Representation Theory of Finite and p-adic Groups of Lie Type

CIRM, November 7th – November 10th

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 am	Introduction to p-adic fields Vincent Sécherre	Minicourse 1 part 2 Olivier Dudas	Short talks 1 Kei Yuen Chan	Minicourse 2 part 3 Shaun Stevens	
9:30 am			Short talks 2 Jesua Epequin Chavez		
10:00 am					
10:30 am			Short talks 3 Zhe CHEN		
11:00 am	Minicourse 1 part 1 Olivier Dudas	Minicourse 2 part 2 Shaun Stevens	Short talks 4 Auguste Hébert	Minicourse 1 part 3 Olivier Dudas	
11:30 am					
12:00 am			Short talks 5 Peter Latham		
12:30 am			Short talks 6 William Wong		
1:00 pm					
1:30 pm					
2:00 pm					
2:30 pm					
3:00 pm					
3:30 pm					
4:00 pm	Minicourse 2 part 1 Shaun Stevens	Long talk 2 Corinne Blondel	Long talk 5 Steffen Koenig		
4:30 pm					
5:00 pm		Long talk 3 Jean Michel	Long talk 6 Ramla Abdellatif		
5:30 pm					
6:00 pm	Long talk 1 Maria Chlouveraki	Long talk 4 Guy Henniart	Long talk 7 Donna Testerman		
6:30 pm					
7:00 pm					

Vincent Sécherre (Versailles)

Introduction to p-adic fields

This lecture in an introduction to p-adic fields for non-specialists. I will first describe the basic example of the field of p-adic numbers. I will then give the general definition of a p-adic field, based on the notion of valuation. I will describe the algebraic structure of these fields and study their finite extensions. If time permits, I will present the main results of local class field theory, which classifies all abelian finite extensions of a p-adic field.

Olivier Dudas (Paris)

Representations of finite reductive groups

Representations of finite reductive groups (such as $GL_n(q)$, $Sp_{2n}(q)$, $E_8(q)$...) are constructed by (parabolic) induction from proper Levi subgroups. Some of the representations - the so-called cuspidal representations - do not appear in any such induced representation. The Harish-Chandra theory aims to classify the irreducible representations using this approach. It consists of solving two problems :

— determining the cuspidal irreducible representations ;
— decomposing induced representations.

The solution to the latter involves the representation theory of other classical algebras, the Iwahori-Hecke algebras.
The purpose of these lectures is to present a modern approach to these problems. After reviewing the classical theory, we will explain how Harish-Chandra induction and restriction yield an action of an affine Lie algebra on the category of representations of the finite classical groups. In that setting, the cuspidal representations are now highest weight vectors and can be explicitly determined. We will focus on the case of $GL_n(q)$ and $Sp_{2n}(q)$, which will involve the combinatorics of partitions and bipartitions."

Shaun Stevens (East Anglia)

Representations of p-adic groups and Hecke algebras

This mini-course will give an introduction to the smooth representation theory of p-adic Lie groups, mostly over the complex numbers (or at least an algebraically closed field of characteristic different from p). As well as giving some of the basic results in the theory, there will be an emphasis on the links with the representation theory of finite Lie groups, as well as on the Hecke algebras which arise.

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Ramla Abdellatif (Amiens)

Title : Extensions between Iwahori-Hecke modules for $SL_2(F)$ in characteristic p

Abstract : Let p be a prime number and F be a non-archimedean field with finite residue class field of characteristic p. Understanding the category of Iwahori-Hecke modules for $SL_2(F)$ is of great interest in the study of p-modular smooth representations of $SL_2(F)$, as these modules naturally show up as spaces of invariant vectors under the action of the standard pro-p-Iwahori subgroup. In this talk, we will discuss a work in progress in which we aim to classify all non-trivial extensions between these modules and to compare them with their analogues for p-modular smooth representations of

$SL_2(F)$ and with their Galois counterpart in the setting of the local Langlands correspondences in natural characteristic.

Corinne Blondel (Paris)

Title : An instance of involvement of Hecke algebras : a special case of base change for unitary groups. (joint work with Kam Fai Tam)

Abstract :
For a ramified unitary group over a p-adic field (p odd), some supercuspidal representations are relatively easy to construct and the study of reducibilities of representations parabolically induced from those can be performed entirely using Hecke algebras. This leads to an explicit description of quadratic base change for these representations.

Maria Chlouveraki (Versailles)

Title : Yokonuma-Hecke algebras

Abstract : Yokonuma-Hecke algebras were introduced by Yokonuma in the 60's as generalisations of Iwahori-Hecke algebras. We will see how we can obtain similar generalisations for affine Hecke algebras. We will study the structure of these affine Yokonuma-Hecke algebras, as well as that of their cyclotomic quotients, and discuss their interesting applications in knot theory.

