Persistence probabilities for processes with stationary increments

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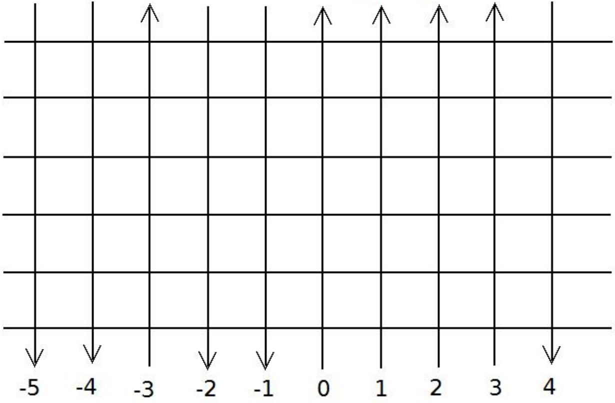
2-d Random walk with random orientation of the vertical lines

Model introduced by [Matheron,de Marsilly] for the displacement of a fluid in a stratified porous media.



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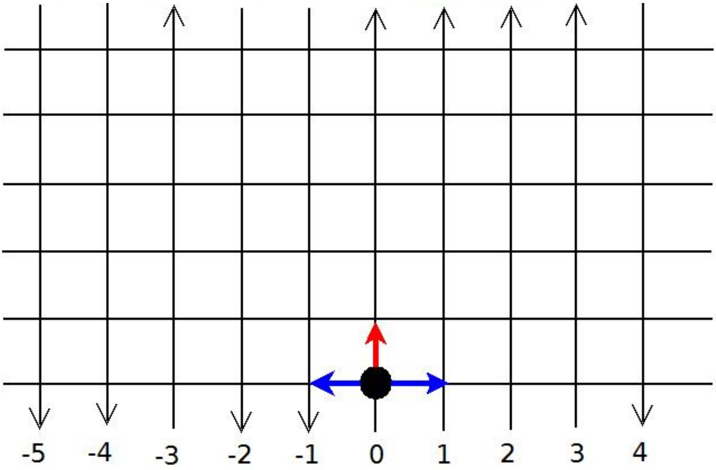

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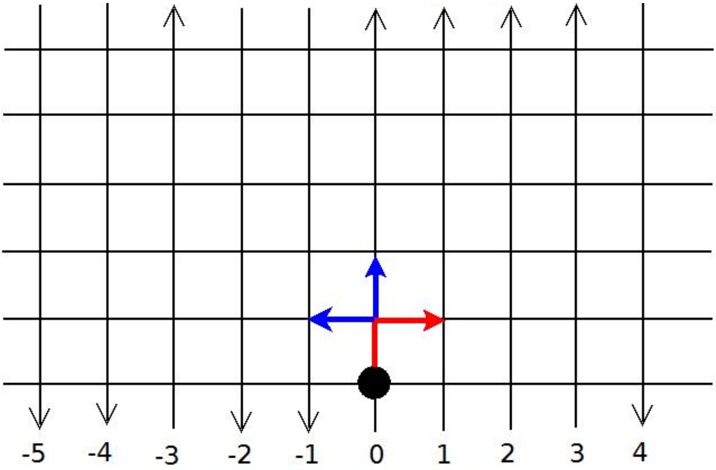

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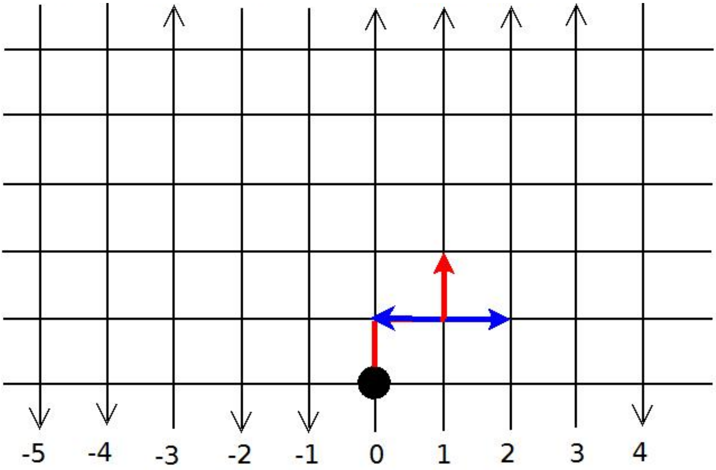

