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Renormalization in Spin Foam Quantum Gravity with Coherent States

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14th November 2016, Marseille

I States in (loop) quantum gravity

- Hilbert space

II Complexifier coherent states (canonical framework)

- properties
- application: semiclassical limit

III Livine-Speziale coherent states (‘spin foam models’)

- properties
- application: semiclassical limit

IV Application of LS cohernt states: renormalization group flow

V Summary & outlook

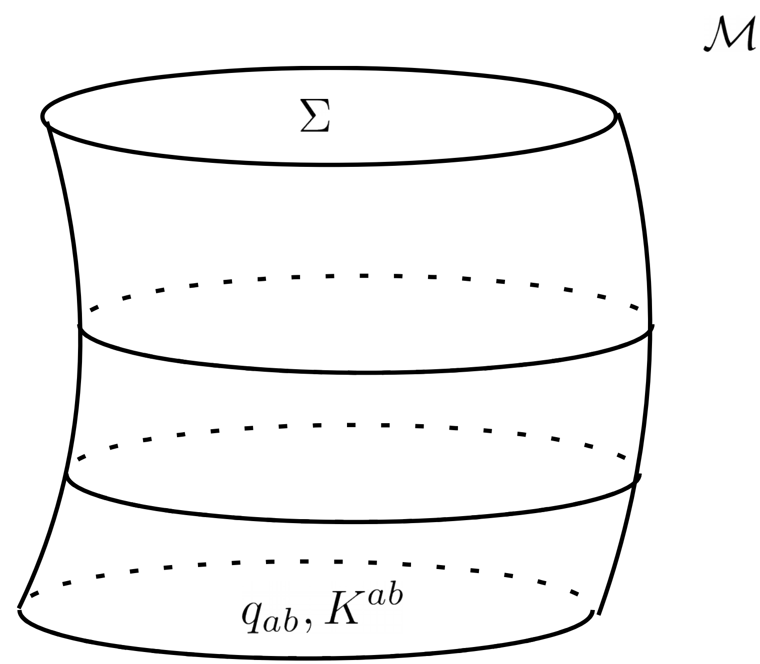
I: The loop quantum gravity Hilbert space

I Loop quantum gravity: variables

Quantization of general relativity: field = <u>geometry</u>

Canonical language: foliation

$$\mathcal{M} \simeq \Sigma \times \mathbb{R}$$



$$q_{ab} = \delta_{IJ} e_a^I e_b^J$$

variables:

'position': Ashtekar connection

$$A_a^I = \Gamma_a^I + \beta e^{b\,I} K_{ba}$$

'momentum': densitized dreibein

$$E_I^a = \det(e)\, \epsilon_{IJK} \epsilon^{abc} e_b^J e_c^K$$

$$\beta \in \mathbb{R} \backslash \{0, \pm 1\}$$

Barbero-Immirzi parameter

$$\{A, A\} = \{E, E\} = 0$$

$$\{A_a^I(x),\ E_J^b(y)\} \sim \delta_a^b \delta_J^I \delta(x, y)$$

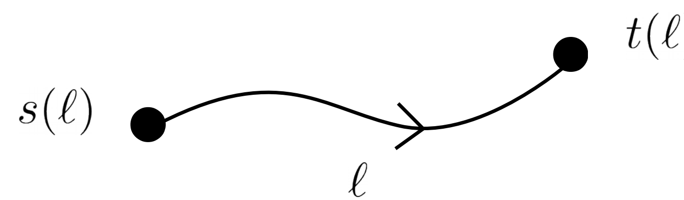
[Ashtekar '92, Ashtekar, Lewandowski '95, Ashtekar, Lewandowski, Marolf, Mourao, Thiemann '95, Rovelli, Smolin '95]

I Loop quantum gravity: variables (smeared)

Smearing of fields:

