“Algebraic combinatorics in Representation Theory”

CIRM 29.08.16/02.09.16

Titles and abstracts

1 Lectures

1. Philippe Biane

   Title: A probabilistic look at some questions in representation theory

   Abstract: I will talk on two topics where probability ideas are relevant for questions in representations theory:

   the first is the characters of symmetric groups. In order to understand the value of characters of large symmetric groups it is useful to introduce the noton of ”free cumulants” of a Young diagram. These quantities arise naturally in the theory of free probability and random matrices and they provide the right asymptotics for characters. More, one can even use them to give exact formulas for characters which are independent of the size of the symmetric group.

   The second is the Littelmann path model in representation theory. I will show how this model is closely related to a famous theorem on Brownian motion due to J. Pitman.

2. Masaki Kashiwara

   Title: Categorification of quantum groups by quiver Hecke algebras, R-matrices and Cluster algebras

   Abstract: Quiver Hecke algebras are a family of graded algebras introduced by Rouquier and Khovanov-Lauda for the purpose of categorifying quantum groups and their representations.

   The category of modules over a quiver Hecke algebra is an abelian category with a monoidal category structure. Its Grothendieck group is isomorphic to the half of the quantum group. By this isomorphism, the set of isomorphic classes of simple modules corresponds to the upper global basis (under certain conditions).

   Moreover, the monoidal category structure has remarkable properties: the tensor product of a pair of (real) simple objects has a simple socle and a simple head.

   In this mini-course, I explain these properties, starting from the definition of quiver Hecke algebras. If the time permits, I explain the monoidal categorification of cluster algebras by quiver Hecke algebras.

3. Bernard Leclerc

   Title: Cluster algebras and representations of quantum affine algebras

   Abstract: (joint work with David Hernandez) Representation theory is one of the most interesting area of applications for the combinatorics of cluster algebras. In 2008, it was found that certain categories of finite-dimensional representations of quantum affine algebras have a cluster structure. Since then, these results have been extended in several directions ((Nakajima, Kimura-Qin, HL, Qin). In particular, they yield new algorithms and new q-character formulas for some large class of irreducible representations. This mini-course will present a survey of the main results and conjectures in this direction.

4. Piotr Śniady

   Title: Jeu de taquin and asymptotic representation theory

   Abstract : We consider an infinite version of Schtzenberger's jeu de taquin (sliding game): from a random infinite Young tableau we remove the corner box and slide the remaining boxes so that the gap after the missing box disappears. What can we say about the trajectory of this avalanche? What can we say about the resulting transformation on the set of infinite Young tableaux and the corresponding dynamical system? The answers to these questions are related to problems of the representation theory of the symmetric groups. This is a joint work with Dan Romik.

2 Long talks

1. Jean-Christophe Aval

   Title: Rectangular parking functions

   Abstract : Parking functions are classical objects in combinatorics, either enumerative or algebraic. They may be seen as labelings of Dyck paths, which are paths with steps $(1, 0)$ and $(0, 1)$ usually embedded in a square $n \times n$, and constrained to be over the diagonal. Their number is known to be $(n + 1)^{n-1}$. The symmetric group acts on parking functions by permuting the labels. This action is nicely encoded in terms of symmetric functions through the Frobenius characteristic. In this talk, we will present generalizations of this: for parking functions embedded in a rectangle $m \times n$, and for Schroder parking functions, defined as labelings of Schroder paths for which a third type of step $(1, 1)$ is allowed.

2. Chris Bowman

   Title: Kronecker Tableaux

   Abstract : The Kronecker problem asks for an algorithmic understanding of the coefficients arising in the decomposition of the tensor product of two simple modules for the symmetric group.

   We provide an algorithm for calculating Kronecker coefficients labelled by so-called co-Pieri triples of partitions. This, in some sense, solves half of the Kronecker problem.

   This is joint work with Maud De Visscher and John Enyang.

