Stability of Phases and Interacting Particle Systems

Nick Crawford; The Technion

CIRM Luminy; May 2017

Based on joint work w/ W. De Roeck

Setup

- $\Omega = \{-1,1\}^{\mathbb{Z}^d}$ or $\Omega_N = \{-1,1\}^{\Lambda_N}$ with $\Lambda_N = [-N,N]^d$.
- Finite Range Markov process $\sigma_t \in \Omega$ or $\in \Omega_N$.
  No Conservation Laws!
- Semigroup $\mathrm{e}^{t\mathcal{L}} : C(\Omega) \to C(\Omega)$.
  Generator $\mathcal{L}$ acting on local functions:

$$\mathbb{E}_\sigma[f(\sigma_t)] = \mathrm{e}^{t\mathcal{L}} \cdot f(\sigma).$$

Main Example of Interest: Ising Glauber Dynamics

1. $-H_N(\sigma|\eta) =$

$$1/2 \sum_{|x-y|=1, x,y\in\Lambda_N} \sigma_x\sigma_y + \sum_{|x-y|=1, x\in\Lambda_N, y\in\Lambda_N^c} \sigma_x\eta_y + h\sum_{x\in\Lambda_N}\sigma_x$$

2. Spin Flip Operator:

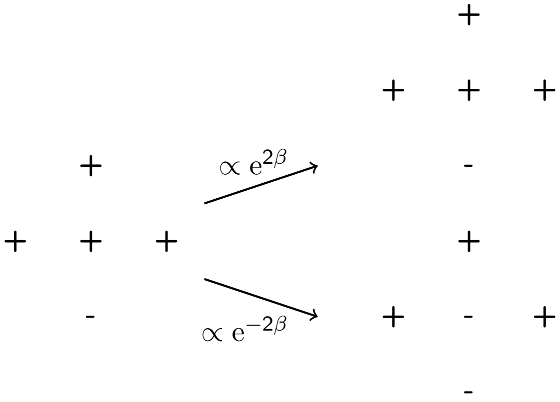
$$\sigma_y^x = \begin{cases} -\sigma_x & \text{if } y = x \\ \sigma_y & \text{o/w.} \end{cases}$$

3. For $\sigma \in \Omega_N$,

$$c_x(\sigma|\eta) = (1 + \exp(\beta[H(\sigma^x|\eta) - H(\sigma|\eta)]))^{-1}$$

and $\mathcal{L}_N^\eta \cdot f(\sigma) = \sum_{x\in\Lambda_N} c_x(\sigma|\eta)[f(\sigma^x) - f(\sigma)]$.

Similar on Ω....


$\propto e^{2\beta}$
$\propto e^{-2\beta}$

Invariant Measures

1. ν a measure on Ω. Invariance:

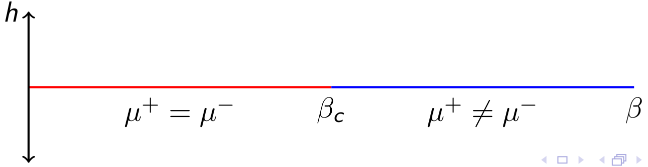
$$\mathbb{E}_\nu[f(\sigma_t)] = \mathbb{E}_\nu[f(\sigma_0)]$$

2. For Glauber, β fixed:
   All Gibbs measures invariant. Weak limits of

$$\mu^\eta_{N,\beta}(\sigma) \propto e^{-\beta H_N(\sigma|\eta)}.$$

   Others?

Structure of Gibbs measures:



Perturbations and Stability of Phases

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- Caution: Glauber with Nonzero External Fields.
- **Current Understanding for Glauber:**
  1. $\beta < \beta_c$ or $h \neq 0$ **Yes. (C-DR '17)**
  2. $\beta > \beta_c$ and $h = 0$ **Don't Know. Very Interesting.**

Examples:

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Important Remark Not reversible in finite volume! (Cycle Condition)

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Previous Related Work

Perturb around independent spin-flips $\beta = 0$.
Everything is true, all techniques work

1. Unique invariant measure
2. Exponential decay of all truncated correlations.
3. All is analytic
4. Invariant measure is *Gibbsian*

Attractivity

Attractive dynamics: Processes started from σ, η with $\sigma \leq \eta$ can be coupled such that $\sigma_t \leq \eta_t$ almost surely. Ising Glauber has this property, perturbations possibly not.

