

Vitaly Bergelson (Ohio State University)

Mini-course *Mutually enriching connections between ergodic theory and combinatorics*

- The early results of Ramsey theory.
Hilbert's irreducibility theorem, Dickson-Schur work on Fermat's equation over finite fields, van der Waerden's theorem, Ramsey's theorem and its rediscovery by Erdos and Szekeres.
- Three main principles of Ramsey theory
First principle: Complete disorder is impossible. Second principle: Behind every 'Partition' result there is a notion of largeness which is responsible for a 'Density' enhancement of this result. Third principle: The sought-after configurations which are always to be found in large sets are abundant.
- Furstenberg's Dynamical approach.
Partition Ramsey theory and topological dynamics Dynamical versions of van der Waerden's theorem, Hindman's theorem and Graham-Rothschild-Spencer's geometric Ramsey.
Density Ramsey theory and Furstenberg's correspondence principle
Furstenberg's correspondence principle. Ergodic Szemerédi's theorem. Polynomial Szemerédi theorem. Density version of the Hales-Jewett theorem.
- Stone-Cech compactifications and Hindman's theorem
Topological algebra in Stone-Cech compactifications. Proof of Hindman's theorem via Poincaré recurrence theorem for ultrafilters.
- IP sets and ergodic Ramsey theory
Applications of IP sets and idempotent ultrafilters to ergodic-theoretical multiple recurrence and to density Ramsey theory. IP-polynomial Szemerédi theorem.
- Open problems and conjectures

If time permits:

- The nilpotent connection
- Ergodic Ramsey theory and amenable groups