On the entropy algorithmics of block gluing two dimensional subshifts of finite type.

Silvere Gangloff, Mathieu Sablik

CIRM 2016

29 novembre 2016

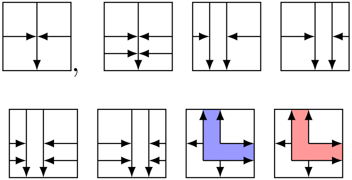
A **two dimensional subshift** is some **compact** subset of $A^{\mathbb{Z}^2}$, where A is some finite set, with the discrete product topology, and which is **invariant** by the action of the shift.

A **two dimensional SFT** is such a subshift defined by a finite set of forbidden (finite) patterns.

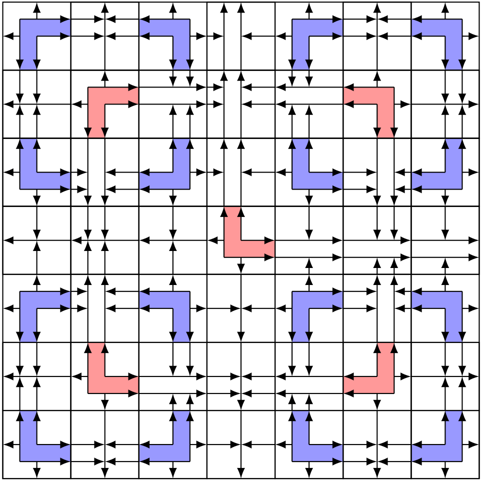
A **neighborhood subshift** is some SFT defined by 2×1 and 1×2 patterns.

An **example of two dimensional SFT** : The Robinson subshift.

Alphabet A (up to some multiples rotations by $\pi/2$) :



Forbidden patterns : 2×1 and 1×2 patterns where arrows don't match, and 2×2 patterns with no blue symbol.



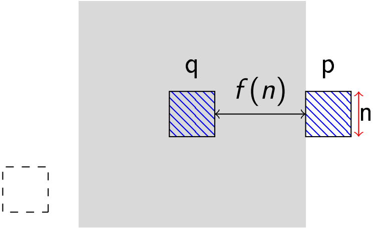
Let X be some two dimensional subshift. The (topological) **entropy** of X is

$$h(X) = \inf_n \frac{\log(N_n(X))}{n^2}$$

where N_n is the number of $n \times n$ patterns that appear in X.

A real number is **computable** when its base two decomposition is a computable sequence. A real number Π_1-**computable** when the infimum of a computable sequence of rational numbers.

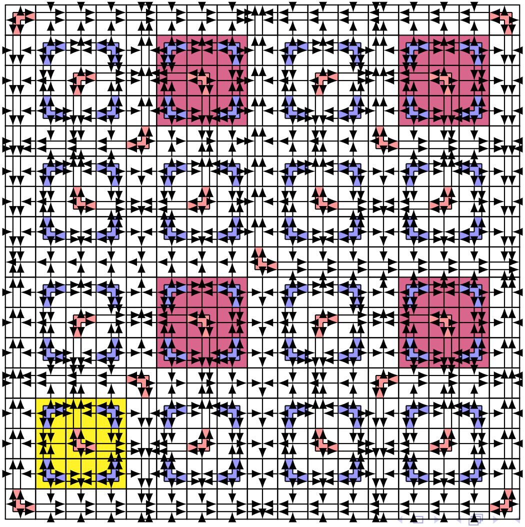
Let $f : \mathbb{N} \to \mathbb{N}$. A two dimensional subshift X is f-**block gluing** when for all $n > 0$, for all p, q two $n \times n$ patterns in the language of X, for all $(u, v) \in \mathbb{Z}^2$ such that $||u - v||_\infty \geq n + f(n)$, there exists $x \in X$ such that $x_{u+[\![0,n-1]\!]^2} = p$ and $x_{v+[\![0,n-1]\!]^2} = q$.



The subshift X is f-transitive when two $n \times n$ patterns can be "glued" (in at least one position) with distance $f(n)$.

Proposition

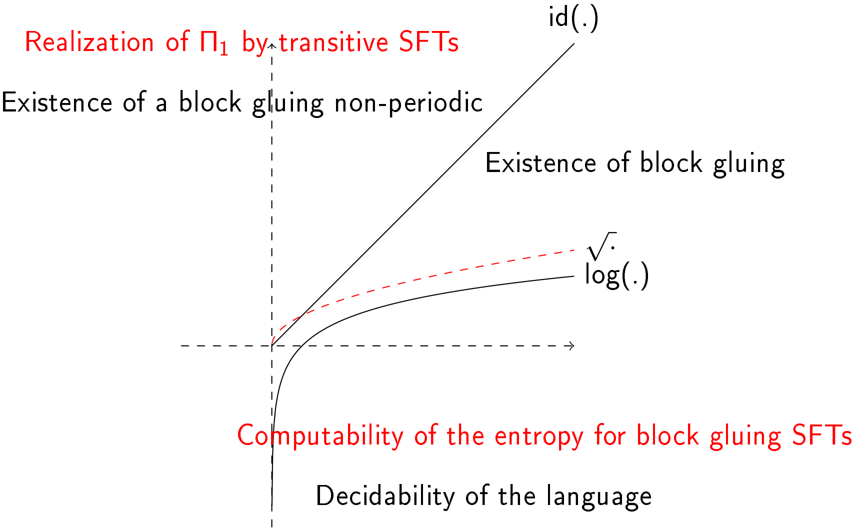
The Robinson subshift is $O(id)$-transitive.



