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Nonlinear Schrödinger equation over $\mathbb{R}^d$:

$$iu_t + \Delta u + \lambda |u|^\sigma u = 0, \quad u(0) = u_0 \in E$$

Problem: Local and global well-posedness on nonstandard spaces (not inside $L^2(\mathbb{R}^d)$).

The techniques applied are fairly general (should be extendible to other dispersive PDE's, some works on diffusive PDE's (Complex Ginzburg-Landau, Navier-Stokes, etc.))

1. Spaces of spatial plane waves: given speeds $\underline{c} = \{c_n\}_{n\in\mathbb{N}}$,

$$X_{\underline{c}} = \{\phi(x,y) = \sum_{n\in\mathbb{N}} f_n(x - c_n y), \{f_n\}_{n\in\mathbb{N}} \text{ in suitable space}\}$$

$$E = H^1(\mathbb{R}^2) \oplus X_{\underline{c}}.$$

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2. Spaces of continuous plane waves:

$$Tf(x,y) = \int_{\mathbb{R}} f(x - cy, c)dc \text{ - Plane wave transform}$$

Generalizes Fourier transform; $Tf \notin L^2(\mathbb{R}^2)$ if $f > 0$; etc.

$$E = H^1(\mathbb{R}^2) + \{Tf : f \text{ in suitable space}\} \hookrightarrow L^4(\mathbb{R}^2).$$

3. Spaces of infinite mass:

$$E = \dot{H}^1(\mathbb{R}^d) \cap L^p(\mathbb{R}^d), \quad 2 < p < 2^*$$

LWP for large ranges of p, including the "true"energy space

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Open problems: GWP, new blow-up solutions, stability of standing waves, scattering,...

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Suggestions and comments are welcome! In person or by sfcorreia@fc.ul.pt