

About periodic waves of dispersive equations

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5 minutes CIRM

Theme 1 : dynamics near periodic waves.

General **aims** :

- 1 With suitable meanings
spectral stability implies dynamical stability
for periodic waves of dispersive equations.
- 2 Describe asymptotic behavior through averaged dynamics
obeying modulation equations.

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- 1 With suitable meanings
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Known facts :

- 1 Periodic waves of parabolic systems.
See Johnson-Noble-Rodrigues-Zumbrun, *Inventiones Math.* 2014.
- 2 Linearized (KdV).
See Rodrigues, *forthcoming* 2017.

Space-modulated stability.

Allow for **local** resynchronization

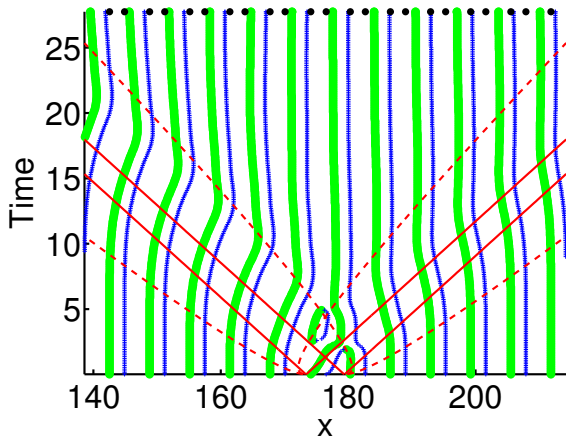
$$\delta_{\mathcal{H}}(u, v) = \inf_{\Psi \text{ one-to-one}} \|u \circ \Psi - v\|_{\mathcal{H}} + \|\partial_x(\Psi - \text{Id}_{\mathbf{R}})\|_{\mathcal{H}}.$$

Direct simulation of a **diffusive** case : space-time diagramm.

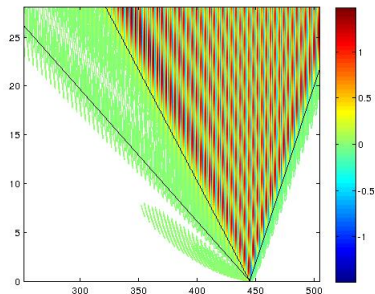
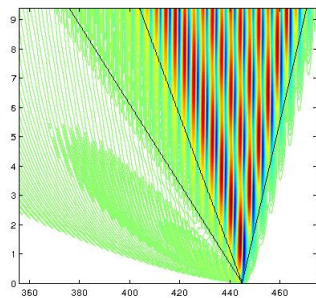
About a stable wave of a parabolic equation.

Barker-Johnson-Noble-Rodrigues-Zumbrun, *Phys. D* 2013.

Peaks.
Troughs.
Averaging.



Linearized dynamics of (KdV) : a direct simulation.

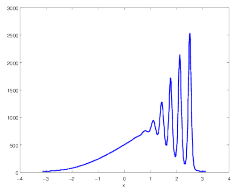
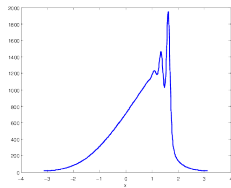
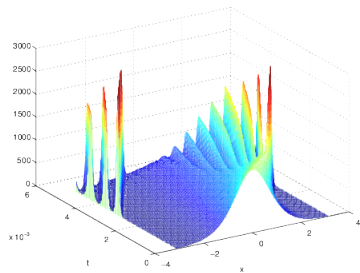
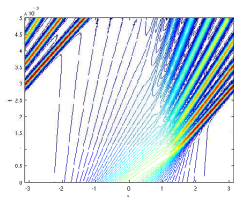


Rodrigues, *forthcoming* 2017.

Right : larger observation scale.

Left : smaller observation scale.

Theme 2 : dispersive shocks.



Known facts : completely **integrable** equations.

See **Lax-Levermore**, **Deift-Zhou**, **Grava**, **Teschl**...

First goal : build **ansatz** in **general systems**. With **Benzoni-Gavage**.