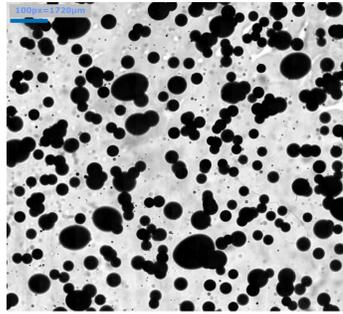


Motivation

- **Industrial problematic:** need to control and optimize two-phase processes (liquid-liquid extraction, gaz-liquid reactor, etc.).

Objectives: Determine useful characteristics (particle size distribution, exchange surface, morphology) of the dispersed phase at high hold-up

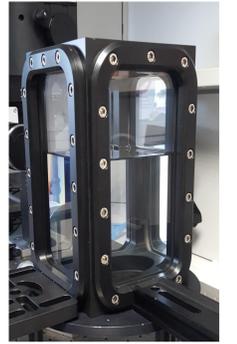
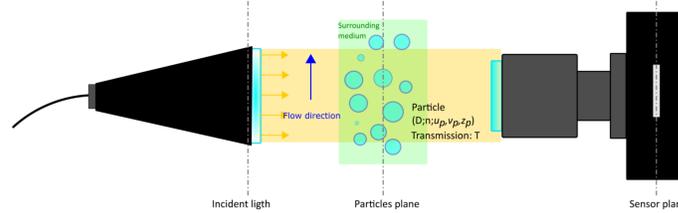
- **Difficulty:** dense two-phase flows leads to overlaps of projected particles.



Gaz-liquid flow (air bubbles in water)

Experimental Setup

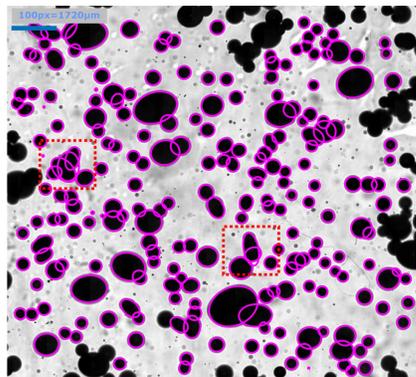
- A glass reactor (plane and sintered)
 - Gaz-liquid flow (e.g.: air+water)
- Collimated light source
- Rapid CMOS camera with high resolution (1MPixel, 12bits)



Methodology

1) Image processing method (de Langlard et al. (2016))

- Pre-processing to reduce noise and enhance the contrast of the image (median filtering, subtraction of the mean image, etc.),
- Edge detection of cluster of particles
- Segmentation (individualization) of particles within the clusters.



Limitations:

- Case with very high hold-up hardly handled (+60% of the image),
- Retrieval of 2D information only.
- Bad detections when the particle shape deviate from an ellipse,
- Complete superimposition of projections not processed,

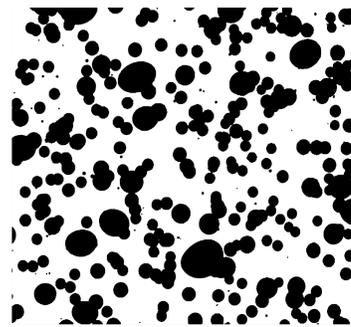
=> Use of stochastic geometry

2) Modelling through stochastic geometry

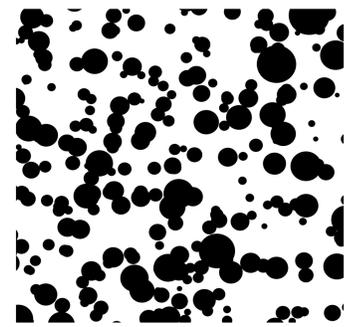
The main idea: find a 3D model that replicates the geometry and spatial characteristics of the 2D observed data

Several steps

- Use a 3D finite Matèrn model
- Make inference on the model either directly (trial-and-error estimation or method of moments) or through a 2D Quermass-interaction model ((Moller et al. (2010), Dereudre et al. (2014))
- Evaluate the accuracy of the model on calibrated silica spheres images



Real binary image



Orthogonal projection of 3D Matèrn model ($W = 10 \times 10 \times 10$, $\lambda = 0.09$, $r \sim U([0.1, 0.45])$)

Finite Matèrn Model

Problem: how to define the projection of a point process?

- Stationary point process => Garcia (2000)
- Use a finite point process

Define a 3D marked point process of hardcore spheres with random radius in a bounded window following Matèrn thinning procedure

- Consistent with the application
- Flexible for defining interaction (repulsion) with the boundary of the window (through a function U)