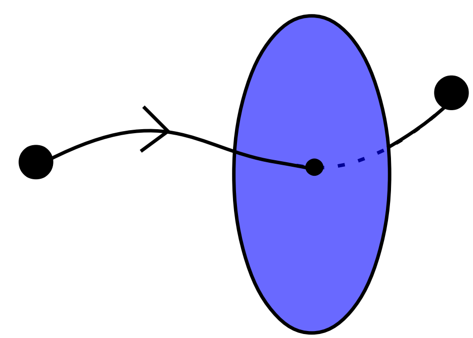
holonomies: parallel transport of connection along lines ℓ

$$h_\ell \ := \ \mathcal{P} \exp \int_\ell A \quad \in SU(2)$$



fluxes: densitized dreibein integrated over surfaces S

$$E_S \ := \ \int_S d^2x \ Ad_{h_{\ell_x}} E \quad \in \mathfrak{su}(2)$$



commutation relations:

$$\{h_\ell, h_{\ell'}\} \ = \ 0$$

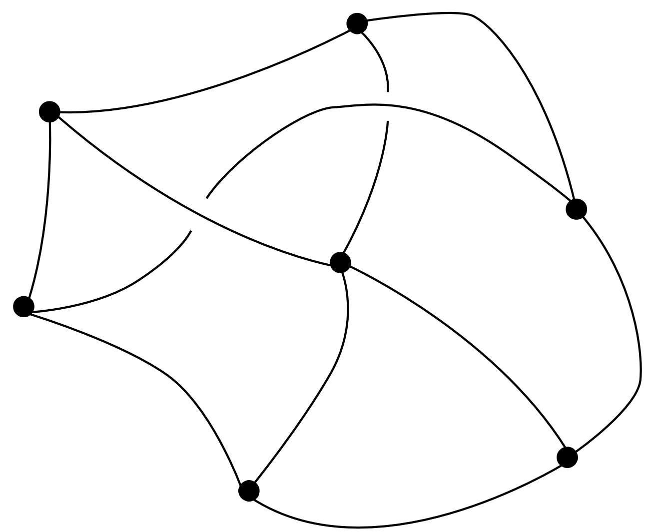
$$\{E_S^I, \ E_S^J\} \ = \ \epsilon^{IJ}{}_K E_S^K$$

$$\{h_\ell, E_S^I\} \ = \ \begin{cases} \tau^I \, h_\ell & \ell \text{ and } S \text{ intersect} \\ 0 & \text{else} \end{cases}$$

I Loop quantum gravity: Hilbert spaces

Hilbert space associated to one graph

$$\mathcal{H}_\gamma \;=\; L^2\big(SU(2)^L/SU(2)^N\big)$$

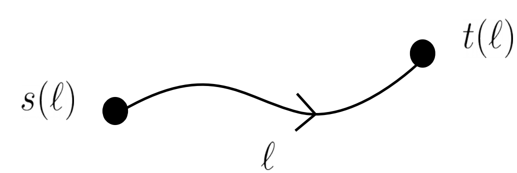


gauge transformation at nodes of the graph:

$$\alpha : SU(2)^N \times SU(2)^L \;\to\; SU(2)^L$$

$$\alpha_{\vec{k}} h_\ell \;:=\; k_{s(\ell)} h_\ell k^{-1}_{t(\ell)}$$

$$\psi(\vec{h}) \;=\; \psi(\alpha_{\vec{k}} \vec{h})$$



Projector to gauge-invariant subspace:

