RANDOM WALKS WITH MEMORY, CIRM
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Gérard Ben Arous (Courant Institute): Spectral gap estimates for spin glasses.

Abstract: This is a joint work with Aukosh Jagannath (Toronto). We show that mixing for mean field spin glasses is exponentially slow in the low temperature regime for a wide class of Ising spin and spherical models and general local reversible dynamics. We introduce a notion of free energy barriers for the overlap, and, using Cheeger type bounds, prove that their existence imply that the spectral gap is exponentially small, and thus that mixing is exponentially slow. We then exhibit sufficient conditions on the equilibrium Gibbs measure which guarantee the existence of these barriers, using the notion of replicon eigenvalue and 2D Guerra Talagrand bounds. We show how these sufficient conditions cover large classes of Ising spin models for Metropolis dynamics and spherical models for Langevin dynamics. Finally, in the case of Ising spins, the 2D Guerra-Talagrand bound enable us to prove a quenched LDP for the overlap distribution, which gives us a wider criterion for slow mixing and a direct relation with the Franz-Parisi potential.

Noam Berger (Technische Universität München): Harnack inequlity for balanced environments.

Abstract: We consider random balanced, not necessarily elliptic, difference equations, and prove a Harnack inequality in the i.i.d. case. We discuss the relation of this result with random walk and percolation. We then discuss non-i.i.d. cases, and, time permitting, discuss the conjectured continuous analogue of this result. Based on joint work with M. Cohen, J.-D. Deuschel and X. Guo.

Erwin Bolthausen (Universität Zürich): How to replace random walk representations in models that don't have one?

Abstract: Random walk representations for the correlations of random fields have played a considerable role in the past. For instance, the correlations of gradient models have such a representation if the interactions are given by a convex function of the gradient. Also, correlations for gradient models with local pinning have been analyzed in great details using random walks with traps, see e.g. [1], where very detailed properties of the range of a random walk played a crucial role. In recent years, there had been interest in random fields which dont́ admit a direct random walk representation, for instance so-called membrane models. In a recent paper with Alessandra Cipriani and Noemi Kurt [2], we analyzed the decay of correlations for such fields with local pinning which is based on analytic methods. Although the method does not use random walks, it is actually close in spirit to the methods using them. Presently, the results are however much less precise than those obtained for gradient models.

References:
[1] Bolthausen, E., and Velenik, Y.: Critical behavior of the massless free field at the depinning transition. Commun. Math. Phys. 223, 161203 (2001).
[2] Bolthausen, E., Cipriani, A., and Kurt, N.: Exponential Decay of Covariances for the Supercritical Membrane Model. Comm. Math. Phys. 2017,

Manuel Cabezas (Universidad Católica de Chile): The ant in the tree.

Abstract: We will analyze the behavior of the simple random walk on 3 different random critical trees: the incipient infinite cluster, the invasion percolation cluster and a critical Galton Watson tree. In particular, we will be interested in the projection of the random walk to certain simple subtrees. In the case of the IPC and IIC we will analyze the projection onto the backbone, while in the case of the critical Galton-Watson trees, we will analyze the projection to some reduced-subtrees.

Nicholas Crawford (Technion Israel Institute of Technology): Stability of the uniqueness phase under non-equilibrium perturbations.

Abstract: In this talk, I will discuss recent work with W. de Roeck on the following natural question: Given an interacting particle system are the stationary measures of the dynamics stable to small (extensive) perturbations? In general, there is no reason to believe this is so and one must restrict the class of models under consideration in one way or another. As such, I will focus in this talk on the simplest setting for which one might hope to have a rigorous result: attractive Markov dynamics (without conservation laws) relaxing at an exponential rate to its unique stationary measure in infinite volume. In this case we answer the question affirmatively.

As a consequence we show that ferromagnetic Ising Glauber dynamics is stable to small, non-equilibrium perturbations in the entire uniqueness phase of the inverse temperature/external field plane. It is worth highlighting that this application requires new results on the (exponential) rate of relaxation for Glauber dynamics defined with non-zero external field.

Jean-Dominique Deuschel (Technische Universität Berlin): Quenched tail estimate for the random walk in random scenery and in random layered conductance.

Abstract: We discuss the quenched tail estimates for the random walk in random scenery. The random walk is the symmetric nearest neighbor walk and the random scenery is assumed to be independent and identically distributed, non-negative, and has a power law tail. We identify the long time aymptotics of the upper deviation probability of the random walk in quenched random scenery, depending on the tail of scenery distribution and the amount of the deviation. The result is in turn applied to the tail estimates for a random walk in random conductance which has a layered structure.

Margherita Disertori (Universität Bonn): Some results on history dependent stochastic processes.