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1. In 1d, uniqueness phase for attractive nearest neighbour dynamics (Gray '82).
2. In general, (Holley '85) shows:
   Attractive perturbations of attractive dynamics have unique invariant measures if orginal process has exp. decay of correlations.

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$$\nu_N^+(\sigma(0)) - \nu_N^-(\sigma(0)) \leq C\mathrm{e}^{-cN}.$$

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Weak Spatial Mixing implies the unique stationary ν satisfies

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Generalizable to finite range, attractive particle systems.
Big problem in Non-reversible setting...

Main Result

- $c_x(\sigma)$ rates of σ_t, a finite range attractive particle system.
- $d_x(\sigma)$ rates of perturbed process $\bar{\sigma}_t$. **No need for attractivity!**
- $M = \sup_{\sigma,x} |c_x(\sigma) - d_x(\sigma)|$.

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Theorem (C-DR '17)

If σ_t has Weak Spatial Mixing, then for M small enough $\bar{\sigma}_t$ has ! stationary $\bar{\nu}$ and

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Corollary

First conclusion holds for Ising Glauber in entire uniqueness phase.

Information (Non)-Percolation

Individual Reservoirs: Fix $\epsilon, \beta > 0$ and $|\beta_x - \beta| < \epsilon$.

$$d_x(\sigma) = (1 + \exp(\beta_x [H(\sigma^x) - H(\sigma)]))^{-1}$$

The Basic Coupling:

Indep., to each x attach

1. Rate 1 Poisson process $N_t(x)$.
2. An i.i.d. sequence $U_{x,i}$ of [0, 1] uniform variables.

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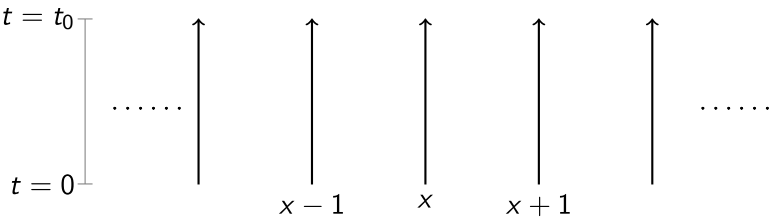

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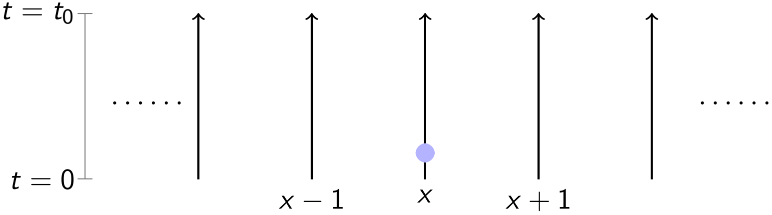

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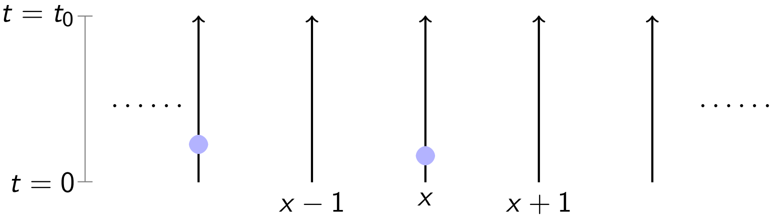

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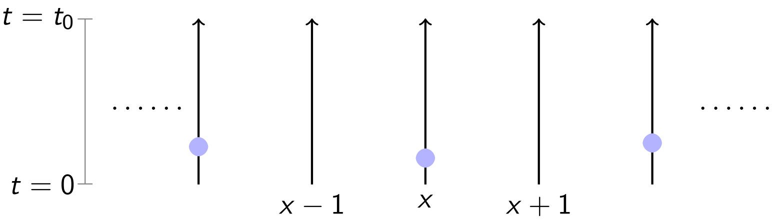

