RATIONAL POINTS AND ALGEBRAIC GEOMETRY

CIRM, SEPTEMBER 26–30, 2016

1. Schedule

	Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
09 : 30	Campana	Schindler	Poonen	Charles	Szamuely
11 : 00	Dokchitser	Voloch	Tian	Stoll	Loughran
14 : 30	Matthiesen	Wittenberg		Harpaz	
16 : 00	Zarhin	Taelman		Beheshti	

2. Titles and abstracts

R. Beheshti:
`Spaces of rational curves on Fano hypersurfaces`

I will discuss some aspects of the geometry of moduli spaces of rational curves on Fano hypersurfaces focusing on a few questions on dimension, irreducibility, and the Kodaira dimension of these spaces.

F. Campana:
`Orbifoldes, aspects géométriques, hyperboliques et arithmétiques`

Conjecturalement selon Lang une variété projective lisse X définie sur un corps de nombres n'est pas potentiellement dense si elle possède une application rationnelle dominante sur une variété Y de type général. Les variétés 'spéciales' sont celles n'admettant, plus généralement, pas d'application sur une 'orbifolde' (Y, D) de type général. Les 'orbifoldes' (Y, D) interpolent entre les variétés projectives ($D = 0$) et quasi-projectives (D réduit), et sont indispensables pour décrire la structure des variétés (quasi)-projectives. Elles sont naturellement munies des mêmes invariants géométriques, arithmétiques et hyperboliques que les variétés (quasi)-projectives Y. On conjecture, pour toute orbifolde, et donc toute variété (quasi)-projective X, une équivalence à la Lang entre ses propriétés géométriques, hyperboliques et arithmétiques. Lorsque X est de type général, on retrouve les conjectures de Lang-Vojta. Dans la situation opposée, X est conjecturalement potentiellement dense, et a une pseudo-métrique de Kobayashi nulle, si et seulement si elle est 'spéciale'. Cette conjecture (de 'Mordell orbifolde') est ouverte (mais résulte de abc) déjà en dimension 1.

F. Charles:
`Bertini theorems in arithmetic geometry`

The classical Bertini irreducibility theorem states that if X is an irreducible projective variety of dimension at least 2 over an infinite field, then X has an irreducible hyperplane section. The proof does not apply in arithmetic situations, where one wants to work over the integers or a finite fields. I will discuss how to amend the theorem in these cases (joint with Bjorn Poonen over finite fields).

T. Dokchitser:
`Rational points on curves under field extensions`

There are many surprisingly difficult questions about the behaviour of rational points on curves and their Jacobians in extensions of number fields. I would like to give a small overview of some of these questions, in particular focusing on the case of elliptic curves.

Y. Harpaz:
`Second 2-descent and rational points on Kummer surfaces`

A powerful method pioneered by Swinnerton-Dyer allows one to study rational points on pencils of curves of genus 1 by combining the fibration method with a sophisticated form of descent along isogenies. A variant of this method, first used by Skorobogatov and Swinnerton-Dyer in 2005, can be applied to study rational points on twisted Kummer surfaces. In this talk we will explain how to incorporate into the method an additional step of second

2-descent. Assuming finiteness of the relevant Tate-Shafarevich groups, this idea can be applied to show that the Brauer-Manin obstruction is the only obstruction to the Hasse principle on certain twisted Kummer surfaces associated to abelian surfaces with all rational 2-torsion. In the case when the abelian surface splits into a product of two elliptic curves one can reproduce in this way the results of Skorobogatov and Swinnerton-Dyer mentioned above under weaker assumptions.

D. Loughran:
`Pseudo-split varieties and arithmetic surjectivity`

A classical theorem of Ax and Kochen (1965) states that given $d > 0$, for all but finitely many primes p every homogeneous form in at least $d^2 + 1$ variables admits a non-trivial p-adic zero. This theorem was originally proved using tools from model theory, however Denef, following a strategy suggested by Colliot-Thélène, recently found a purely geometric proof that moreover gives results for more general families of varieties. In this talk we build upon Denef's work and give a criterion that completely classifies those families of varieties for which an analogue of the Ax–Kochen theorem holds, using the new notion of a "pseudo-split variety". This work is joint with Arne Smeets and Alexei Skorobogatov.

