 2D unbounded domains. Under a certain condition on the asymptotic behavior at infinity, we prove that the vorticity and its gradient of solutions are both globally square integrable. As their applications, Loiuville-type theorems are obtained.
This is the joint work with Prof. Yutaka Terasawa at Nagoya University and Prof. Yuta Wakasugi at Ehime University.

References

[1] D. Chae. Remarks on the Liouville type problem in the stationary 3D Navier-Stokes equations. *arXiv:1502.04793v1*

[2] Y. Giga, P.-Y. Hsu, and Y. Maekawa. A Liouville theorem for the planer Navier-Stokes equations with the no-slip boundary condition and its application to a geometric regularity criterion. *Comm. Partial Differential Equations*, 39:1906–1935, 2014.

[3] G. Koch, N. Nadirashvili, G.Seregin, and V. Sverák. Liouville theorems for the Navier-Stokes equations and applications. *Acta Math.*, 347(5):1533–1589, 1995.

[4] H. Kozono, Y. Terasawa, and Y. Wakasugi. A Remark on Liouville-type theorems for the stationary Navier-Stokes equations in three space dimensions. *J. Funct. Anal.*, 272: 804–818, 2017.

Ondřej Kreml

On the Riemann problem for the 2D compressible isentropic Euler equations

Abstract: We consider the Riemann problem for the two-dimensional compressible isentropic Euler system, i.e. we solve the equations on the whole 2D space and the initial data consists of two pairs of constants, each on one of the halfplanes. This classical problem possesses a one-dimensional self-similar admissible solution which is unique in the BV class. We study uniqueness of admissible solutions in the broader class of essentially bounded functions. We summarize the up-to-date results concerning uniqueness and nonuniqueness of admissible weak solutions in certain cases of initial data. The nonuniqueness results are based on the theory of De Lellis and Székelyhidi for incompressible Euler equations.

Mads Kyed

On L^p estimates for time-periodic solutions to parabolic boundary value problems of Agmon-Douglis-Nirenberg type

Abstract: A celebrated result of Agmon, Douglis and Nirenberg states that if an elliptic operator A satisfies the so-called complementing condition with respect to a number of boundary operators, then a solution to the corresponding boundary value problem satisfies an a priori L^p estimate. In my talk, I will consider the related parabolic operator $\partial_t - A$ and present a new result, which states that a time-periodic solution to the corresponding parabolic boundary value problem satisfies a similar L^p estimate. Since time-independent solutions are trivially time-periodic, I will be able to present the result in such a way that it contains the theorem of Agmon, Douglis and Nirenberg as a special case. Finally, I will discuss applications to models that describe rotating fluids.

Pierre Gilles Lemarié-Rieusset

On stability of dissipative solutions and the role of vorticity

Abstract: Assume that $\vec{u}_n$, p_n and $\vec{f}_n$ are smooth functions such that they satisfy the Navier-Stokes equations

$$\partial_t \vec{u}_n + \vec{u}_n.\vec{\nabla}\vec{u}_n = \Delta\vec{u}_n - \vec{\nabla}p_n + \vec{f}_n, \quad \operatorname{div}\vec{u}_n = 0.$$

If on a domain $]t_1, t_2[\times B(x_0, r) = I \times \Omega$ we have the following control: $\vec{u}_n$ is bounded in $L^\infty(I, L^2(\Omega)) \cap L^2(I, H^1(\Omega))$, p_n is bounded in $L^{3/2}(I \times \Omega)$ and $\vec{f}_n$ is bounded in $L^2(I, H^{-1}(\Omega))$ then we may find subsequences that converge *-weakly to $\vec{u}$ (in $L^\infty(I, L^2(\Omega)) \cap L^2(I, H^1(\Omega))$), p (in $L^{3/2}(I \times \Omega$) and $\vec{f}$ (in $L^2(I, H^{-1}(\Omega))$) and the limits still satisfy the Navier-Stokes equations on $I \times \Omega$. Moreover, the limit $\vec{u}$ is suitable in the sense of Caffarelli, Kohn and Nirenberg.
What can be done if we remove the assumption on the control of p_n?

References

[1] Chamorro, Diego, Lemarié-Rieusset, Pierre-Gilles and Mayoufi, Kawther. The role of the pressure in the partial regularity theory for weak solutions of the Navier–Stokes equations. *Preprint arXiv:1602.06137*.