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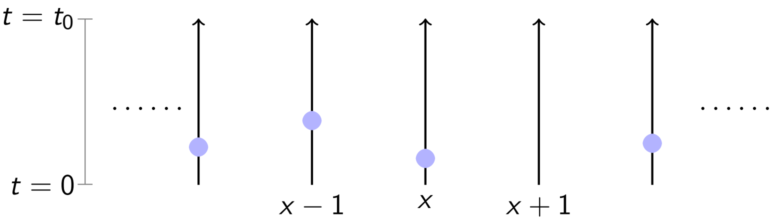

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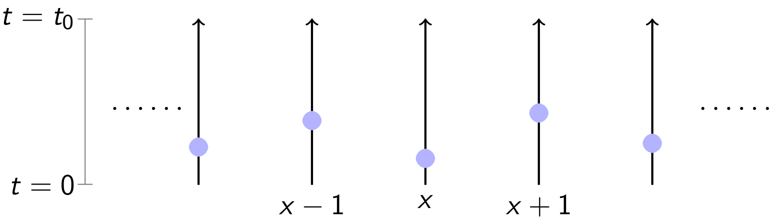

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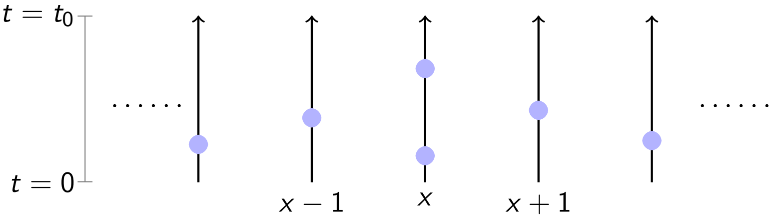

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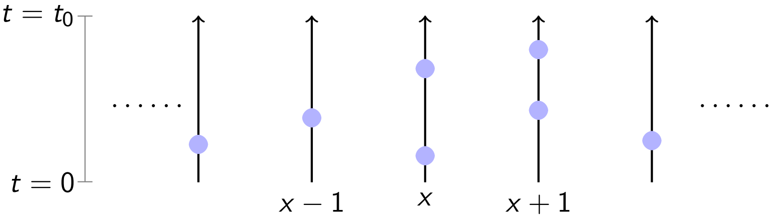

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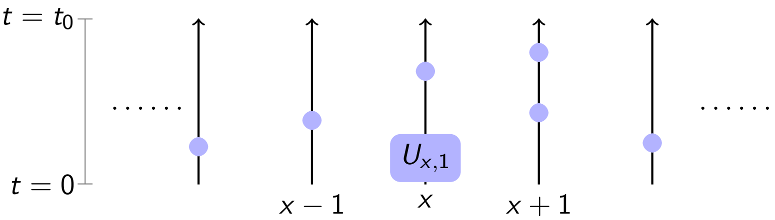

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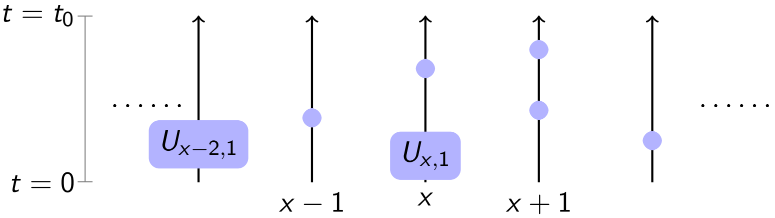

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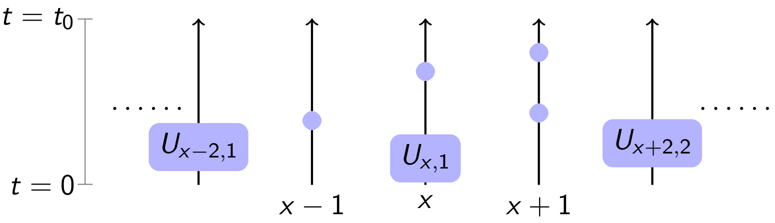

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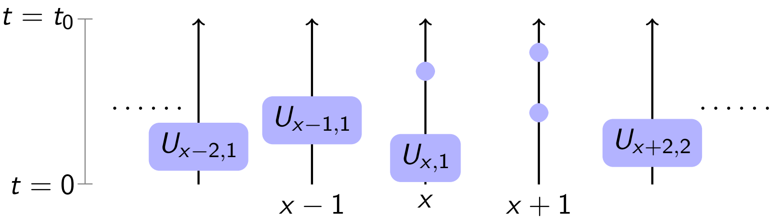

