Flash Presentation

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Background

- **Affiliation** University of Texas at Austin
- **Position** Graduate student
- **Year** Rising third-year
- **Supervisor** Professor Nataša Pavlović

Current Research Interests

- Asymptotic analysis
- Dispersive PDE arising from physical contexts (e.g. water waves)

The Davey-Stewartson System

The Davey-Stewartson system on $\mathbb{R} \times \mathbb{R}^2$ reads in simplied form

$$ic_t + \partial_{x_1}^2 c \pm \partial_{x_2}^2 c = \mu |c|^2 c - d_{x_1} c \tag{2.1}$$

$$\partial_{x_1}^2 d \pm \partial_{x_2}^2 d = -(|c|^2)_{x_1} \tag{2.2}$$

where $\mu \in \{\pm 1\}$.
Above, *c* is a complex-valued amplitude and *d* is a real-valued velocity potential.

Ghidalia and Saut classification of the system:

- **(+,+)** Elliptic-Elliptic
- **(-,+)** Hyperbolic-Elliptic
- **(+,-)** Elliptic-Hyperbolic
- **(-,-)** Hyperbolic-Hyperbolic

Small data theory

We may rewrite the elliptic-elliptic DS system as a nonlocal nonlinear Schrödinger-type equation

$$iu_t + \Delta u = \mu|u|^2 u + \frac{\partial_1^2}{\Delta} u \tag{2.3}$$

The equation has L^2-critical scaling $u_\lambda(t,x) := \lambda u(\lambda^2 t, \lambda x)$.

Small-data theory and LWP established by Ghidalia and Saut via fixed-point argument using Hölder's inequality, Strichartz estimates, and the Calderón-Zygmund theorem. In particular, one has the blow-up criterion

$$\lim_{T\uparrow T_{max}} \|u\|_{L^4_{t,x}([0,T]\times\mathbb{R}^2)} = \infty \tag{2.4}$$

GWP and Scattering of EE DS system at Critical Regularity

Theorem

If $\mu = 1$ or if $\mu = -1$ and $M(u) < \|Q\|_{L^2}$, then solutions of the EE DS system are global and satisfy the uniform space-time estimate

$$\|u\|_{L^4_{t,x}} \leq C(\|u_0\|_{L^2}) \tag{2.5}$$

1. **Strategy of proof**
   - Concentration compactness/rigidity roadmap of Kenig-Merle
   - Long-time Strichartz estimate technique of Dodson
   - Improved bilinear Strichartz estimates
   - Frequency-localized interaction Morawetz-type estimate
2. **Difficulties**
   - Application of bilinear Strichartz estimates to terms of form $u_{hi}^2 \frac{\partial_1^2}{\Delta}(u_{lo}^2)$
   - No interaction Morawetz estimate for DS system

3D Gravity-Capillary Waves

The Zakharov/Craig-Sulem formulation of the water waves problem for an incompressible, irrotational fluid in a domain of infinite horizontal expanse and finite depth is

$$\partial_t h = G(h)\psi \tag{3.1}$$

$$\partial_t \psi = -gh + \sigma H(h) - \frac{1}{2}|\nabla\psi|^2 + \frac{(G(h)\psi + \nabla h \cdot \nabla\psi)^2}{2(1+|\nabla h|^2)} \tag{3.2}$$

The linearization of the system about the rest state $(\underline{h}, \underline{\psi}) = (0, 0)$ is the complex-valued dispersive equation

$$\partial_t u + i\Lambda u = 0 \tag{3.3}$$

where $\Lambda := \sqrt{|\nabla|\tanh(|\nabla|)(g + \sigma|\nabla|^2)}$ and $u := h + i\Lambda^{-1}|\nabla|\tanh(|\nabla|)\psi$.

Rigorous Justification of Modulation Approximation to GWW

Normalize $g = 1$ and ignore surface tension. Let $0 < \epsilon \ll 1$ be a small parameter. By the method of multiple scales, one can seek a wave packet solution of the Z/CS system with basic wave number $k_0 = (|k_0|, 0)$ and dispersion relation $\omega_0 = |k_0| \tanh(|k_0|)$, which is slowly modulated in space and time and travelling parallel to the x-axis.

(3.4) $$\epsilon \begin{pmatrix} i\omega_0 c(\tau, X_1, X_2) e^{i(k_0 \cdot x - \omega_0 t)} + c.c. \\ c(\tau, X_1, X_2) e^{i(k_0 \cdot x i \omega_0 t)} + c.c. + d(\tau, X_1, X_2) \end{pmatrix} + \mathcal{O}(\epsilon^2),$$

where c_g is the group velocity and $\tau = \epsilon^2 t$, $X_1 = \epsilon(x - c_g t)$, and $X_2 = \epsilon x_2$ are slow variables.

Plugging in ansatz to WW equation, one finds (c, d) satisfies a hyperbolic-elliptic Davey-Stewartson system with respect to the slow variables.

Goal

Prove that that the multiple scales approximate solution approximates a true solution of the WW equations with small error over long times.