Xian Liao

Global regularity of two-dimensional density patch for inhomogeneous incompressible viscous flow

Abstract: e consider the density patch problem for the two dimensional inhomogeneous incompressible Navier-Stokes system, that is, the initial density is taken of the form $\rho_0 = \eta_1 \mathbf{1}_{\Omega_0} + \eta_2 \mathbf{1}_{\Omega_0^c}$ with any pair of positive constants (η_1, η_2). Then for any positive time, the density behaves as $\eta_1 \mathbf{1}_{\Omega(t)} + \eta_2 \mathbf{1}_{\Omega(t)^c}$. We aim to show the persistence of the $W^{k+2,p}$-regularity of the boundary of the domain $\Omega(t)$. The analysis relies heavily on the (time-weighted) energy estimates for the velocity vector field v as well as its tangential derivative $\partial_X v$ (here X denotes the tangent vector field of the domain $\Omega(t)$). This is a joint work with Ping Zhang (Chinese Academy of Science).

References

[1] X. Liao and P. Zhang. Global regularities of two-dimensional density patch for inhomogeneous incompressible viscous flow with general density. *arXiv:1604.07922.*

Vaclav Macha

Inviscid limit for the compressible system with non-local interactions

Abstract: The collective behavior of animals can be modeled by a system of equations of continuum mechanics endowed with extra terms describing repulsive and attractive forces between the individuals. This system can be viewed as a generalization of the compressible Euler equations with all of its unpleasant consequences, e.g., the non-uniqueness of solutions. We analyze the equation describing a viscous approximation of a generalized compressible Euler system and we show that its dissipative measure-valued solutions tend to a strong solution of the Euler system as viscosity tends to 0, provided that the strong solution exists.

Josef Málek

On the analysis of a class of thermodynamically compatible viscoelastic fluids with stress diffusion

Abstract: We first summarize the derivation of viscoelastic (rate-type) fluids with stress diffusion that generates the models that are compatible with the second law of thermodynamics and where no approximation/reduction takes place. The approach is based on the concept of natural configuration that splits the total response between the current and initial configuration into the purely elastic and dissipative part. Then we restrict ourselves to the class of fluids where elastic response is purely spherical. For such class of fluids we then provide a mathematical theory that, in particular, includes the long-time and large-data existence of weak solution for suitable initial and boundary value problems. This is a joint work with Miroslav Bulicek, Vit Prusa and Endre Suli.

Nader Masmoudi

Stability of the 3D Couette Flow

Abstract: We will discuss the dynamics of small perturbations of the plane, periodic Couette flow in the 3D incompressible Navier-Stokes equations at high Reynolds number. For sufficiently regular initial data, we determine the stability threshold for small perturbations and characterize the long time dynamics of solutions near this threshold. For rougher data, we obtain an estimate of the stability threshold which agrees closely with numerical experiments. The primary linear stability mechanism is an anisotropic enhanced dissipation resulting from the mixing caused by the large mean shear; the main linear instability is a non-normal instability known as the lift-up effect. Understanding the variety of nonlinear resonances and devising the correct norms to estimate them form the core of the analysis we undertake. This is based on joint works with Pierre Germain and Jacob Bedrossian.

Evelyne Miot

An asymptotic regime for the Vlasov-Poisson system

Abstract: We investigate the gyrokinetic limit for the two-dimensional Vlasov-Poisson system in a regime studied by F. Golse and L. Saint-Raymond. First we establish the convergence towards the Euler equation under several assumptions on the energy and on the norms of the initial data. Then we provide a first analysis of the asymptotics for a Vlasov-Poisson system describing the interaction of a bounded density with a moving point charge.

Sárka Nečasová

Weak-strong uniqueness for fluid-rigid body interaction problem with slip boundary condition

Abstract: We shall consider the problem of the motion of a rigid body in an incompressible viscous fluid filling a bounded domain. This problem was studied by several authors. They mostly considered classical non-slip boundary conditions, which gave them very paradoxical result of no collisions of the body with the boundary of the domain. Only recently there are results when the Navier type of boundary are considered.

We shall consider the Navier condition on the boundary of the body and the non-slip condition on the boundary of the domain. This case admits collisions of the body with the boundary of the domain. We shall prove the global existence of weak solution of the problem. Secondly, we prove existence of strong solution and finally we show weak-strong uniqueness .?

References

[1] Chemetov, Nikolai V. Nečasová, Šárka. The motion of the rigid body in the viscous fluid including collisions. Global solvability result. *Nonlinear Anal. Real World Appl.* 34 (2017), 416?-445.