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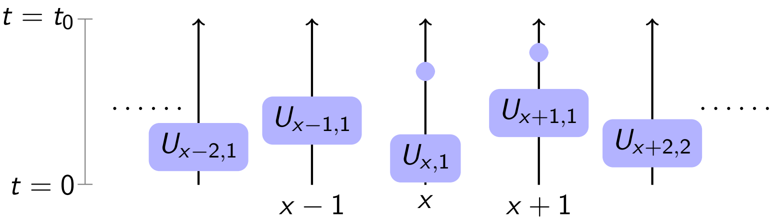

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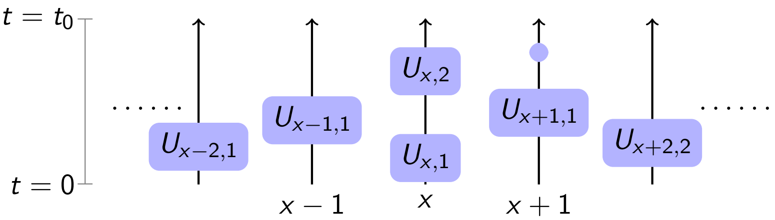

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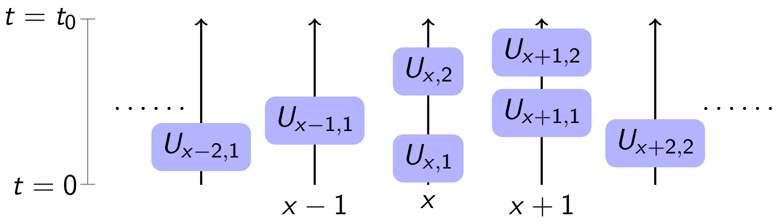
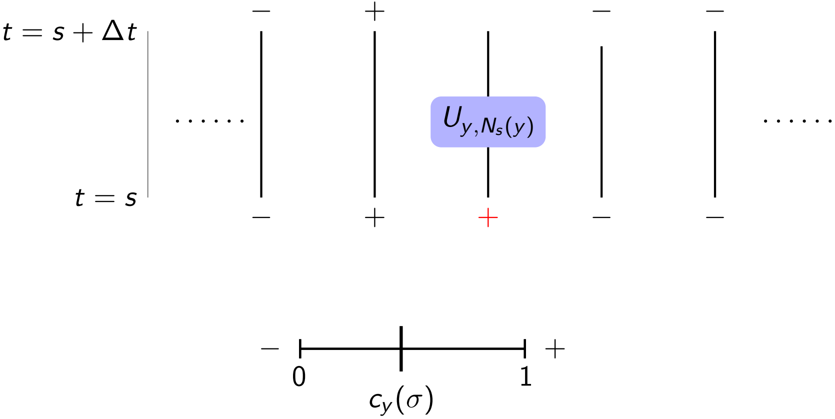


If $\sigma_s(y) = +$



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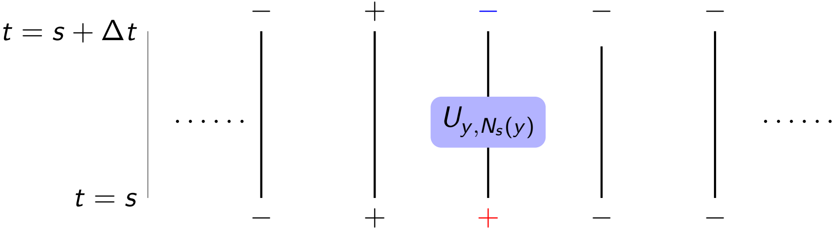