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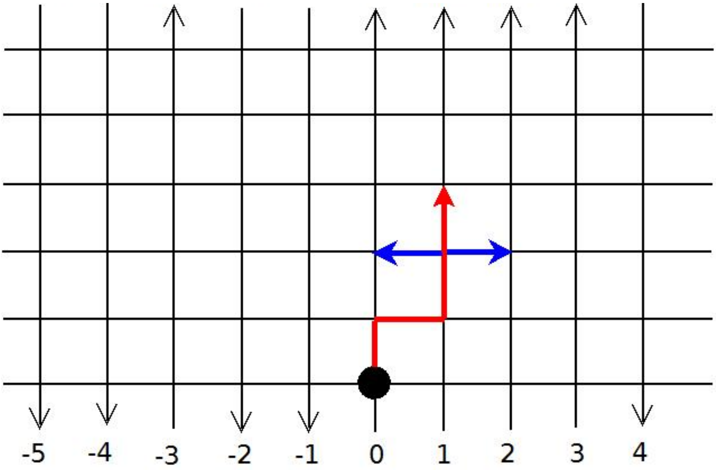

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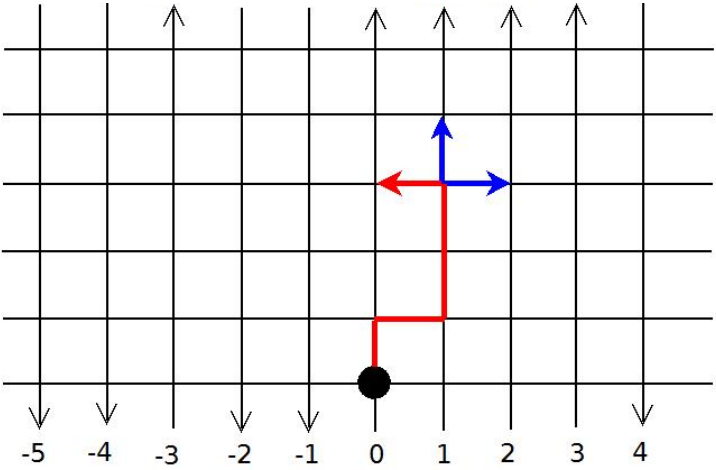

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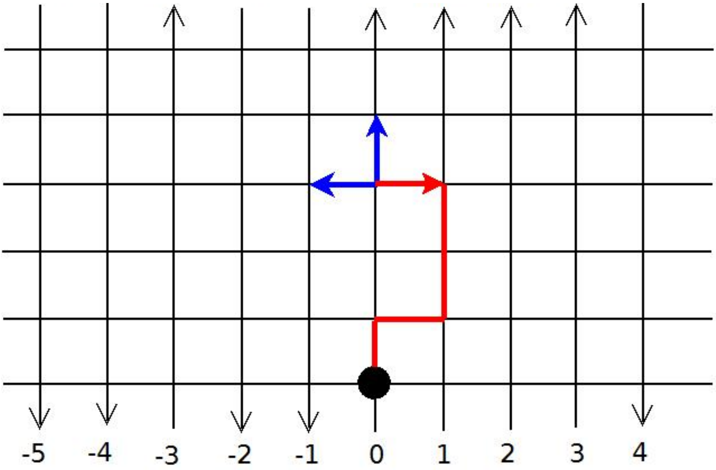