[2] Chemetov, Nikolai V. Nečasová, Šárka, Muha, Boris. Weak-strong uniqueness for fluid-rigid body interaction problem with slip boundary condition. Preprint 2017.

Charlotte Perrin

A model of fluid with pressure dependent viscosity

Abstract: In this talk, I will present a new mathematical approach to deal with a complex rheology where the viscosity coefficient depends on the pressure. If such constitutive law is established from the physical point of view, for instance for granular media, it greatly complicates the mathematical analysis of the equations compared to the well-known case of constant viscosities. I will present an original result of existence of global weak solutions to an incompressible system with a nonlocal dependence of the viscosity with respect to the pressure. The non-locality represents memory effects in the flow that are due in granular media to networks of contacts between grains. I will show that the solutions can be obtained as singular limits of solutions of a compressible system with singular viscosities.

Konstantin Pileckas

On Singular Solutions of Time-Periodic Stokes Problems in a Power Cusp Domain

Abstract: The time-periodic Stokes problem with the boundary value having a nonzero flux is considered in the power cusp domain:

$$\begin{cases} \mathbf{u}_t(x,t) - \nu\Delta\mathbf{u}(x,t) + \nabla p(x,t) = \mathbf{f}(x,t), \quad x \in \Omega, \\ \operatorname{div}\mathbf{u}(x,t) = 0, \quad x \in \Omega, \\ \mathbf{u}(x,t)|_{\partial\Omega} = \mathbf{a}(x,t), \\ \mathbf{u}(x,0) = \mathbf{u}(x,2\pi), \end{cases}$$

in a bounded domain $\Omega = \Omega_H \cup \Omega_0$, $\Omega_H = \{x \in \mathbb{R}^n : |x'| < \varphi(x_n), x_n \in (0,H)\}$, $\varphi(x_n) = \gamma_0 x_n^\lambda$, $\gamma_0 = \text{const}$, $\lambda > 1$, $n = 2,3$, and $\partial\Omega_0$ is Lipschitz. Here $\mathbf{f}$ and $\mathbf{a}$ are time-periodic functions, $x' = (x_1, ..., x_{n-1})$. We assume that $\operatorname{supp}\mathbf{a} \subset \partial\Omega_0 \cap \partial\Omega$ and

$$\int\limits_{\partial\Omega_0\cap\partial\Omega} \mathbf{a}\cdot\mathbf{n}\,ds = -F(t).$$

We look for the solution $\mathbf{u}$ which is singular in the cusp point O (its singularity depends on the cusp's power λ). In order to find the solution we construct an asymptotic expansion near the singularity point and reduce the problem to the case where the energy solution exists. The solution of the problem is found then as the sum of the asymptotic expansion and the term with finite energy.

Milan Pokorný

Steady equations describing flow of chemically reacting heat conducting compressible mixtures

Abstract: We consider the following system of equations

$$\begin{aligned} \operatorname{div}(\rho\mathbf{u}) &= 0, \\ \operatorname{div}(\rho\mathbf{u}\otimes\mathbf{u}) - \operatorname{div}\mathbf{S} + \nabla\pi &= \rho\mathbf{f}, \\ \operatorname{div}(\rho E\mathbf{u}) + \operatorname{div}(\mathbf{Q} + \pi\mathbf{u} - \mathbf{S}\mathbf{u}) &= \rho\mathbf{f}\cdot\mathbf{u} \\ \operatorname{div}(\rho Y_k\mathbf{u}) + \operatorname{div}\mathbf{F}_k &= m_k\omega_k, \quad k = 1,\dots,L \end{aligned} \tag{0.1}$$

which describes the steady flow of a compressible heat conducting mixture of gases whose component may chemically react, where only the barycentric velocity is taken into account. Recalling results from [1] and [2] we present existence results of weak and variational entropy solutions to this system provided the molar masses of all constituent are the same. Assuming the mixture to be contained in a dilutant, we may remove the assumption of equal molar masses and prove existence of weak or variational entropy solutions in dependence on the parameters of the problem.

References

[1] V. Giovangigli, M. Pokorný and E. Zatorska. On the steady flow of reactive gaseous mixture. *Analysis (Berlin)*, 35(4):319-341, 2015.

[2] T. Piasecki and M. Pokorný. Weak and variational entropy solutions to the system describing steady flow of a compressible reactive mixture. *to appear in Nonlinear Analysis TMA*, 2017.