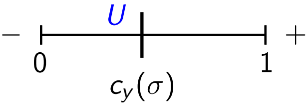

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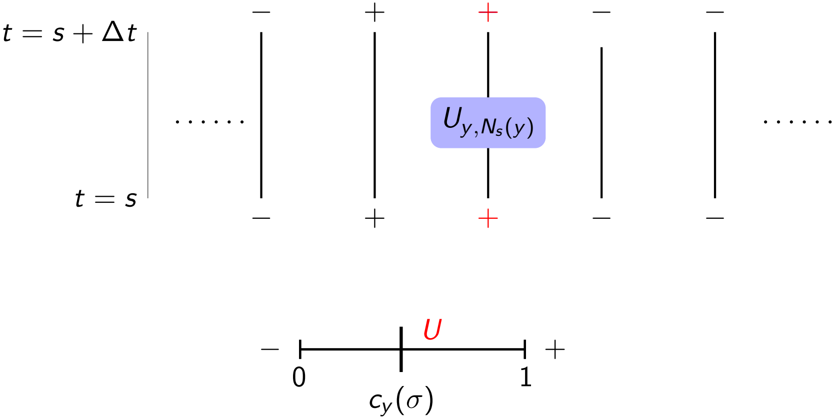

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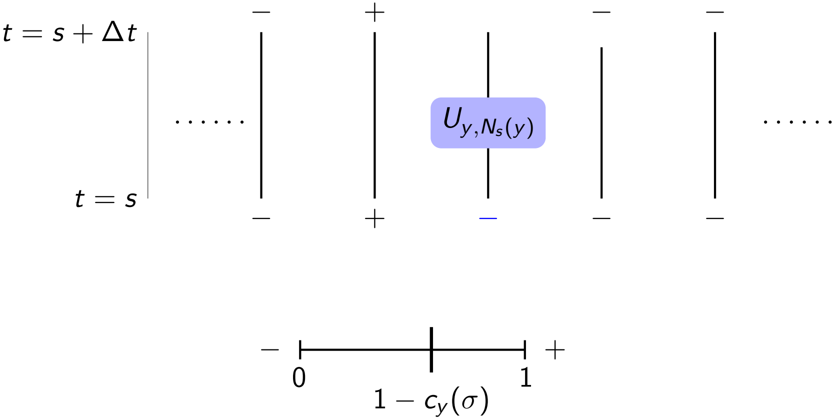

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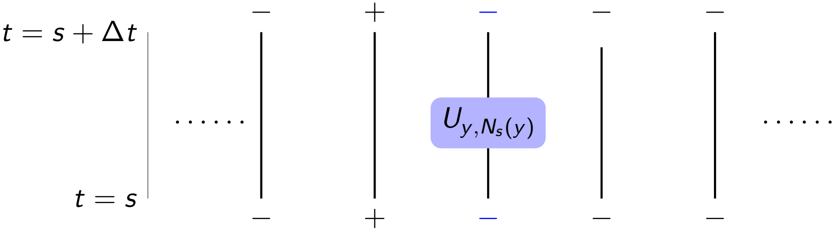

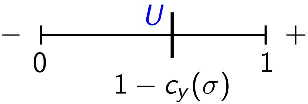

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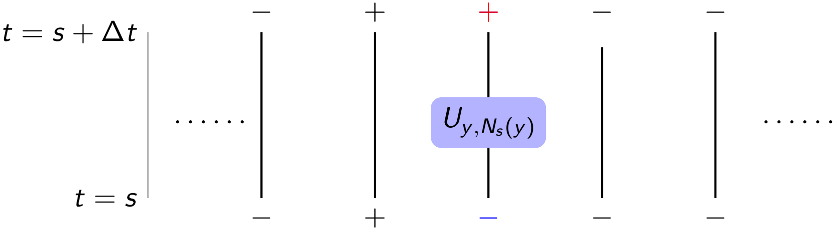
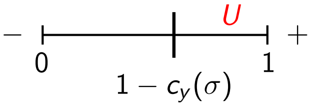
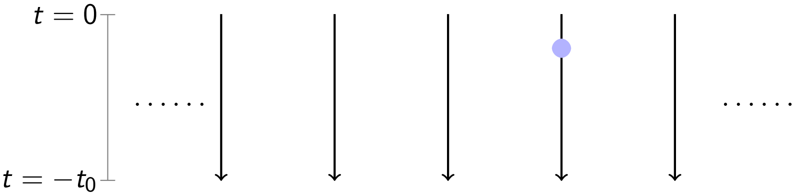




Coupling from Past (Propp, Wilson '96)