p

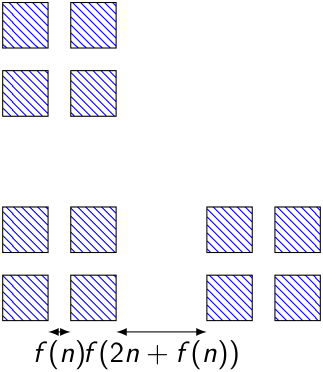
q

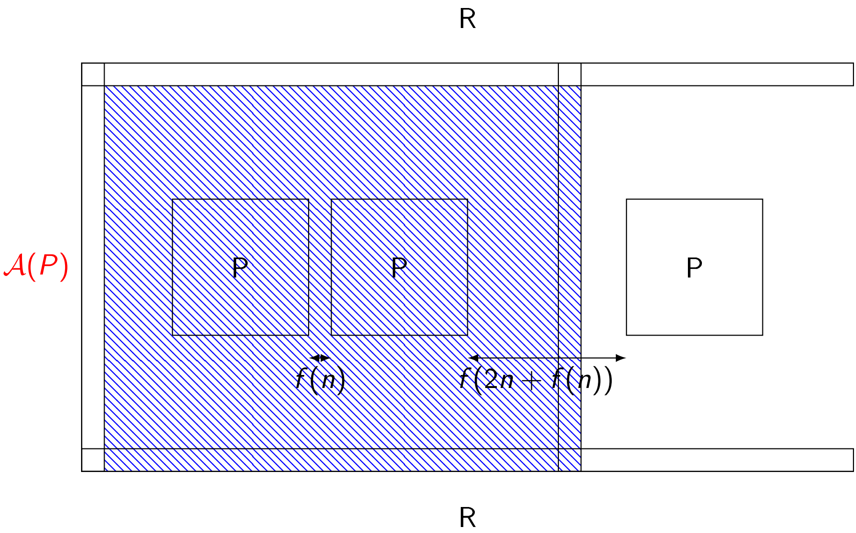
Question : how is the influence of the dynamical property of gluing on the computability of the entropy ?


id(.)
Realization of Π_1 by transitive SFTs
Existence of a block gluing non-periodic
Existence of block gluing
$\sqrt{\cdot}$
log(.)
Computability of the entropy for block gluing SFTs
Decidability of the language

Proposition

There exists some $c > 0$ such that any f-block gluing two-dim. neighborhood subshift with $\forall n, f(n) \leq c\log(n)$ has a computable entropy.




R
$\mathcal{A}(P)$
P
P
P
$f(n)$
$f(2n+f(n))$
R

Theorem (1)

The entropies of $O(id)$-transitive two dimensional SFTs are exactly the Π_1-computable numbers.

which is an amelioration of **Hochman and Meyerovitch (07)** theorem :

Theorem (2)

The entropies of two dimensional SFTs are exactly the Π_1-computable numbers.

Ideas of the proof of the second theorem : given some h a Π_1 computable number,

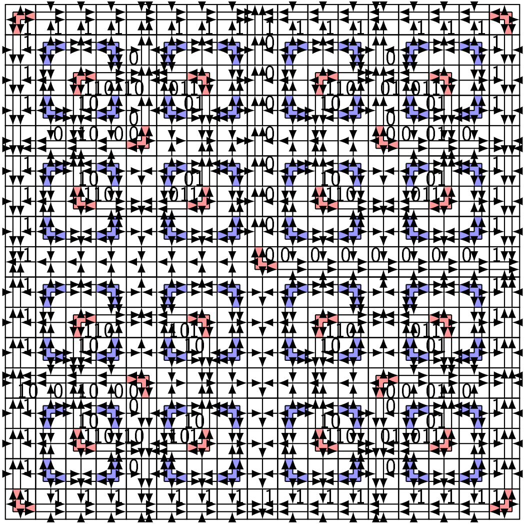
- Construction of an SFT that have four layers.
- The first has alphabet $\{0, 1, 1'\}$, and the bits of a same line are all 0 or $1/1'$ (**identification of lines**).
- The second one permits to identify the types of bits (0 or 1/1') of sets of lines in a **'toeplitz' structure**. These groups of lines are the 'levels' $1, 2, 3..$

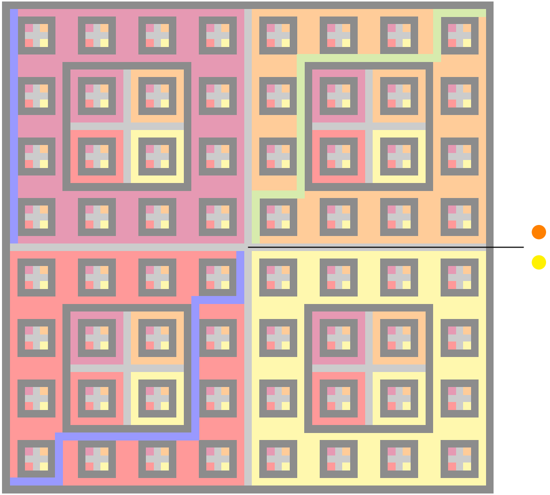
- The third one is the **Robinson subshift**, which induce frames for computations of turing machines that will control the frequency of $1/1'$ symbols so that the entropy is h (Thanks to some technical lemma that links the entropy to the frequency).
- The fourth layer is the support of **machines computations**.

The **obstacles for gluing** are the following : identification of bits $0, 1, 1'$ in the first and second layer, and the computations of machines that happen only in **infinite computation areas**.

The **modifications** we done to this proof are the following :

- A different way to **identify bits** that take place not in the whole plane but **inside computation frames**.
- The **simulation** of the behavior of the machines that happen in infinite computation areas into every finite one.





Question : Can we realize every Π_1-computable number as the entropy of a linearly block gluing SFT ?