3. Giovanni Cerulli-Irelli

   Title: Quiver Grassmannians of Dynkin type.

   Abstract : Given a finite-dimensional representation M of a Dynkin quiver Q (which is the orientation of a simply-laced Dynkin diagram) we attach to it the variety of its subrepresentations. This variety is stratified according to the possible dimension vectors of the subresentations of M. Every piece is called a quiver Grassmannian. Those varieties were introduced by Schofield and Crawley Boevey for the study of general representations of quivers. As pointed out by Ringel, they also appeared previously in works of Auslander. They reappered in the literature in 2006, when Caldero and Chapoton proved that they can be used to categorify the cluster algebras associated with Q. A special case is when M is generic. In this case all the quiver Grassmannians are smooth and irreducible of "minimal dimension". On the other hand, in collaboration with Markus Reineke and Evgeny Feigin, we showed that interesting varieties appear as quiver Grassmannians associated with non-rigid modules. In this talk I will survey on recent progresses on the subject. In particular I will provide another proof of the key result of Caldero and Chapoton.

4. Ghislain Fourier

   Title: Linear degenerations of flag varieties

   Abstract : In 2011, Evgeny Feigin introduced the degenerate flag variety as the highest weight orbit of a PBW degenerate highest weight module. He has provided, similarly to the classical case, a description in terms of sequences of subspace, a "flag description". I will generalize the notion of "degenerate flag variety" by considering all linear flag descriptions, hence introducing the universal linear degeneration of the flag variety. I will provide descriptions of the flat, the irreducible and the normal locus of these degenerations. Finally, to see the analogy to the classical situation, I will explain how these loci are related to Schubert varieties and (partially) PBW degenerate highest weight modules. This is joint work with G. Cerulli-Irelli, X.Fang, E.Feigin, M.Reineke.

5. Myungho Kim

   Title: Supersymmetric polynomials and the center of the walled Brauer algebra

   Abstract: In this talk, I will introduce a family of commuting elements of the walled Brauer algebra, called the Jucys-Murphy elements. This family is a variant of the ones introduced by Brundan-Stroppel and Sartori-Stroppel. As similar in the case of symmetric groups, the supersymmetric polynomials in the Jucys-Murphy elements belong to the center of the walled Brauer algebra. We show that if the walled Brauer algebra is semisimple, then these supersymmetric polynomials generate the center. This fact enables us to mimic the approach of Okounkov-Vershik on the representation theory of symmetric groups. Furthermore, we have an analogue of the Jucys-Murphy elements for the quantized walled Brauer algebra and a similar connection between the supersymmetric polynomials in the Jucys-Murphy elements and the center. Interestingly enough, this connection was already observed by H. Morton in 2001 in terms of HOMFLY skein on the annulus, and our result gives a proof of a conjecture of Morton. This is a joint work with Ji Hye Jung and was posted on arXiv:1508.06469

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6. Martina Lanini

   Title: Parking spaces and Catalan combinatorics for complex reflection groups.

   Abstract: Recently, Armstrong, Reiner and Rhoades associated to any (well generated) complex reflection group two parking spaces, and conjectured their isomorphism. This has to be seen as a generalisation of the bijection between non-crossing and non-nesting partitions, both counted by the Catalan numbers. In this talk, I will review the conjecture and discuss a new approach towards its proof, based on the geometry of the discriminant of a complex reflection group and the corresponding Lyashko-Looijenga map. This is an ongoing joint project with Iain Gordon."

7. Cédric Lecouvey

Title: Harmonic functions on multiplicative graphs: applications to representation theory and random walks

Abstract: To each simple module of a complex Lie algebra is associated a multiplicative graph. The talk aims to describe the extremal harmonic functions associated to the graphs so obtained in terms of Weyl characters. These harmonic functions have interesting applications, not only in probability theory (conditioning of natural random walks), but also in representation theory itself (positive morphims of character rings, asymptotic of tensor multiplicities and canonical bases).