Abstract: Edge reinforced random walk and vertex reinforced jump processes are history dependent stochastic processes, where the particle tends to come back more often

on sites it has already visited in the past. For a particular scheme of reinforcement these processes are mixtures of reversible Markov chains whose mixing measure can be related to a non-linear sigma model introduced in the context of random matrices. This relation allows to prove, in particular, transience for weak reinforcement of both processes, in dimension larger or equal to 3. I will give an overview on these models and explain some recent results.

Anna Erschler (Ecole Normale Superieure): Almost invariance for n-step dis-rtibutions of random walks.

Alessandra Faggionato (Sapienza Universitâ di Roma): 1d Mott variable range hopping.

Abstract: Mott variable range hopping is a basic mechanism of electron transport in strongly disordered solids in the regime of strong Anderson localization. In a mean field approximation, the mathematical model is given by a suitable random walk on a simple point process of $\mathbb{R}^d$ with points marked by energy random variables. We will focus on the 1d case and on the physical Mott law concerning the conductivity at low temperature. We recall some previous results obtained with P. Caputo and discuss more in detail the effect of applying an external uniform force field, by presenting results recently obtained in collaboration with N. Gantert and M. Salvi. In particular, we present conditions assuring ballisticity and sub-ballisticity, which reduce to a full characterization in the case of a renewal simple point process. Moreover, we derive the Einstein relation and discuss the linear response for a suitable class of observables.

Alex Fribergh (Université de Montréal): The ant in the labyrinth.

Abstract: One of the most famous open problem in random walks in random environments is to understand the behavior of a simple random walk on a critical percolation cluster, a model known as the ant in the labyrinth. I will present new results on the scaling limit for the simple random walk on the critical branching random walk in high dimension. In the light of lace expansion, we believe that the limiting behavior of this model should be universal for simple random walks on critical structures in high dimensions. Joint with Gérard Ben Arous and Manuel Cabezas.

Alan Hammond (University of California, Berkeley): The weight, geometry and coalescence of scaled polymers in Brownian last passage percolation.

Abstract: In last passage percolation (LPP) models, a random environment in the two-dimensional integer lattice consisting of independent and identically distributed weights is considered. The weight of an upright path is said to be the sum of the weights encountered along the path. A principal object of study are the polymers, which are the upright paths whose weight is maximal given the two endpoints. Polymers move in straight lines over long distances with a two-thirds exponent dictating fluctuation. It is natural to seek to study collective polymer behaviour in scaled coordinates that take account of this linear behaviour and the two-third exponent-determined fluctuation. We study Brownian LPP, a model whose integrable properties find an attractive probabilistic expression. Building on a study arXiv:1609.02971 concerning the decay in probability for the existence of several near polymers with common endpoints, we demonstrate that the probability that there exist k disjoint polymers across a unit box in scaled coordinates has a superpolynomial decay rate in k. This result has implications for the Brownian regularity of the scaled polymer weight profile begun from rather general initial data.

Dmitry Ioffe (Technion): Non-colliding random walks under area tilts.

Abstract: Ensembles of non-colliding walks under area tilts naturally show up as effective models of level lines in various low temperature 2+1 SOS-type interfaces. Interesting and physically relevant scaling regimes correspond to tilts tending to zero and number of walks tending to infinity. I shall try to describe existing results, conjectures and open questions regarding the matter.

Based on joint works and discussions with Pietro Caputo, Senya Shlosman, Yvan Velenik and Vitali Wachtel.

Gady Kozma (Weizmann Institute): Internal diffusion-limited aggregation with random starting points.

Abstract: We consider a model for a growing subset of a euclidean lattice (an "aggregate") where at each step one choose a random point from the existing aggregate, starts a random walk from that point, and adds the point of exit to the aggregate. We show that the limiting shape is a ball. Joint work with Itai Benjamini, Hugo Duminil-Copin and Cyril Lucas.

Takashi Kumagai (Kyoto University): Time changes of stochastic processes : Convergence and heat kernel estimates.

Abstract: In recent years, interest in time changes of stochastic processes according to irregular measures has arisen from various sources. Fundamental examples of such time-changed processes include the so-called Fontes-Isopi-Newman (FIN) diffusion and fractional kinetics (FK) processes, the introduction of which were partly motivated by the study of the localization and aging properties of physical spin systems, and the two-dimensional Liouville Brownian motion, which is the diffusion naturally associated with planar Liouville quantum gravity.

This FIN diffusions and FK processes are known to be the scaling limits of the Bouchaud trap models, and the two-dimensional Liouville Brownian motion is conjectured to be the scaling limit of simple random walks on random planar maps.

