French-American Conference on Nonlinear Dispersive PDEs - CIRM 2017

Jorge Drumond Silva

CAMGSD

Instituto Superior Técnico

Universidade de Lisboa, Portugal

jsilva@math.tecnico.ulisboa.pt

Schrödinger-Debye (w/ A. Corcho)

$$\begin{cases} iu_t + \frac{1}{2}\Delta u = uv, \\ \mu v_t + v = \lambda |u|^2, \quad \mu > 0,\ \lambda = \pm 1 \end{cases}$$

Quadratic Schrödinger System (w/ A. Corcho, S. Correia, F. Oliveira)

$$\begin{cases} iu_t + \Delta u + \overline{u}v = 0 \\ 2iv_t + \Delta v - \beta v + \frac{1}{2}u^2 = 0 \end{cases}$$

NLS with Potential (w/ R. Carles)

$$iu_t + \frac{1}{2}\Delta u = V(t,x)u + |u|^{2\sigma}u$$

The Einstein-Maxwell-scalar field system

w/ J. Costa, P. Girão, J. Natário

- Simplest hyperbolic matter model that does not form singularities in the absence of gravity: massless scalar field

The equations for a gravitating massless scalar field ϕ in a sourceless electromagnetic field F with a cosmological constant Λ are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 2T_{\mu\nu}$$

$$\Box_g \phi = 0$$

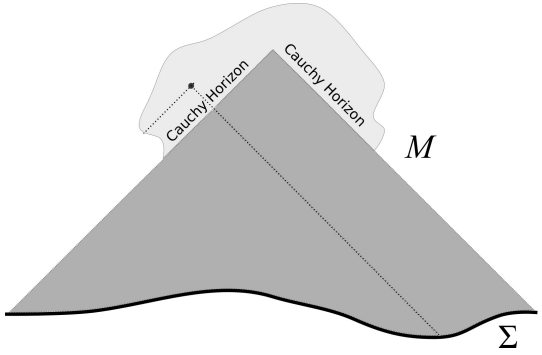
$$dF = d^* F = 0$$

$$T_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} \partial_\alpha \phi \, \partial^\alpha \phi \, g_{\mu\nu} + F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu}$$

Determinism and Global Uniqueness

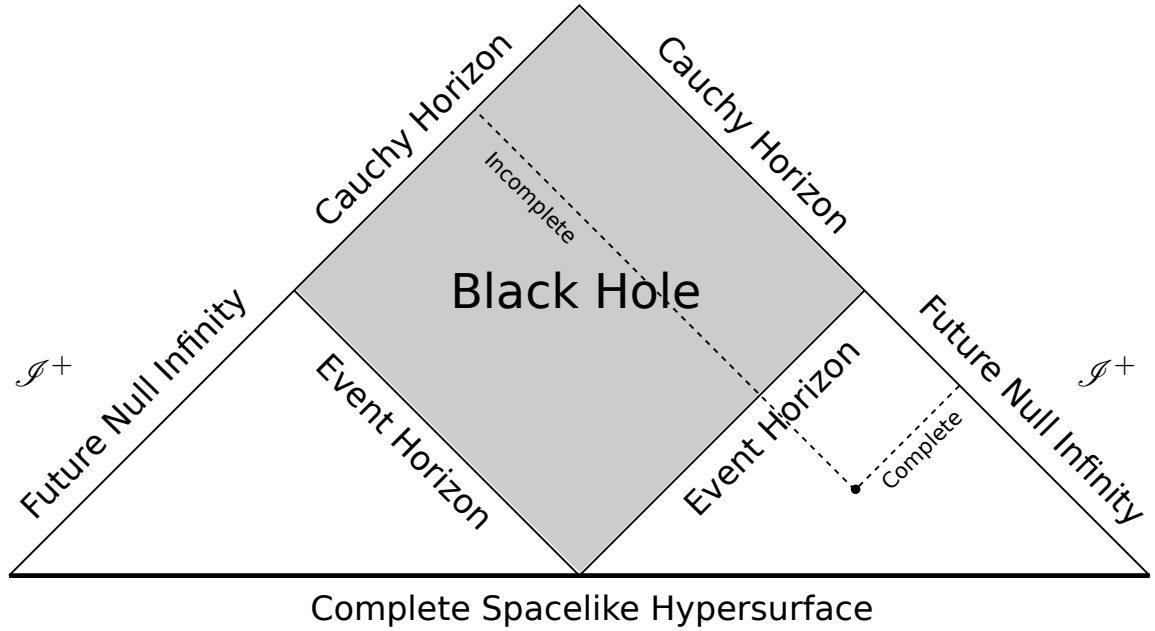
Uniqueness of the solutions of the Einstein equations, as for any hyperbolic PDEs, is a result of a domain of dependence property.