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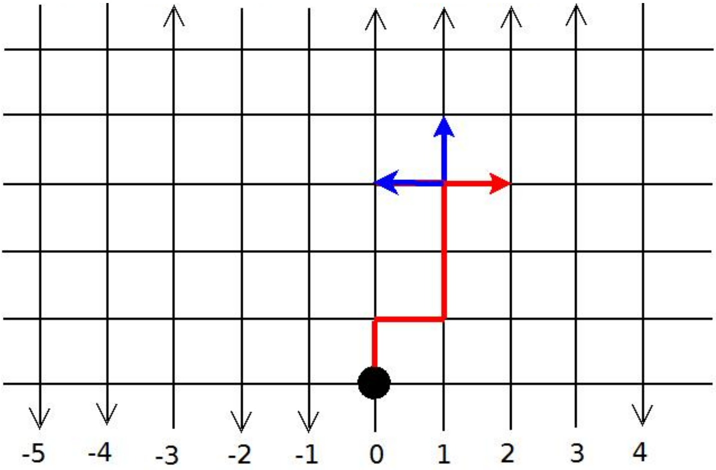

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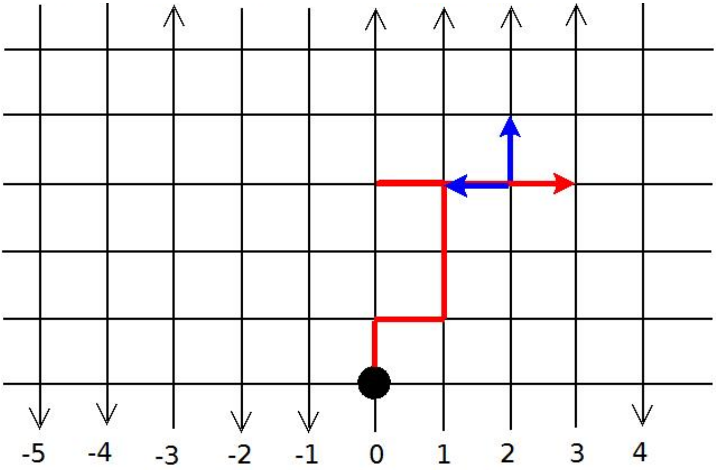

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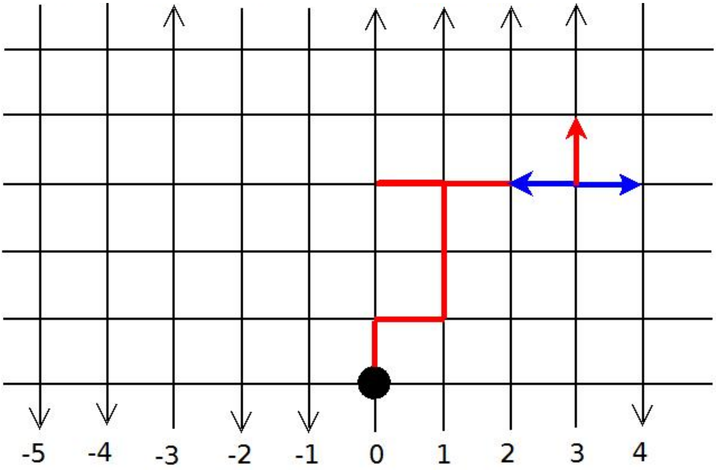

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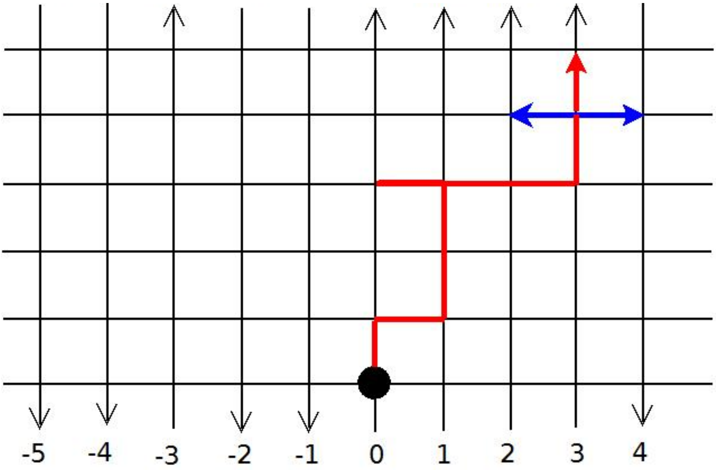

