Joël Rivat (Aix-Marseille University) Mini-course *Introduction to Analytic Number Theory*

By taking as a common thread some famous results and conjectures concerning prime numbers, the aim of this mini-course is to present mathematical tools which allow us to cope with them. To obtain the Prime Number Theorem, we will study arithmetic functions, Dirichlet series, properties of Riemmann ζ function and its zeros. We will show the optimal form of the large sieve using the Beurling-Selberg function and we will apply it to twin prime numbers. We will show Vinogradov's theorem (each odd number sufficiently large is the sum of three prime numbers) using the Hardy and Littlewood method. We will learn to estimate trigonometric sums using van der Corput's method which has numerous applications (complexity calculations, asymptotic formulas, discrepancy estimates). Finally, we will speak about Möbius disjointness of dynamical systems in the context of Sarnak's conjecture.

1 Prime numbers

- Tchebychev's inequality,
- Arithmetic functions,
- Dirichlet series,
- Riemann zeta function,
- Riemann's functional equation for zeta,
- The zeros of zeta,
- The Prime Number Theorem.

2 The large sieve

- The analytic forme of the large sieve,
- The Beurling-Selberg function,
- The sieve,

- The arithmetic form of the large sieve,
- Applications: twin prime numbers, sums of characters.

3 The van der Corput method

- Estimates of exponential sums,
- Applications: asymptotic formulas, discrepancy estimates.

4 Vinogradov's theorem

- The circle method,
- Major and minor arcs,
- Vaughan's identity,
- Sums of type I and II.

5 Sarnak's conjecture

- Möbius disjointness of dynamical systems - Sarnak's conjecture