In the first part of my talk, I will provide a general framework for studying such time changed processes and their discrete approximations in the case when the underlying stochastic process is strongly recurrent, in the sense that it can be described by a resistance form, as introduced by J. Kigami. In particular, this includes the case of Brownian motion on tree-like spaces and low-dimensional self-similar fractals.

In the second part of my talk, I will discuss heat kernel estimates for (generalized) FIN diffusions and FK processes on metric measure spaces.

This talk is based on joint works with D. Croydon (Warwick) and B.M. Hambly (Oxford) and with Z.-Q. Chen (Seattle), P. Kim (Seoul) and J. Wang (Fuzhou).

Françoise Pène (Université de Bretagne Occidentale): Persistence probabilities for processes with stationary increments.

Abstract: We develop a new approach to study the asymptotics of persistence probabilities. Our approach is adapted to the case of processes with stationary increments. It enables the study of the persistence exponent without exponential moment assumptions. It also provides sharp asymptotics for persistence probabilities when the increments are bounded. This is a joint work with Frank Aurzada and Nadine Guillotin-Plantard.

Yuval Peres (Microsoft Research): Self-interacting martingales and uniform spanning forests.

Abstract: In the first half of the talk, I will survey results and open problems on transience of self-interacting martingales. In particular, I will describe joint works with S. Popov, P. Sousi, R. Eldan and F. Nazarov on the tradeoff between the ambient dimension and the number of different step distributions needed to obtain a recurrent process. In the second, unrelated, half of the talk, I will present joint work with Tom Hutchcroft, showing that the component structure of the uniform spanning forest in $\mathbb{Z}^d$ changes every dimension for $d > 8$. This sharpens an earlier result of Benjamini, Kesten, Schramm and the speaker (Annals Math 2004), where we established a phase transition every four dimensions. The proofs are based on a the connection to loop-erased random walks.

Timo Seppäläinen (University of Wisconsin, Madison): Random walk in random environment and the Kardar-Parisi-Zhang class.

Abstract: This talk describes 1+1 dimensional directed random walks in correlated random environments that obey the Kardar-Parisi-Zhang 2/3 fluctuation exponent. Such RWRE processes arise from two sources: (i) as limits of quenched polymer measures when the length of the polymer path is taken to infinity, and (ii) as limits of RWRE in an IID environment conditioned on an atypical velocity. In both cases results currently exist only for exactly solvable models. This talk is based on joint papers with Márton Balázs (Bristol), Nicos Georgiou (Sussex), Firas Rassoul-Agha (Utah), and Atilla Yılmaz (Koç).

Vladas Sidoravicius (IMPA Brazil): The mathematics of DLA.

Perla Sousi (University of Cambridge): Random walks on dynamical percolation.

Abstract: We study the behaviour of random walk on dynamical percolation. In this model, the edges of a graph are either open or closed and refresh their status at rate Œº, while at the same time a random walker moves on Gat rate 1, but only along edges which are open. On the d-dimensional torus with side length n, when the bond parameter is subcritical, the mixing times for both the full system and the random walker were determined by Peres, Stauffer and Steif. I will talk about the supercritical case, which was left open, but can be analysed using evolving sets (joint work with Y. Peres and J. Steif).

Thomas Spencer (Institute for Advanced Study, Princeton): Introduction to hyperbolic sigma models.

Abstract: This talk will introduce two statistical mechanics models on the lattice. The spins in these models have a hyperbolic symmetry. Correlations for these models can be expressed in terms of a random walk in a highly correlated random environment. In the SUSY hyperbolic case these walks are closely related to the vertex reinforced jump process and to the edge reinforced random walk. (Joint work with M. Disertori and M. Zirnbauer.)

Balint Virag (University of Toronto): Random Sorting Networks.

Abstract: A sorting network is a shortest way to get from the reverse permutation to the identity using nearest neighbor swaps. There has been a lot of recent work on our conjecture with Angel, Holroyd and Romik: the permutation matrix of a uniformly chosen sorting network looks like a sphere. I will explain.

Ofer Zeitouni (Weizmann Institute): Homogenization for controlled random walk in random potential - the one dimensional case.

Abstract: We consider a control problem, suggested by Elena Kosygina, where the objective is the exponential of a sum over path $\{X_i, i = 1, ..n\}$ of a random potential $V(\theta_{X_i}\omega)$, plus θX_n. Taking the normalized log of the objective, we prove homogenization, with an effective hamiltonian depending on the parameters of the problem as well as θ, and given in terms of the tilted free energy for the problem without control. We identify the regimes in which the effective Hamiltonian is not convex (as function of θ). Based on joint work with Atilla Yilmaz.