Chenyin Qian
Asymptotic behavior for the quasi-geostrophic equations with fractional dissipation in $\mathbb{R}^2$

Abstract: We consider the asymptotic behavior of solutions of the following 2D quasi-geostrophic equations (QGE) with fractional dissipation in $\mathbb{R}^2$:

$$\theta_t + u \cdot \nabla\theta + \kappa(-\Delta)^\alpha \theta = F(x, \theta) \tag{0.2}$$

with $1/2 < \alpha \leqslant 1$, and $\theta(x, 0) = \theta^0$, $F(x, \theta)$ a given function. Here the real scalar function θ is the so-called potential temperature, and the incompressible velocity field $u = (u_1, u_2) = (-\mathcal{R}_2\theta, \mathcal{R}_1\theta)$ is determined from θ by applying Riesz operators. Motivated by previous results of N. Ju and other authors on the existence of the global attractor for the autonomous system (0.2) involving external force on bounded domains with smooth boundary or a spatial domain with periodic boundary conditions, we analyze the problem in $\mathbb{R}^2$ with external force $F(x, \theta)$. Besides, we also intend to discuss finite Hausdorff and fractal dimensions of the global attractor by the fractional Lieb-Thirring inequality (introduced by Lundholm-Thanh Nam-Portmann).

References

[1] R. Farwig and C. Qian. Asymptotic behavior for the quasi-geostrophic equations with fractional dissipation in $\mathbb{R}^2$. *Preprint*, TU Darmstadt, 2017.

Michael Růžička
Analysis of Fluid Models with Microstructure

Abstract: There are several situations where classical fluid models are not sufficient to describe the motion of the fluid. In the case of a Cosserat continuum one has to deal with models with micropolarity. We present some recent results on the existence theory for generalized Newtoniam fluids with micropolarity. Special attention is paid to the underlying weighted function spaces.

Jonas Sauer

Partially Periodic Instationary Generalized Stokes Equations and an Application to Stability of Viscoelastic Poiseuille-Type Flows

Abstract: We consider an instationary generalized Stokes system with nonhomogeneous divergence data under a periodic condition in only some directions. The problem is set in the whole space, the half space or in (after an identification of the periodic directions with a torus) bounded domains with sufficiently regular boundary. We show unique solvability for all times in Muckenhoupt weighted Lebesgue spaces. The divergence condition is dealt with by analyzing the associated reduced Stokes system and in particular by showing maximal regularity of the partially periodic reduced Stokes operator. The result is used to investigate L^p-stability issues of small periodic viscoelastic Poiseuille-type flows in two dimensions subject to

$$\begin{cases} \partial_t F + u \cdot \nabla F &= F\nabla u, \\ \operatorname{div} u &= 0, \\ \partial_t u - \nu\Delta u + u \cdot \nabla u + \nabla \pi &= \operatorname{div} F^T F, \end{cases}$$

where F denotes the deformation tensor, u the velocity, π the pressure and ν the viscosity. This model has been considered by Lin, Liu, and Zhang (2005). We show global existence and exponential decay of perturbations of the Poiseuille-type flows whenever the initial perturbation and the height of the layer are sufficiently small. One of the main ideas is to make a change of variables in the perturbed model to reveal the hidden dampening term in the transport equation. This however destroys the divergence-free condition for the velocity, which is why we need to investigate nonhomogeneous divergence data. Another important ingredient is a fixed point argument due to Kreml and Pokorný (2009), which only asks for contraction in weak norms and therefore is tailor-made for the hyperbolic/parabolic type of the problem.

Andreas Schmidt

The Navier-Stokes equations with the Coulomb boundary condition

Abstract: On my poster the Navier-Stokes equations on manifolds with the Coulomb friction law boundary condition are considered. This condition means that the solution can slip at the boundary once the tangential stress exceeds a given threshold. If that is the case, the tangential component of the solution has the opposite direction of the tangential stress. This system, that has already been studied in the Euclidean case, is formulated on manifolds and the existence of weak solutions is discussed. Also the dependence of the solution on the threshold is treated.

Gregory Seregin

Remarks on Liouville Type Theorems for Steady-State Navier-Stokes Equations

Abstract: Liouville type theorems for the stationary Navier-Stokes equations are proven under certain assumptions. These assumptions are motivated by conditions that appear in Liouvile type theorems for the heat equations with a given divergence free drift.

