Politics is for present, but an equation is something for eternity.
(Albert Einstein)

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Motivation

Master's thesis:

"*Exponential bases on two dimensional trapezoids*" (with L. Decarli),
Proceedings of AMS 143 (2015), 2893-2903

Generalized Hartree equation

Result: Dichotomy below the threshold (S. Roudenko - A., 2016)

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$$iu_t + \Delta u + \left(|x|^{-(N-\gamma)} * |u|^p\right)|u|^{p-2}u = 0 \quad x \in \mathbb{R}^n, t \in \mathbb{R} \quad 0 < \gamma < N$$

with $u_0 \in H^1(\mathbb{R}^N)$ and $0 < s < 1$. Assume that $M^{1-s}E^s[u_0] < M^{1-s}E^s[Q]$.

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- $||u_0||_{L^2}^{1-s}||\nabla u_0||_{L^2}^{s} < ||Q||_{L^2}^{1-s}||\nabla Q||_{L^2}^{s}$
  $\implies u(t)$ exists globally in time, and scatters in H^1 for all $t \in \mathbb{R}$.

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Work in progress

- Scattering without using concentration - compactness as in Dodson-Murphy.

Blow-up criteria

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To ∞ and beyond - STAY MAGICAL