$$\Pi \;:\; L^2(SU(2)^E) \;:\longrightarrow\; \mathcal{H}_\gamma$$

$$\Pi \;=\; \int_{SU(2)^V} d^V k \; \alpha_{\vec{k}}$$

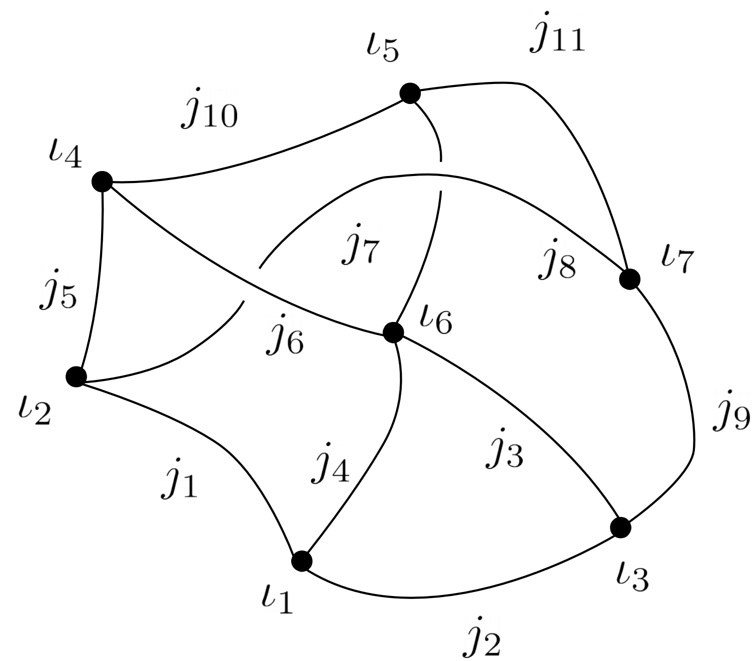
Orthonormal basis of graph Hilbert space:

spin network functions

$$\psi_{\vec{j},\vec{\iota}}(\vec{h}) = \left\langle \bigotimes_{\text{nodes } n} \iota_n, \bigotimes_{\text{links } \ell} D_\ell(h_\ell) \right\rangle$$

$$j_\ell \in \frac{1}{2}\mathbb{N}$$

$$\iota_n \in \mathrm{Inv}_{SU(2)}\left(\bigotimes_{\ell \supset n} V_{j_\ell}\right)$$



geometric interpretation:

nodes = quanta of space
links = intersection faces between neighbouring quanta

II: Complexifier coherent states

II CCS: Hall's construction of wavepackets

Relation between phase space & complexified configuration space

first: for one edge only → configuration variable: $h \in SU(2)$

$$\hat{C} \ := \ -\frac{t}{2}\Delta$$

complexifier (in this case: Laplace operator)

$$t \ = \ \frac{\hbar\kappa}{(\text{reference length scale})^2}$$

$$h^{\mathbb{C}} \ = \ \sum_{n=0}^{\infty} \underbrace{\{\hat{C},\{\hat{C},\{\cdots,\{\hat{C},h\}\}\cdots\}}_{n \text{ times}} \ = \ g \ = \ e^{\frac{t}{\hbar\kappa}E^I\tau_I}h \ \in \ SL(2,\mathbb{C})$$

$$\psi_g(h) \ = \ \left(e^{-t\hat{C}}\delta_{h_0}(h)\right)_{h_0\to g}$$

polar decomposition

$$\boxed{\psi_g^t(h) \ = \ \sum_{j\in\frac{1}{2}\mathbb{N}}(2j+1)e^{-j(j+1)\frac{t}{2}}\mathrm{tr}_j\left(gh^{-1}\right)}$$

[Hall '94, Ashtekar. Lewandowski, Marolf, Mourao, Thiemann '95, Thiemann '00, Thiemann, Winkler '01, Sahlmann, Thiemann, Winkler '01]