8. Petra Schwer

   Title : Studying affine Deligne Lusztig varieties via folded galleries in buildings

   Abstract: We present a new approach to affine Deligne Lusztig varieties which allows us to study the so called "non-basic" case in a type free manner. The central idea is to translate the question of non-emptiness and the computation of the dimensions of these varieties into geometric questions in the Bruhat-Tits building. All boils down to understand existence of certain positively folded galleries in affine Coxeter complexes. To do so, we explicitly construct such galleries and use, among other techniques, the root operators introduced by Gaussent and Littelmann to manipulate them.

3 Short talks

1. Thomas Gerber

   Title: Triple crystal structure for Fock spaces

   Abstract: Combinatorial Fock spaces play a key role in the modular representation theory of complex reflection groups (and related structures) and finite classical groups. They have a triple module structure: for two quantum groups of affine type A, and for a Heisenberg algebra. The Kashiwara crystal arising from a quantum group action is known to have several important interpretations. I will explain how to define a suitable notion of crystal for the Heisenberg algebra, intertwining the two Kashiwara crystals. This gives new results about the representation theory of finite unitary groups and rational Cherednik algebras.

2. Xin Fang

   Title: Feigin-Fourier-Littelmann-Vinberg polytopes and string polytopes

   Abstract : For Lie algebras of type A and C, both Feigin-Fourier-Littelmann-Vinberg s(FFLV) polytopes and string polytopes parametrize bases of their irreducible representations. G Fourier and P. Littelmann have provided explicit unimodular maps between these polytopes, showing that they are isomorphic. In this talk I will explain a geometric proof by realizing both of them as Newton-Okounkov bodies. This talk is based on a joint work with G. Fourier.

3. Ruslan Maksimau

   Title : Categorical actions and KLR algebras

   Abstract : The affine Lie algebra $sl(n+1)$ contains a subalgebra isomorphic to the affine Lie algebra $sl(n)$. It is natural to ask if we can restrict categorical representations from affine $sl(n+1)$ to affine $sl(n)$. We prove that a category with an action of affine $sl(n+1)$

contains a subcategory with an action of affine $sl(n)$. To prove this statement, we construct an isomorphism between the KLR algebra associated with the n-cycle and a subquotient of the KLR algebra associated with the $(n+1)$-cycle.

4. Huafeng Zhang

   Title Representations of quantum affine superalgebra

   Abstract : We study tensor product properties of finite-dimensional representations of quantum affine superalgebras associated to general linear Lie superalgebras $gl(m|n)$: cyclicity and R-matrices. One essential ingredient in the proofs is the notion of Weyl module.

Algebraic COmbinatorics for Representation Theory, 2016

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 am	Welcome	Minicourse 3 part 1	Long Talk 5	Minicourse 1 part 2	Minicourse 3 part 2
9:15 am	Minicourse 1 part 1				
9:30 am					
9:45 am					
10:00 am			Coffee Break		
10:15 am			Long Talk 3		
10:30 am		Coffee Break		Coffee Break	Coffee Break
10:45 am	Coffee Break	Minicourse 4 part 1		Minicourse 2 part 2	Minicourse 4 part 2
11:00 am	Minicourse 2 part 1				
11:15 am			Coffee Break		
11:30 am			Long Talk 6		
11:45 am					
12:00 am					
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4:15 pm	Long Talk 1	Long Talk 3		Long Talk 7	
4:30 pm					
4:45 pm					
5:00 pm					
5:15 pm	Coffee Break	Coffee Break		Coffee Break	
5:30 pm	Short Talk 1	Short Talk 2		Short Talk 4	
5:45 pm					
6:00 pm	Coffee Break	Coffee Break		Coffee Break	
6:15 pm	Long Talk 2	Long Talk 4		Long Talk 8	
6:30 pm					
6:45 pm					
7:00 pm					
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8:00 pm					