Timofey Shilkin

On the local properties of weak solutions to elliptic equations with divergence-free drifts

Abstract: We discuss the local properties of weak solutions to the equation $-\Delta u + b \cdot \nabla u = 0$ describing the diffusion in a stationary incompressible flow. The corresponding theory is well-known in the case of the general (not necessary divergence-free) sufficiently regular drift (namely, for $b \in L_n$, where n is the dimension of the space). Our main interest is focused on the case of b with limited regularity (namely, $b \in L_2$). In this case the structure assumption $\operatorname{div} b = 0$ turns out to be crucial. In our paper we recall some known properties of weak solution in the case of the divergence-free drifts $b \in L_2$ and also establish some new results on the local boundedness of weak solutions.

Senjo Shimizu

Strong solutions of the Navier-Stokes equations based on the maximal Lorentz regularity theorem in Besov spaces

Abstract: We show existence and uniqueness theorem of local strong solutions to the Navier-Stokes equations with arbitrary initial data and external forces in the homogeneous Besov spaces with both negative and positive differential orders which are invariant under the change of scaling. If the initial data and external forces are small, then the local solutions can be extended globally in time. Our solutions also belong to the Serrin class in the usual Lebesgue space. The method is based on the maximal Lorentz regularity theorem of the Stokes equations in the homogeneous Besov spaces. This is a joint work with Prof. H. Kozono (Waseda, Japan).

Maria Specovius

Remark about the Helmholtz-decomposition in a cone

Abstract: The Helmholtz-decomposition is one of the essential tools in the theory of fluid mechanics. This lecture deals with the Helmholtz -decomposition of weighted L^q - spaces in an n-dimensional cone $C = \{x \,|\, x/|x| \in \omega\}$ where ω is a subset of the unit sphere with sufficiently smooth boundary. Using a relatively elementary toolbox we construct the decomposition $\mathbf{L}^q_\beta(C) = X^q_\beta C \oplus G^q_\beta(C)$ for all $\beta \in \mathbb{R} \setminus E$, where

- E is a discrete countable set of real numbers (forbidden values)
- $\mathbf{L}^q_\beta(C) = \{\mathbf{u} \in L^q_{loc}(C) \,|\, \int_C |u|^q |x|^{\beta q} dx < \infty\}$
- X^q_β is the closure of $C^\infty_{0,\sigma}(C)$ in $\mathbf{L}^q_\beta(C)$
- $G^q_\beta = \{\nabla p \in \mathbf{L}^q_\beta(C), p \in L^q_{\beta-1}(C)\}$

Yasushi Taniuchi

Brezis-Gallouet-Wainger type inequality and its application to Navier-Stokes equations in unbounded domains

Abstract: We shall find the largest Banach space that satisfies the Brezis-Gallouet-Wainger type inequality. As an application, we shall establish Serrin type regularity criteria of smooth solutions to the 3-D Navier-Stokes equations in unbounded domains Ω, when Ω is an exterior domain with $\partial\Omega \in C^\infty$, R^3 or R^3_+. It is known that if smooth L^3-solutions u of the Navier-Stokes equations on $(0, T)$ satisfies $\int_0^T \|u\|^2_{L^\infty(\Omega)} d\tau < \infty$, then u can be continued to the smooth solution on $(0, T')$ for some $T' > T$. In this talk, we shall slightly relax this condition for continuation of smooth solutions.

This is a joint work with Kohei Nakao (Shinshu university).

Marius Tucsnak

Motion of solids in a viscous gas: wellposedness and long-time behavior

Abstract: The mathematical models for the motion of solids in a fluid furnish a class of problems which have been interesting scientists for a long time. Among the early references, the work of Kirchhoff, considering the motion of a rigid body in an inviscid fluid filling the space, is still of mathematical interest and it is used, for instance, in marine engineering. In the case of viscous fluids, the first results in this field go back to Stokes but the study of the coupling of the equations of solid motion with the Navier-Stokes equations (essentially incompressible) is more recent, see, for instance, [1], [2], [3]. An important difficulty arising in these problems is the fact the domain filled by the fluid is one of the unknowns of the problem, so we have, to a certain extent, a free boundary problem.

In this presentation we consider the system coupling the equations of a viscous compressible fluid with those of a rigid body immersed in it. Moreover, the fluid is supposed to be heat conducting. An important example of application is the *adiabatic piston* problem, which has been extensively studied in the statistical physics literature. Our main results assert:

- Global in time existence and uniqueness and convergence to equilibrium in one space dimension.
- Local existence in an $L^p - L^q$ setting in three space dimensions.

From a methodological viewpoint, we present a systematic way of proving that the the maximal regularity properties of the linearized fluid system are "inherited" by the linearized fluid-structure system. This is obtained using some new perturbation results for $R-$ sectorial operators.