II CCS: properties

eigenvectors of annihilation operator:

$$\hat{g}_{AB}\,\psi^t_g \;=\; g_{AB}\,\psi^t_g$$

completeness relation:

$$\int_{SL(2,\mathbb{C})} d\nu^t(g)|\psi^t_g\rangle\langle\psi^t_g| \;=\; \mathrm{id}_{L^2(SU(2))}$$

coherent state transform: see talk by Hall

Ehrenfest properties for polynomials $\mathcal{O}(h,E)$:

$$\left\langle \mathcal{O}(\hat{h},\hat{E})\right\rangle_{\psi^t_g} \;=\; \mathcal{O}(h(g),\,E(g))\;+\;O(t)$$

peakedness properties:

$$\frac{|\langle\psi^t_g|\psi^t_{g'}\rangle|^2}{\|\psi^t_g\|^2\|\psi^t_{g'}\|^2} \;=\; \begin{cases} 1 & g=g' \\ O(t^\infty) & g\neq g' \end{cases}$$

$$\frac{|\langle\psi^t_h|\psi^t_{h'}\rangle|^2}{\|\psi^t_h\|^2\|\psi^t_{h'}\|^2} \;=\; \sum_{\gamma:h\to h'} f(d(\gamma))\exp-\frac{d(\gamma)^2}{t}$$

sum over geodesics γ from h to h'

II CCS: semiclassical limits of constraints

Complexifier coherent states on the whole graph:

$$\psi^t_{\vec{g}} \;:=\; \Pi\big(\otimes_\ell \, \psi^t_{g_\ell}\big)$$

[BB, Thiemann '07]

On a cubic graph with "spacing" ϵ : master constraint $\hat{M}$

$$\left\langle \hat{M} \right\rangle_{\psi^{t,\epsilon}_{\vec{g}}} = M_{\text{class}} + O(t,\epsilon)$$

[Giesel, Thiemann '06]

$\rightarrow$ consistency check of quantization of dynamics

"semiclassical perturbation theory": expansion in t

[Giesel, Thiemann '06
Stottmeister, Thiemann '15]

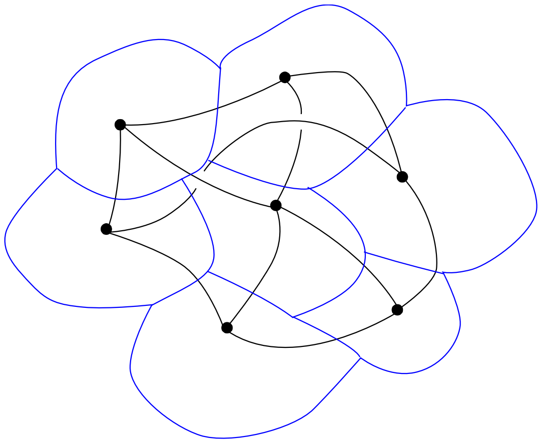
"dipole cosmology": computation of transition between homogenous, isotropic metrics: ~ FRW

[Rovelli, Vidotto '10, Bianchi, Rovelli, Vidotto '10]

III: Livine-Speziale coherent states

III LS states: geometric interpretation of spin network data

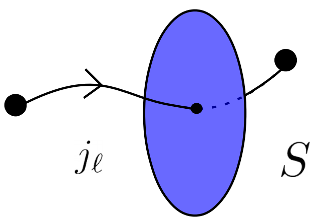
Spin network functions:



$$\psi_{\vec{j},\vec{\iota}}$$

$$j_\ell \in \frac{1}{2}\mathbb{N} \qquad \iota_n \in \mathrm{Inv}_{SU(2)}\Big(\bigotimes_{\ell \supset n} V_{j_\ell}\Big)$$

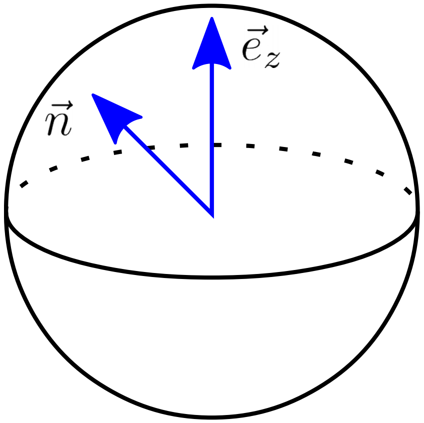
area operator:



$$\hat{A}r_S \psi_{\vec{j},\vec{\iota}} \sim 8\pi\beta\hbar\kappa \, j_\ell \, \psi_{\vec{j},\vec{\iota}}$$

group coherent states (up to a phase): $|j\ m\rangle \in V_j$

$$|j\ \vec{n}\rangle := \pi_j(h_{\vec{n}})|j\ j\rangle$$



$$\vec{n} = h_{\vec{n}} \triangleright \vec{e}_z$$

[Livine, Speziale '07]