L. Matthiseen:
`Correlations of multiplicative functions`

In joint work with Browning and Skorobogatov from 2014 we studied weak approximation on conic bundle surfaces. On the analytic side of this work, the key ingredient was an asymptotic evaluation of linear correlations of a specific multiplicative function, namely the function r that counts representations of an integer as a sum of two squares. Here, we discuss a generalisation of this result to a larger class of multiplicative functions.

B. Poonen:
`TBA`

D. Schindler:
`Representations of integers by systems of three quadratic forms`

In this talk we discuss representations of triples of integers by systems of three quadratic forms $\{Q_1, Q_2, Q_3\}$ in k variables. We improve on classical results by reducing the number of required variables to $k \geqslant 10$, for 'almost all' tuples and under a non-singularity assumption on the quadratic forms. This is joint work with Lillian Pierce and Melanie Matchett Wood.

M. Stoll:
The generalized Fermat equation $x^2 + y^3 = z^{11}$

Generalizing Fermat's original problem, equations of the form $x^p + y^q = z^r$, to be solved in coprime integers, have been quite intensively studied. It is conjectured that there are only finitely many solutions in total for all triples (p, q, r) such that $1/p + 1/q + 1/r < 1$ (the 'hyperbolic case'). The case $(p, q) = (2, 3)$ is of special interest, since several solutions are known. To solve it completely in the hyperbolic case, one can restrict to $r = 8, 9, 10, 15, 25$ or a prime ≥ 7. The cases $r = 7, 8, 9, 10, 15$ have been dealt with by various authors. In joint work with Nuno Freitas and Bartosz Naskrecki, we are now able to solve the case $r = 11$ and prove that the only solutions (up to signs) are $(x, y, z) = (1, 0, 1), (0, 1, 1), (1, -1, 0), (3, -2, 1)$. We use Frey curves to reduce the problem to the determination of the sets of rational points satisfying certain conditions on certain twists of the modular curve $X(11)$. A study of local properties of mod-11 Galois representations cuts down the number of twists to be considered. The main new ingredient is the use of the 'Selmer group Chabauty' techniques developed recently by the speaker to finish the determination of the relevant rational points.

T. Szamuely:
`Geometry of Severi--Brauer varieties (after Kollár)`

Recently, János Kollár gave new and elegant geometric proofs for many of the classical statements about Severi–Brauer varieties. In my lecture I shall present some of the lollipops.

L. Taelman:
`Complex multiplication of K3 surfaces`

A complex K3 surface X with complex multiplication is algebraic and defined over a number field. In this talk we will classify the models of X over number fields. This has applications to K3 surfaces over finite fields, and in particular we will explain how it can be used to classify the possible zeta functions of K3 surfaces over finite fields, strengthening earlier results which only gave information 'up to a finite field extension'.

Z. Tian:
`Weak approximation for cubics over global function fields`

I will talk about a geometric approach to weak approximation of cubic surfaces at places of good reduction over global function fields. This is based on joint work with Letao Zhang.

F. Voloch:

`Differential descent obstructions`

We will discuss a new obstruction to the existence of rational and integral points on algebraic varieties over function fields obtained by considering covers described by differential equations.

O. Wittenberg:

`Sur la conjecture de Hodge entière pour les solides réels`

Nous formulons un analogue de la conjecture de Hodge entière pour les variétés réelles. Celui-ci possède des liens étroits avec des propriétés classiques: existence d'une courbe réelle de genre pair, algébricité de l'homologie du lieu réel. Comme dans le cas complexe, la conjecture de Hodge entière réelle peut tomber en défaut mais est plausible pour les 1-cycles sur les variétés dont la géométrie est assez simple. Nous l'établissons pour plusieurs familles de solides uniréglés. Il s'agit d'un travail en commun avec Olivier Benoist.

Y. Zarhin:

Compatible systems of ℓ-adic representations arising from abelian varieties

Famous (and still unproven in full generality) conjectures of Serre–Grothendieck, Tate and Fontaine-Mazur describe ℓ-adic representations that arise from the action of the absolute Galois group of a number field K on the (twisted) ℓ-adic cohomology groups of projective algebraic varieties that are defined over K. Assuming all these conjectures (and the Hodge conjecture), we discuss the following question: which ℓ-adic representations correspond to the ℓ-adic Tate modules of an abelian variety? We give an answer for abelian varieties without nontrivial endomorphisms. This is a report on a joint work with Stefan Patrikis and Felipe Voloch .