References

[1] B. Desjardins and M. Esteban. *Existence of weak solutions for the motion of rigid bodies in a viscous fluid*. Arch. Ration. Mech. Anal., 146.

[2] J. A. San Martín, V. Starovoitov, and M. Tucsnak. *Global weak solutions for the two-dimensional motion of several rigid bodies in an incompressible viscous fluid*. Arch. Ration. Mech. Anal., 161 (2002), pp. 113–147.

[3] D. Serre. *Chute libre d'un solide dans un fluide visqueux incompressible. Existence*. Japan J. Appl. Math., 4 (1987), pp. 99–110.

Erika Ushikoshi

Hadamard variational formula for the eigenvalue of the Stokes equations and its applications

Abstract: We consider the domain perturbation problem of the stationary Stokes equations with the Dirichlet boundary condition.

Hadamard variational formula is well known to represent the first variation of the Green function or the eigenvalue under a certain domain perturbation. Jimbo-Ushikoshi(2015) succeeded to establish such a representation formula for the multiple eigenvalue of the Stokes equations with the Dirichlet boundary condition.

In this talk, we consider the application of this formula to determine the topological type of the domain.

This work is a joint work with Prof. Shuichi Jimbo (Hokkaido Univ.), Hideo Kozono (Waseda Univ.) and Yoshiaki Teramoto (Setsunan Univ.).

David Wegmann

Existence of Strong Solutions and Decay of Turbulent Solutions of Navier-Stokes Flow with Nonzero Dirichlet Boundary Data

Abstract: We consider the Navier-Stokes equations in a domain with compact boundary and nonzero Dirichlet boundary data β. A solution is constructed as a sum of a very weak solution b to the instationary Stokes equations with nonzero boundary data and a weak solution v to a system of Navier-Stokes type with zero boundary data.

Assuming that $\beta(t) \to 0$ as $t \to \infty$, we proved in [2] that there exists a solution v which fulfills $\|v(t)\|_2 \to 0$ as $t \to \infty$. Furthermore, in a bounded domain the solution v tends exponentially to 0 if the corresponding data is exponentially decreasing.
As a last result, we calculated a lower polynomial bound for the decay rate if Ω is unbounded. Therefore, we used a suitable spectral decomposition of the Stokes operator as introduced in [1] and Duhamel's formula.

Recently, we proved the same decay result for an arbitrary turbulent solution. The main tool for the proof is to show the existence of a strong solution after some time $T > 0$ and to identify the turbulent solution with the strong solution in the time interval $[T, \infty)$.

This is a result of a collaboration with Prof. Dr. Reinhard Farwig and Prof. Dr. Hideo Kozono.

References

[1] W. Borchers and T. Miyakawa. Algebraic L^2 decay for Navier-Stokes flows in exterior domains. II. *Hiroshima Math. J.*, **3**, 621–640,1991.

[2] R. Farwig, H. Kozono, and D. Wegmann. Decay of non-stationary Navier-Stokes flow with nonzero Dirichlet boundary data. *Indiana Univ. Math. J.*, 2016, in press.

[3] R. Farwig, H. Kozono, and D. Wegmann. Existence of Strong Solutions and Decay of Turbulent Solutions of Navier-Stokes Flow with Nonzero Dirichlet Boundary Data. *Journal of Mathematical Analysis and Applications*, 2017, in press.

Minsuk Yang

On singularities for $L^\infty(0,T;L^3_{weak}(\mathbb{R}^3))$ *solutions to the Navier–Stokes equations*

Abstract: We study the possible singularities of a weak solution u to the Cauchy problem of the incompressible Navier–Stokes equations. We first discuss about regularity properties for the weak solution u satisfying the condition $L\infty(0,T;L^3_{weak}(\mathbb{R}^3))$ without any smallness assumption on that scale, where $L^3_{weak}(\mathbb{R}^3)$ denotes the standard weak Lebesgue space. We then discuss about blowup points at a singular time t and the size and distribution of singularities in terms of fractal dimension. The presentation is based on joint works with Hi jun Choe and Jörg Wolf.

Ping Zhang

Large time behavior os solutions to 3-D MHD system with initial data near equilibrium

Abstract: Given initial data (b_0, u_0) close enough to the equilibrium state $(e_3, 0)$, we prove that the 3-D incompressible MHD system without magnetic diffusion has a unique global solution (b, u). Moreover, we prove that $(b(t) - e_3, u(t))$ decay to zero with rates in both L^∞ and L^2 norm. (This is a joint work with Wen Deng).