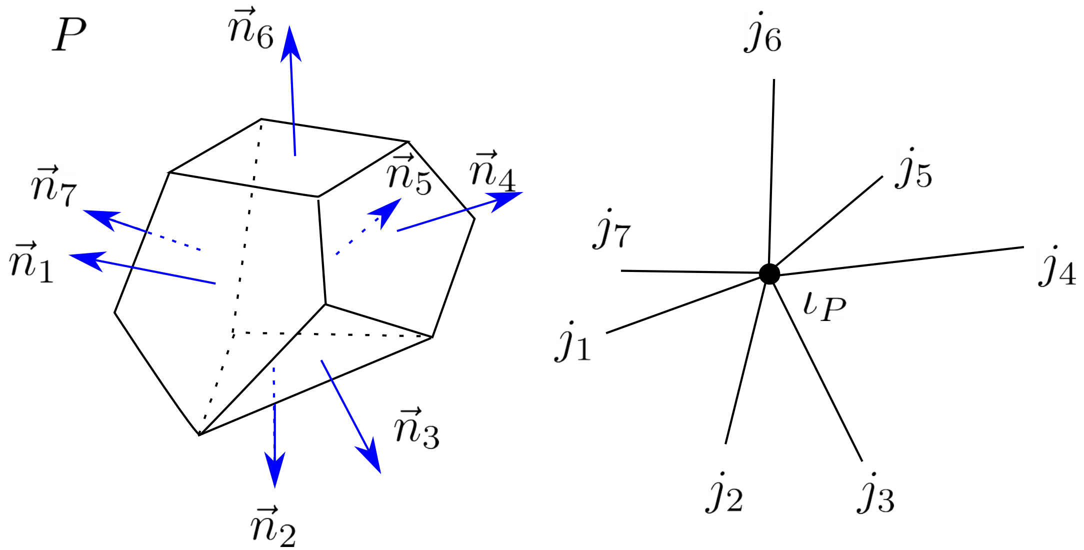
III LS states: “coherent polyhedra”

A convex polyhedron P in $\mathbb{R}^3$ with face areas j_ℓ :

set of vectors $\vec{n}_\ell$ satisfying closure condition

$$\sum_\ell j_\ell \vec{n}_\ell = 0$$

“coherent intertwiners”:



$$\iota_P := \int_{SU(2)} dh\, h \triangleright \left[\otimes_{\ell \supset n} |j_\ell\, \vec{n}_\ell\rangle \right]$$

$\longrightarrow$ see talk by Speziale for more details

III LS states: properties

Coherent intertwiners $\iota_P = \int_{SU(2)} dh\ h \triangleright \left[\otimes_{\ell \supset n} |j_\ell\ \vec{n}_\ell\rangle \right]$

Form an overcomplete basis:

$$\int_{(S^2)^L} d^2 n_1 \cdots d^2 n_L\ |\iota_P\rangle\langle\iota_P| = \frac{1}{\prod_\ell (2j_\ell + 1)} \mathrm{id}_{\mathrm{Inv}_{SU(2)}}[\otimes V_\ell]$$