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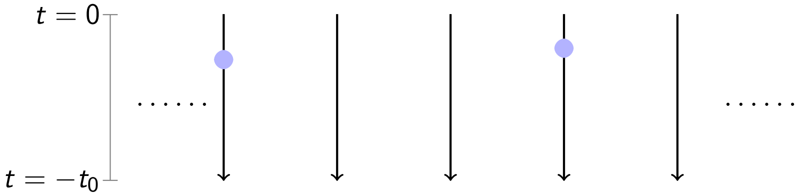

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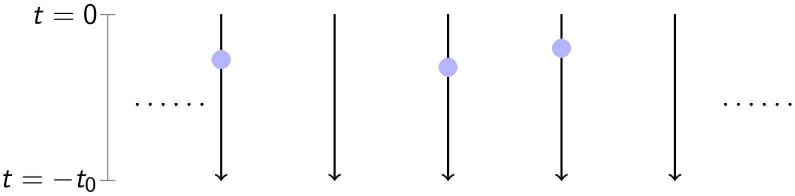

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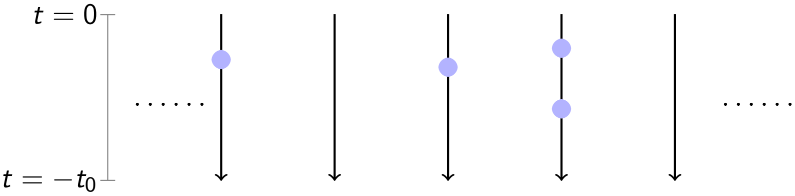

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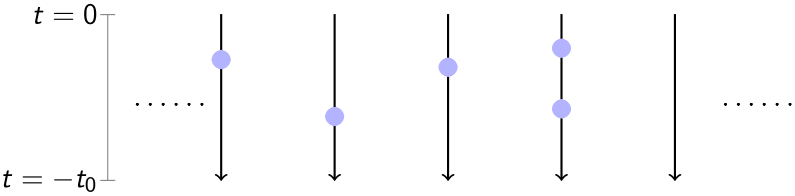

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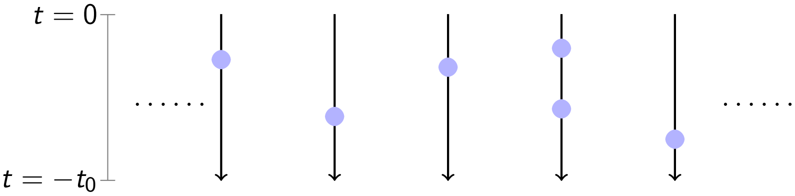

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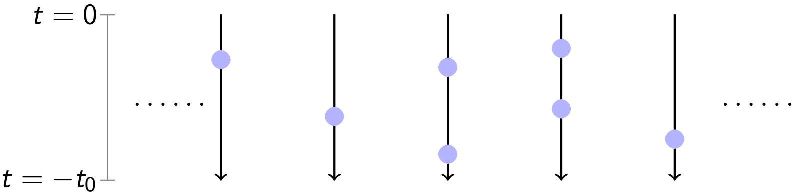

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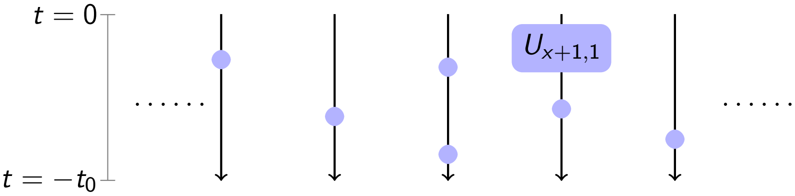

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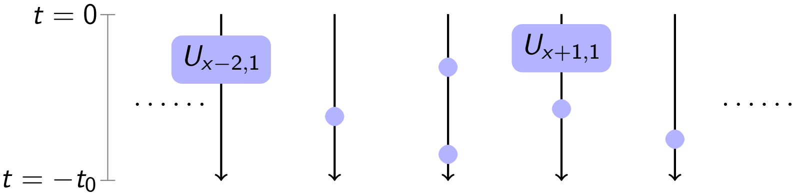

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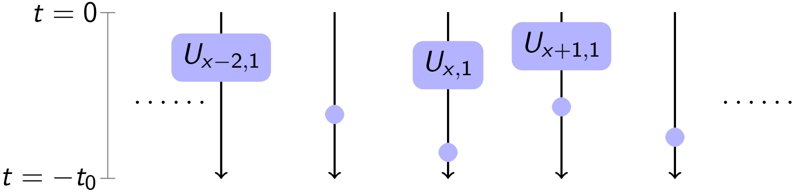

