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- *Current affiliation*: Postdoc at Louisiana State University
- *Previously:*
  - Visiting professor at Universidad Autonoma Metropolitana
  - Postdoc at Mexican Institute of Petroleum
- *Ph.D.* Arizona State University
- Research Interest
  Dispersive equations, Fluid dynamics, Fractals and Controllability

Considering the $\mathrm{NLS}_p^+(\mathbb{R}^n)$

$$\begin{cases} i\partial_t u + \triangle u + |u|^{p-1}u = 0 \\ u_0(x) \in H^1(\mathbb{R}^n) \end{cases}$$

1. Characterization of NLS solutions in the mass-supercritical and energy subcritical regime

$$0 < s < 1 \iff \begin{cases} p > 5 & d = 1 \\ p > 3 & d = 2 \\ \frac{4+d}{d} < p < \frac{d+2}{d-2} & d \geq 3 \end{cases}$$

2. Critical norm concentration phenomena for mass-supercritical energy-subcritical NLS

Project 1: Nonlinear Schrödinger Equation II

3. Polariton-Exciton system

$$\begin{cases} i\partial_t\phi &= (\omega_c - i\kappa_c - \frac{\hbar}{2m_C}\Delta)\phi + \gamma\psi \\ i\partial_t\psi &= (\omega_x - i\kappa_x \pm |\psi|^2)\psi + \gamma\phi \end{cases}$$

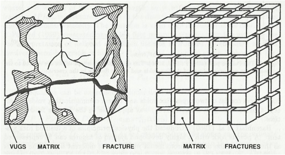
   1. my talk!!!

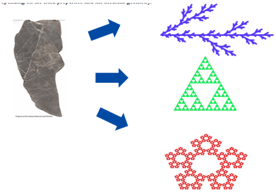
4. The complex Ginzburg–Landau equation $\mathrm{CGL}_{3,5}(\mathbb{R})$

$$i\psi_t + \frac{D}{2}\psi_{xx} - f|\psi|^2\psi + \nu|\psi|^4\psi = i\delta\psi + i\epsilon|\psi|^2\psi + i\beta\psi_{xx} + i\mu|\psi|^4\psi$$

   1. Traveling waves in $\psi(t,x) = e^{i\omega t}v(x-ct) \to$ bifurcation map

1. Fluid dynamics in porous media





$$\tau c_2\phi_2 \frac{\partial^2 p_2}{\partial t^2} + \left(c_2\phi_2 + \frac{\tau\alpha k_1}{\mu}\right)\frac{\partial p_2}{\partial t} = \frac{k_{2x}}{\mu}\frac{\partial^2 p_2}{\partial x^2} + \frac{k_{2y}}{\mu}\frac{\partial^2 p_2}{\partial y^2} - \left(c_1\phi_1 - \frac{\tau\alpha k_1}{\mu}\right)\frac{\partial p_1}{\partial t}$$

$$c_1\phi_1 \frac{\partial p_1}{\partial t} = \frac{\alpha k_1}{\mu}\left(p_2 - p_1\right)$$

2. Numerical simulation vs. models vs real data

Project 2: Diffusion type systems II

3. Approximate controllability of semilinear equations of the form in the Hilbert spaces $\mathcal{U}$ and $\mathcal{Z}$

$$\begin{cases} z' = -\mathbb{A}z + \mathbb{B}u + \displaystyle\int_0^t \mathbb{M}(t,s,z_s)ds + \mathbb{F}(t,z_t,u(s)), & z \in \mathcal{Z},\ t \geq 0, \\ z(s) = \Phi(s), & s \in [-r,0], \\ z(t_k^+) = z(t_k^-) + \mathbb{I}_k(t_k, z(t_k), u(t_k)), & k = 1,2,3,\ldots,p, \end{cases}$$

Example:

1. Heat,
   – bounded operator: $\mathbb{A} = \Delta$
   – bounded operator: $\mathbb{B} = 1_w$
   – nonlinearities: $\mathbb{M} = K * g(t,x)$ and $\mathbb{F}$.
2. Beam equation
   $$w_{tt} - 2\beta\Delta w_t + \Delta^2 w = u(t,x) + f(t, w(t-r), w_t(t-r), u) + \int_0^t M(t-s)g(w(s-r,x))ds$$
3. Strongly damped wave equation
4. Working on Benjamin-Bona-Mahony, Burgers' equations with memory

Project 3: 3D Navier-Stokes I

$$\begin{cases} u_t - \nu \Delta u + u \cdot \nabla u + \nabla p = 0 \\ \text{div } u = 0 \end{cases} \qquad \text{with} \qquad u(x,0) = v(x).$$

1. No Leray's backward self-similar solutions in $L^{12/5}$ or in $L^{q,\infty}(\mathbb{R}^3)$ for $q \in (12/5, 6)$.
2. ϵ–criteria: $\exists \epsilon > 0$, if a suitable weak solution u in Q_1 s.t.

$$\sup_{t \in [-1,0]} \int_{B_1} |u(x,t)|^2 dx + \int_{Q_1} |\nabla u|^2 dy ds + \int_{-1}^{0} \|p\|_{L^1(B_1)} ds \leq \epsilon,$$

then $u \in L^\infty(Q_{1/2})$.

Project 3: 3D Navier-Stokes II

3. $\alpha \in [6/5, 2]$, $\beta = \frac{4\alpha}{7\alpha - 6} \in [1, 2]$. $\exists \epsilon > 0$ s.t. if suitable weak solution u in Q_1 satisfies

$$\int_{-1}^{0} \|u\|_{L^{2\alpha}(B_1)}^{2\beta} \, ds + \int_{-1}^{0} \|p\|_{L^{\alpha}(B_1)}^{\beta} \, ds \leq \epsilon,$$

   then $u \in L^\infty(Q_{1/2})$.

4. Want to explore the magneto-hydro-dynamics (MHD) equations

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + (u \cdot \nabla) u - (b \cdot \nabla) b + \nabla p = 0, \\ \dfrac{\partial b}{\partial t} - \Delta b + (u \cdot \nabla) b - (b \cdot \nabla) u = 0 \\ \mathrm{div} u = 0, \qquad \mathrm{div} b = 0 \end{cases}$$