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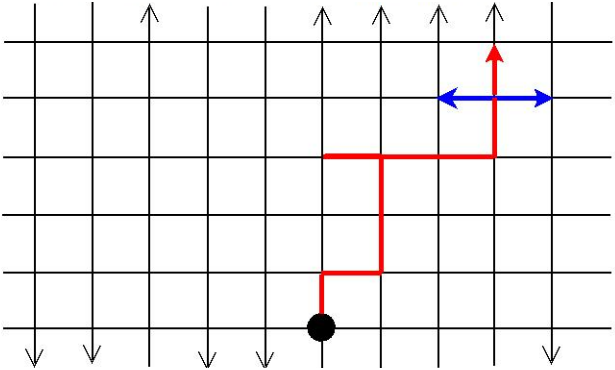

horizontal position: random walk S_n

vertical position: $Z_n = \sum_{k=1}^{n} \varepsilon_{S_k} \mathbf{1}_{\{S_k = S_{k-1}\}}$, with $\varepsilon_\ell = \begin{cases} 1 \text{ if } (\uparrow) \\ -1 \text{ if } (\downarrow) \end{cases}$.

Idea : $Z_n = \sum_{\ell \in \mathbb{Z}} \varepsilon_\ell \# \{k = 1, ..., n \ : \ S_k = S_{k-1} = \ell\}$
$Z_n \sim \frac{1}{3} \sum_{\ell \in \mathbb{Z}} \varepsilon_\ell \# \{k = 1, ..., n \ : \ S_k = \ell\}$.

SSRW vs Matheron-de Marsily model

[Castell,Guillotin-Plantard,P,Schapira]

orientations	No	random
recurrence or transience	recurrence	
mean horizontal displacement	$n^{\frac{1}{2}}$	
mean vertical displacement	$n^{\frac{1}{2}}$	
$P(M_n = 0)$	$\frac{C}{n}$	

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- **Question 1**: [Majumdar]: Persistence probability.

$$\mathbb{P}(Z_n^* < 0)? \quad \text{with } Z_n^* := \max(Z_1, ..., Z_n).$$

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- **Question 2**: Range.

Ambitious question

What happen if horizontal AND vertical lines are randomly oriented?

- Then
$$X_n=\sum_{\ell\in\mathbb{Z}}\varepsilon_\ell\#\{k=1,...,n\ :\ Y_k=Y_{k-1}=\ell\}$$
and
$$Y_n=\sum_{\ell\in\mathbb{Z}}\eta_\ell\#\{k=1,...,n\ :\ X_k=X_{k-1}=\ell\}$$

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- **Conjecture 1**: the walk is transient.
- **Conjecture 2**: the two coordinates are of order $n^{2/3}$.
- Heuristics: $X_n, Y_n \sim n^{-\gamma}$, so $\mathbb{P}(X_n = \ell) \sim n^{-\gamma}$ (LLT) $\mathbb{E}[\#\{k=1,...,n \ : \ X_k = X_{k-1} = \ell\}] \sim \sum_{k=1}^n \mathbb{P}(X_k = \ell) \sim n^{1-\gamma}$ and follow Kesten-Spitzer computation of variance of RWRS or even more heuristically:

$$n^\gamma \sim Y_n \sim \sum_{\ell=1}^{n^\gamma} \eta_\ell \, n^{1-\gamma} \sim n^{\gamma/2}\, n^{1-\gamma}.$$

So $\quad \mathbb{P}(X_n = 0, \ Y_n = 0) \sim \mathbb{P}(X_n = 0)\mathbb{P}(Y_n = 0) \sim n^{-4/9}$.

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- **Question 2**: Range.
- **Answers**: [Aurzada,Guillotin-Plantard,P]