Convergence of infinite branch directions of RST to the uniform distribution

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Introduction

We study the so-called Radial Spanning Tree (RST) introduced by Baccelli & Bordenave '07.

Our aim is to complete known information on semi-infinite branches of RST.

Plan

(1) Radial Spanning Tree
(2) Known facts on semi-infinite branchs
(3) New results
(4) Proof (sketch)

RST

Let $\mathcal{N}$ be an homogeneous PPP with intensity 1.

RST is a directed tree $\mathcal{T}$ rooted at $\mathcal{O}$ and defined by

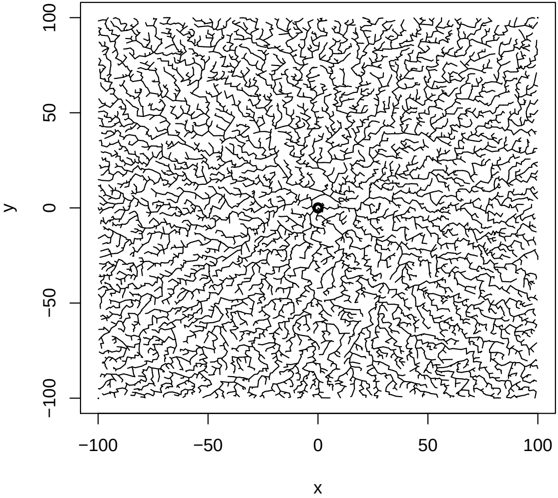
- The vertex set: $\mathcal{N} \cup \{O\}$.
- The edge set: each $X \in \mathcal{N}$ is linked to the closest $Y \in B(O, |X|)$.
  Y is unique a.s.

Notation : $Y = Anc(X)$
T_X^{out} is the subtree of $\mathcal{T}$ rooted at X.

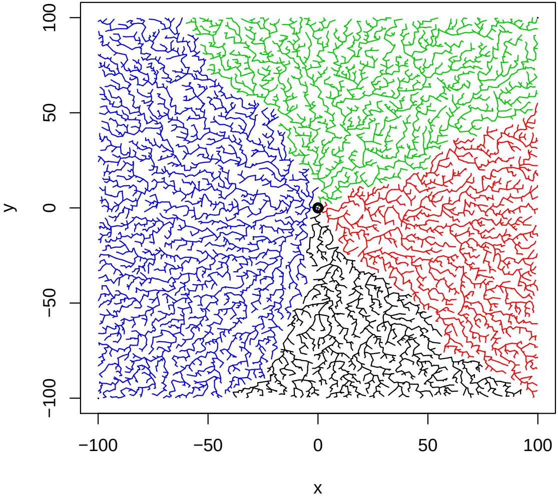
Remark. A.s. each vertex $X \in \mathcal{N} \cup \{O\}$ has a finite degree.

$\Rightarrow$ There exists at least one semi-infinite branche in $\mathcal{T}$.

Simulation of RST



In color...



Semi-infinite branches of RST

$(X_n)_{n\in\mathbb{N}}$ admits an asymptotic direction $\theta \in [0;2\pi)$ if $\lim_{n\to\infty} \frac{X_n}{\|X_n\|} = e^{i\theta}$.

Theorem (Baccelli & Bordenave '07)

RST $\mathcal{T}$ satisfies :

1. *a.s. each semi-infinite branche of $\mathcal{T}$ admits an asymptotic direction.*
2. *a.s. for each $\theta \in [0;2\pi)$, there exists a semi-infinite branche of $\mathcal{T}$ having asymptotic direction θ.*
3. *for each $\theta \in [0;2\pi)$ (determinist), there exists a unique semi-infinite branche of $\mathcal{T}$ having asymptotic direction θ.*

Semi-infinite branches of RST

ν_r : number of semi-infinite branches of $\mathcal{T}$
which intersect $C(O,r) = rS^1 = \{x \mid \|x\| = r\}$.

Rem: • $\nu_r \overset{\text{p.s.}}{\to} \infty$

• The mean number of edges of $\mathcal{T}$ intersecting $C(O,r)$ is of order r.

Theorem (Baccelli, Coupier & Tran '14)

Sub-linearity of the mean number of semi-infinite branches :

$$\mathbb{E}\,\nu_r = o(r) \;\; \textit{quand} \;\; r \to \infty.$$

Conjecture (Coupier '16) : $\forall \varepsilon > 0$, $\nu_r = o(r^{3/4+\varepsilon})$ a.s. and in L^1.

New results

Let A be an arc of S^1. We set

$$Ext(A, \varepsilon, s) = \left\{ x \mid \|x\| \geq s, \ \frac{x}{\|x\|} \in S^1 \setminus A^\varepsilon \right\}.$$

Let $\Omega(r, \varepsilon)$ be the event:

$\forall s \geq r, \forall A$ arc of S^1 and $\forall X \in \mathcal{N}$ such that $[X, Anc(X)] \cap sA \neq \emptyset$ the branche T_X^{out} does not depend on arbitrary changements of $\mathcal{N} \cap Ext(A, \varepsilon, s)$.

Proposition (Coupier & D '16)

For each $\varepsilon > 0 \quad \mathbb{P}\{\Omega(r, \varepsilon)\} \to 1$ *as* $r \to \infty$.

New results

Let μ_r be the measure on S^1 defined by $\mu_r(A) = \frac{\nu_r(A)}{\nu_r}$, where $\nu_r(A)$ is the number of semi-infinite branches of $\mathcal{T}$ which intersect rA.

Theorem (D. & Coupier '17)

The measure μ_r converges in probability, as $r \to \infty$, to the uniform distribution on S^1.

Sketch of the proof

1-st step:
From Th. [Coupier & D '16] it follows that for all disjoint arcs A and B the random variables $\mu_r(A)$ and $\mu_r(B)$ are asymptotically independent as $r \to \infty$.

2-nd step is based on the following

Proposition

Let (μ_n) be a sequence of random probability measures on S^1 such that

- *The distribution of μ_n is invariant under rotations.*
- *For all disjoint arcs A and B the random variables $\mu_n(A)$ and $\mu_n(B)$ are asymptotically independent as $n \to \infty$.*

Then μ_n converges in probability, as $n \to \infty$, to the uniform distribution on S^1.