Consider the future Maximal Globally Hyperbolic Development for the IVP associated to a certain Einstein-matter system starting from a complete spacelike Cauchy initial surface.



If this solution is extendible then global hyperbolicity (i.e. global uniqueness) fails at the extension across the Cauchy horizon.

Determinism breaks down!



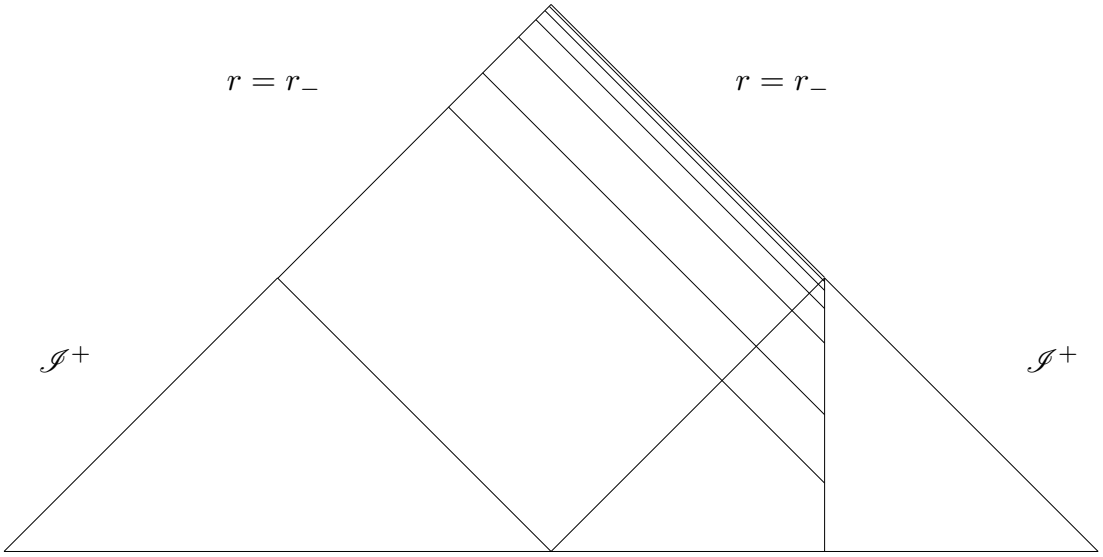
Solutions can be continued smoothly (even analytically!) across the Cauchy horizon.

Global uniqueness fails without loss in regularity.

But...

Blue Shift Effect: R. Penrose, M. Simpson [Int. Jour. Theor. Phys., 1973]

The Cauchy horizon should be unstable to perturbations



Mass Inflation: W. Israel, E. Poisson [Phys. Rev. D, 1990] used nonlinear heuristic analysis to argue that, for more generic solutions, with no cosmological constant $\Lambda = 0$, the Hawking mass $\varpi \to \infty$ at the Cauchy horizon.

The Strong Cosmic Censorship Conjecture

(R. Penrose [1972]) *Generic* complete asymptotically flat initial data for reasonable matter models have MGHD which are locally inextendible as (suitably regular) Lorentzian manifolds.

i.e.

For generic initial data, the solution is globally unique wherever it is appropriately defined.