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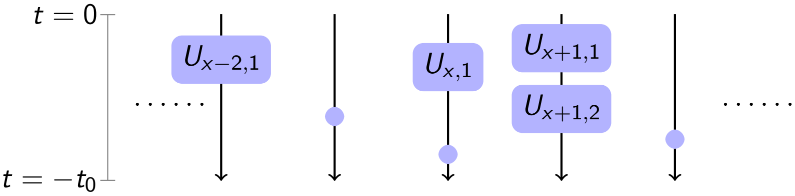

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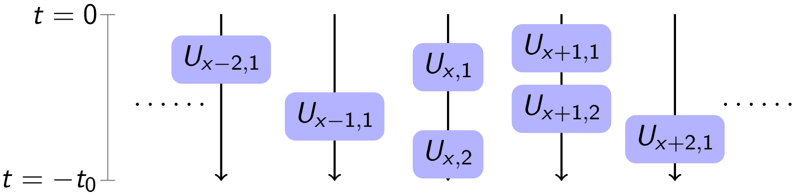
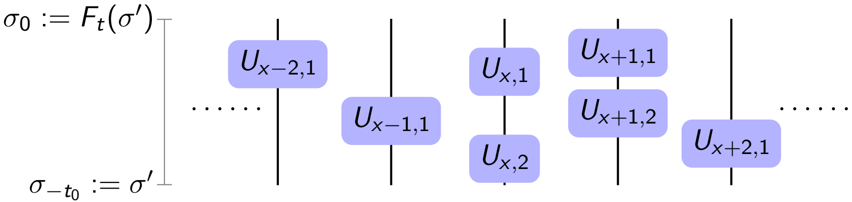
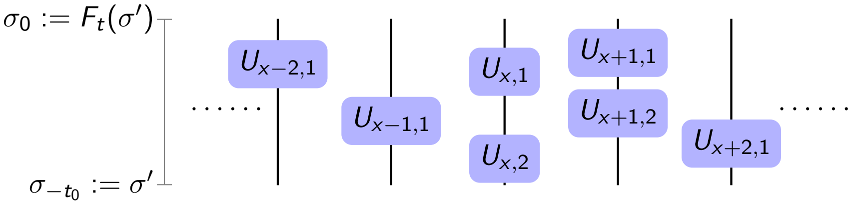


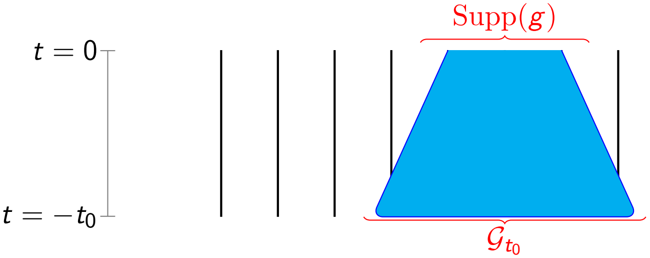


$\sigma_0 := F_t(\sigma')$
$\sigma_{-t_0} := \sigma'$
$U_{x-2,1}$
$U_{x-1,1}$
$U_{x,1}$
$U_{x,2}$
$U_{x+1,1}$
$U_{x+1,2}$
$U_{x+2,1}$



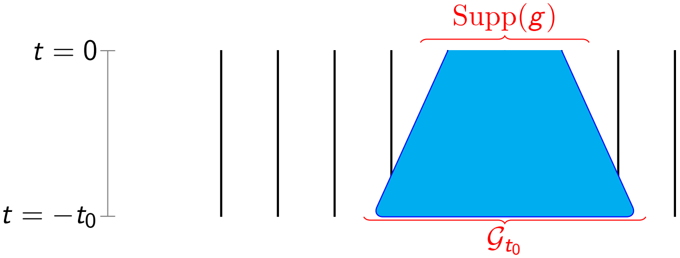
- Given N's and U's, σ_0 **deterministic** function of $\sigma_{-t} = \sigma'$,

$$\sigma_0 = F_t(\sigma')$$



- For any g local, $\mathcal{G}_t := \mathrm{Supp}_{\mathbb{Z}^d} g \circ F_t$

$$\mathcal{G}_t = \varnothing \Leftrightarrow g \circ F_t = \mathrm{cnst},$$



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$$\sup_{\sigma_0} |\mathbb{E}_{\sigma_0}[g(\sigma_t)] - \nu(g)| \leq \mathbb{P}(\mathcal{G}_t \neq \varnothing).$$