Guy Henniart (Orsay)

Title : Mod.p Hecke algebras for reductive p-adic groups (after Abe, Herzig, Henniart, Vignéras)

Abstract : Let F a p-adic field and G a connected reductive group over F. Let C be an algebraically closed field of characteristic p. As over complex numbers, one studies an irreducible smooth C-representation π of $G(F)$ through its restriction to open compact subgroups J. Choosing J to be a maximal parahoric subgroup K, the pro-p radical subgroup $K(1)$ of K has non-zero fixed vectors in π, and π contains an irreducible representation V of $K/K(1)$. Now $K/K(1)$ is a finite reductive group, and its representation theory over C is known ; we shall explain its interplay with the Hecke algebra of V in $G(F)$, and a few of the ideas -with examples- which led the above authors to a classification of admissible smooth C-representations of $G(F)$. this leads to higher dimensional generalizations.

Stephen Koenig (Stuttgart)

Title : Cellular algebras and affine cellular algebras

Abstract : Cellular and affine cellular structures will be explained in terms of generalised matrix algebras. Then, after briefly listing some old and new classes of examples, some structural properties of (affine) cellular algebras will be discussed.

Jean Michel (Paris)

Title : Quasisemisimple classes

Abstract : This is a report on joint work with François Digne. Quasisemisimple elements are a generalisation of semisimple elements to disconnected reductive groups (or equivalently, to algebraic automorphisms of reductive groups). In the setting of reductive groups over an algebraically closed field, we discuss the classification of quasisemisimple classes, including isolated and quasi-isolated ones. The talk will start with the basic theory of non-connected reductive groups.

Donna Testermann (Lausanne)

Title : Multiplicity free actions of simple algebraic group

Abstract : We discuss joint work with Martin Liebeck and Gary Seitz, in which we consider the following question :
Let M be a maximal closed reductive subgroup of a simple algebraic group G, defined over an algebraically closed field k of characteristic 0. Determine all irreducible kG-modules V such that V is a multiplicity-free kM-module.
We report on general techniques for approaching the problem and our results for the case where $G = SL(W)$ and M is a simple group of type A_n acting irreducibly on W.

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Kei Yuen Chan

Title : Bernstein-Zelevinsky derivatives and Hecke algebras

Abstract : Bernstein-Zelevinsky derivatives were introduced for classifying simple representations of the general linear group GL(n,F) over a non-Archmedian local field F. They are also useful tools for the branching problem of the pair $(GL(n+1,F), GL(n,F))$. In this talk, I shall discuss a realization of Bernstein-Zelevinsky derivatives in the affine Hecke algebra level, and its application on the branching problem. This is a joint work with Gordan Savin.

Jesua Epequin Chavez

Title : Reductive dual pair of $Sp_{2n}(q)$

Abstract : If (G, G') is a reductive dual pair of $Sp_{2n}(q)$, the theta correspondance associates a set of irreducible representations of G to an irreducible representation of G'. In this talk we will show how to obtained a âĂİpreferredâĂİ representation out of this set for symplectic-orthogonal pairs and for unipotent representations of the principal series of G'. That is, weâĂŹll see how to extract a bijection out of the theta correspondance. This generalizes the case proved by Aubert and Przebinda, where one of the groups in the pair is symplectic of dimension 4 and the other is a split orthogonal group.

Zhe CHEN

Title : Algebraisation and geometrisation in higher Deligne–Lusztig theory

Abstract : Higher Deligne–Lusztig theory uses l-adic cohomology to produce representations of reductive groups over certain local rings ; in generic case they are parametrised by pairs consisting of rational maximal tori and characters of the rational subgroups. Meanwhile, there are algebraically constructed representations due to Gerardin that based on the same parameters. At even levels, we proved they are isomorphic (joint with Stasinski), which answered a question of Lusztig. Moreover, in this case the higher Deligne–Lusztig characters can thus be geometrised in a similar manner as in the classical case. We report these works.

Auguste Hébert

Title : Gindikin-Karpelevich finiteness for Kac-Moody groups over local fields

Abstract : In 2012, Braverman, Garland, Kazhdan and Patnaik proved a generalization of Gindikin-Karpelevich formula for affine Kac-Moody groups over ultrametric fields. This formula involves cardinals of quotients of intersections of subgroup of the affine Kac-Moody group. The finiteness of this quotients was called "Gindikin-Karpelevich finiteness". Braverman and Kazhdan conjecured that this finiteness was true for general (not necessarily affine) Kac-Moody groups over ultrametric field. We will prove this conjecture. For this, we will use the masure (hovel) associated by Gaussent and Rousseau to this situation, which is a kind of generalization of a Bruhat-Tits building.

Peter Latham

Title : Depth zero supercuspidals of p-adic groups

Abstract : I'd discuss a complete description of the theory of types (in the sense of Bushnell–Kutzko) for depth zero supercuspidal representations of arbitrary p-adic group (those representations obtained via pullback from cuspidals of certain finite reductive groups), showing that the only such types are those induced from the types constructed by Moy–Prasad and Morris. One nice application of this is that, using the DeBacker–Reeder paper, it allows you to extend the "Langlands correspondence" for certain finite groups (GL, SL, PGL) which was constructed by MacDonald in the 80s to a much larger class of p-adic groups (all of those considered by DeBacker and Reeder).

William Wong

Title : On the derived category of special linear group of degree 2 over finite field

Abstract : I managed to create a non-trivial autoequivalence in the derived category of such group algebras which exchange the principal and non-principal blocks (when p is odd). It utilizes a recent notion of perverse equivalence and it can be regard as a generalization of the Morita equivalence between blocks of $SL(2, p)$.