"Flash" Dispersion on Trees

Liviu Ignat

Institute of Mathematics of the Romanian Academy

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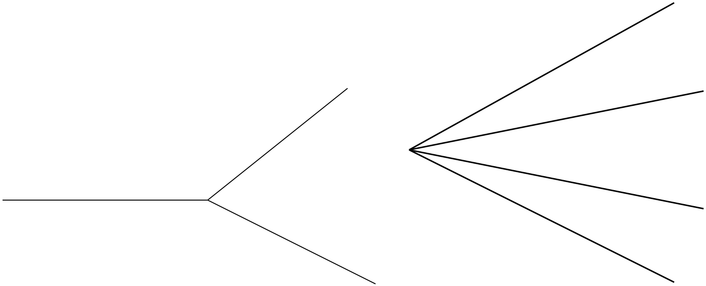

Figure: Star shaped trees

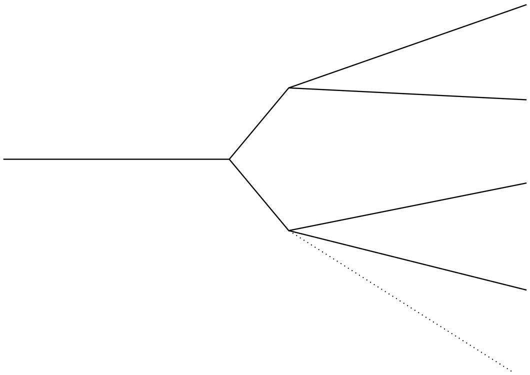

Figure: A regular/general tree

Schrödinger equation on such structure

Schrödinger equation on each segment (finite or infinite) + coupling

Type of couplings:

1. Kirchhoff: continuity & $\sum u_x^k = 0$, Banica & L.I. 2010
2. δ-coupling, continuity & $\sum u_x^k = \delta u$, Banica & L.I. 2014
3. δ'-coupling, cotinuity of u_x & $\sum u^k = \delta u_x$, Adami 2011 for star shaped trees

More general couplings

Kostrykin & Schrader 2000

- $\Delta(A, B)$ self-adjoint operator

$$AU + BU' = 0$$

satisfying

- A, B are $n \times n$ real matrices
- $\mathrm{rank}(A|B) = n$
- $AB^t = BA^t$

New results on the star shaped case. Joint work with Andreea Grecu

To prove the dispersion property it is sufficient to evaluate

$$\int_0^\infty e^{itk^2} e^{ikx} G(k, A, B) dk$$

where

$$G(k, A, B) = (A + ikB)^{-1}(A - ikB)$$

Two important properties:

- $\det(A + ikB) = 0$ has no real root $k \neq 0$
- $G(k, A, B)$ is unitary for any $k \neq 0$: $G\overline{G}^t = I_n$

A short idea of the proof

1. Van der Corput
2. $|G_{ij}(k)| \leq 1$ for all $k \in \mathbb{R} \setminus \{0\}$
3. $G_{ij}(k) = \frac{P_{ij}(k)}{Q_{ij}(k)}$ with $\deg P_{ij} \leq \deg Q_{ij}$
4. $P_{ij}(k) = k^{\alpha_{ij}} \tilde{P}_{ij}(k)$, $Q_{ij}(k) = k^{\beta_{ij}} \tilde{Q}_{ij}(k)$ with $\alpha_{ij} \geq \beta_{ij}$

No idea how to continue in the general case using this approach

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THANKS for your attention !!!