Attractivity $\Rightarrow$

$$\mathbb{P}(\mathcal{G}_t \neq \varnothing) = \mathbb{E}_+[g(\sigma_t)] - \mathbb{E}_-[g(\sigma_t)].$$

for g increasing.

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Lemma (Lubetsky-Sly '13)

For equilibrium Ising Glauber dynamics,

$$\mathbb{E}_+[\sigma_t(0)] - \mathbb{E}_-[\sigma_t(0)] = \mathbb{P}(\mathcal{G}_t = \varnothing)$$

In particular $\mathbb{P}(\mathcal{G}_t = \varnothing) \leq Ce^{-ct}$.

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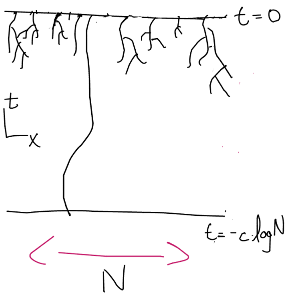
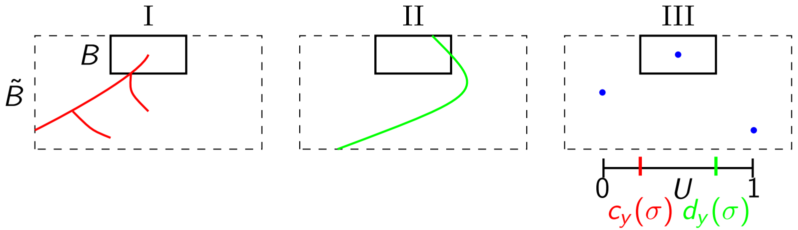


Space-Time non-Percolation

Course Graining: Fix $r, N \in \mathbb{N}$

$$B_0 = \{0, 1, \ldots, rN - 1\}^d \times (0, N],$$
$$B_{\boldsymbol{n}} = B_0 + (rNk, N\ell) \quad \text{for } \boldsymbol{n} = (k, \ell) \in \mathbb{Z}^d \times \mathbb{Z}.$$

Bad Boxes:



A box is good o/w.

Lemma

For N large enough, and ϵ small enough (depending on N)

$$\mathbb{P}(B_{\boldsymbol{n}} \text{ is bad}) \leq \mathrm{e}^{-cN}.$$

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Consider $\mathbf{X} := (\mathbf{1}\{B_{\boldsymbol{n}} \text{ is bad}\})_{\boldsymbol{n}\in\mathbb{Z}^{d+1}}$.

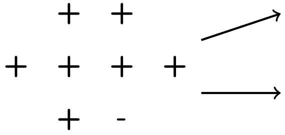
Lemma

For N large enough and ϵ small enough, $\mathbf{X}$ is dominated by subcritical site percolation ($*$-connected sense).

Open Questions

1. In uniqueness phase, is $\bar{\nu}$ Gibbs? (Redig et. al.)

2. Is attractivity essential in uniqueness phase? e.g.



3. Stability of Coexistence.

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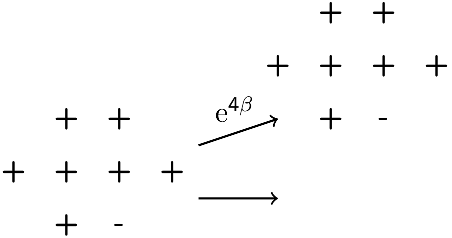

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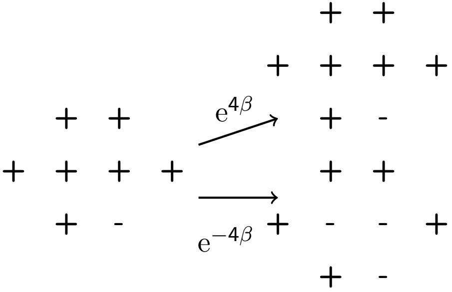

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