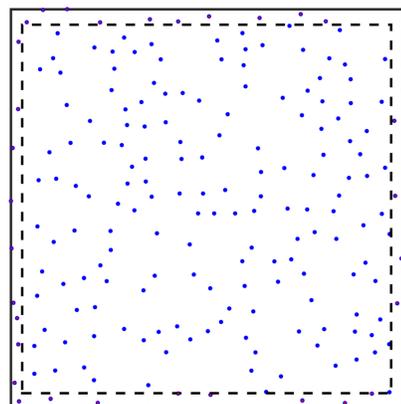
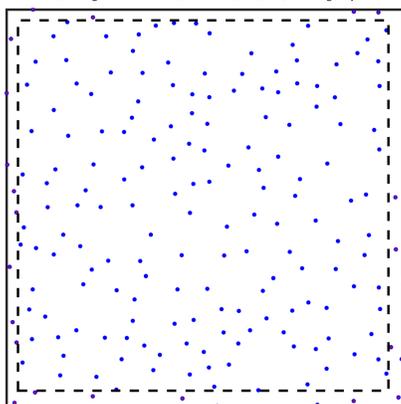
Retention probability of point located at x and with radius r

$$g(x, r) = \int_0^1 \exp\left(-\left(U(x, r, w) + \lambda(1-w) \int_0^{+\infty} v(B(x, r+y) \cap W) F(dy)\right)\right) dw$$

where $U(x, r, w)$ (energy function) represents the (first-order) interaction between the point located at x with radius r and weight w .

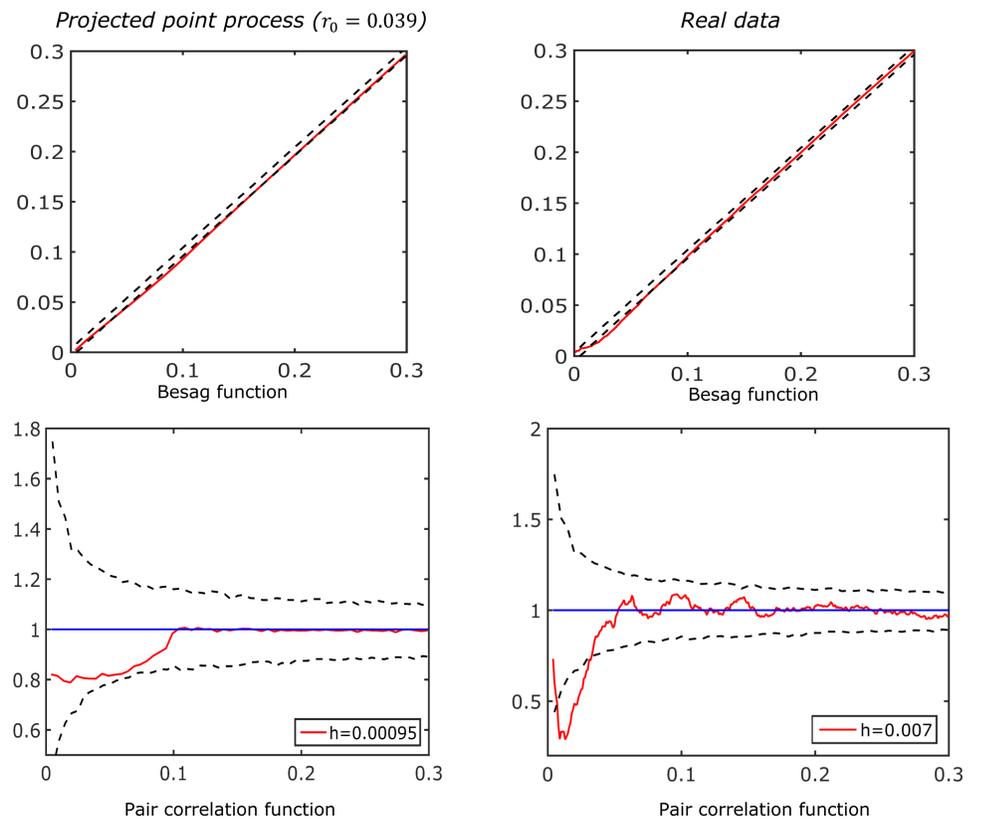
$$U(x, r_0, w) = \lambda(1-w)v(B(x, 2r_0) \setminus W)$$

$$U(x, r_0, w) = 0$$



Exploratory Analysis of the Projected Point Process

Comparison of some second-order characteristics function between the projected process and the point process of detected centers by the proposed algorithm



Conclusions & Outlook

- First estimation achievement of the size distribution of overlapping elliptical particles (in 2D) using segmentation methods
- Definition of a finite Matèrn model including first-order interaction with the boundary
- Exploratory analysis brings confidence on using the proposed 3D model to characterize a two-phase flows
- **Outlook:**
 - Propose an inference procedure to determine parameters of the 3D Matèrn model,
 - Extension to ellipsoidal shape.

References

- [1] de Langlard, M., et al (2016). A Multiscale Method for Shape Recognition of Overlapping Elliptical Particles. 23rd International Conference in Pattern Recognition (ICPR), Cancun, Mexico.
- [2] Moller, J. et al (2010). Likelihood Inference for Unions of Interacting Discs. Scandinavian Journal of Statistics, vol. 37, pp. 365-381.
- [3] Dereudre, D. et al. (2014). Estimation of the Intensity Parameter of the Germ-Grain Quermass-Interaction Model when the Number of Germs is not Observed. Scandinavian Journal of Statistics, vol. 41, pp. 809-829.
- [4] Garcia, N. L. et al. (2000). Spatial Point Processes and the Projection Method. Progress in Probability, vol. 60, pp. 270-298.
- [5] Mansson, M. et al. (2002). Random Patterns of Nonoverlapping Convex Grains. Advances in Applied Probability, vol. 34, n^o4, pp. 718-738.