Minimise fluctuations of operators: normal vectors

$$\frac{\langle\iota_P|\ n^I_\ell \hat{E}_I\ |\iota_P\rangle}{\langle\iota_P|\ \iota_P\rangle} = j_\ell \qquad\qquad \Delta_{\iota_P}\left(n^I_\ell \hat{E}_I\right) = 0$$

can be used to give “quasi-geometric” boundary data

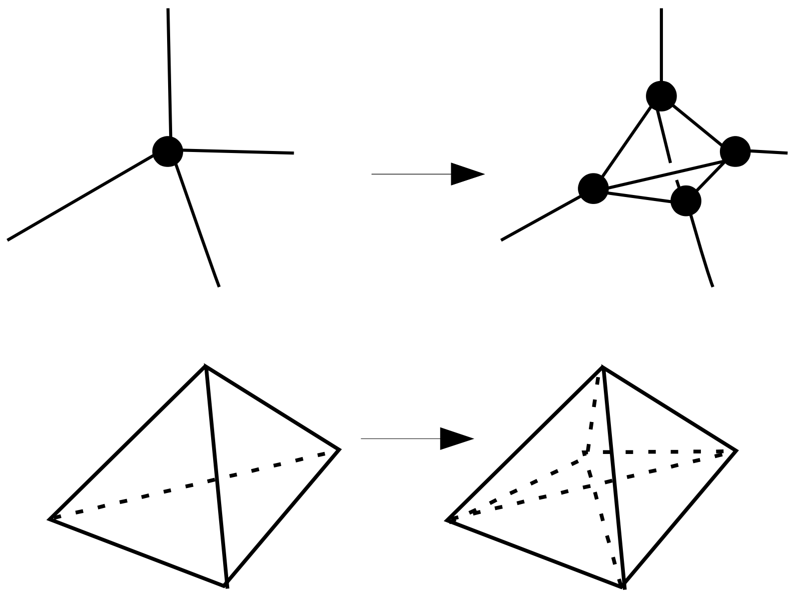
used in a variety of ways in loop quantum gravity, $\longrightarrow$ see talk by Rovelli
e.g. computations of black hole/white hole transition

Christodoulu, Rovelli, Speziale, Vilensky ‘16]

III LS states: application: large-j-limit

Coherent intertwiners $\iota_P = \int_{SU(2)} dh\, h \triangleright \left[\otimes_{\ell \supset n} |j_\ell\, \vec{n}_\ell\rangle \right]$

As boundary data for the path integral formulation “spin foams”:



Initial state: one tetrahedron
Final state: four tetraheda

“Pachner move”

Boundary of a four-simplex

Spin Foam amplitude $\mathcal{A}_v$

$$\mathcal{A}_v \overset{j_\ell \to \infty}{\sim} e^{\frac{i}{\hbar} S_{\mathrm{Regge}}} + \text{c.c.}$$

Regge action appears in limit of large areas:
discrete general relativity!

[Reisenberger '94, Barrett, Crane '99, Livine, Speziale '07, Engle, Pereira, Rovelli, Livine '07, Freidel, Krasnov '07, Kamiński, Kisielowski, Lewandowski '09, Han, Thiemann '10, Oriti Baratin '11, ...]

[Barrett et al '08, Freidel, Conrady '08]

IV: Renormalization with LS coherent states

IV Background-independent renormalization

Idea of renormalization in quantum gravity: notoriously difficult

perturbatively non-renormalizable (see EFT methods, though)

$$g_{\mu\nu} = \underbrace{\eta_{\mu\nu}}_{\text{classical background}} + \underbrace{h_{\mu\nu}}_{\text{quantum fluctuations}}$$

<u>important:</u> renormalization is *not* about “getting rid of infinities”, but about the flow of coupling constants describing how theory behaves effectively at different scales (Wilson).

background-independent renormalization: what is a “scale”? Geometric data is encoded in the field (metric) itself!

basic idea used here: “scale” = fine-ness of used graph

IV Background-independent renormalization in spin foam models

Host of literature:

scale = discretization of space-time

[Oeckl ‘03, Manrique, Oeckl, Weber, Zapata ‘06]

“summing over transitions between spin network functions” ~ “continuum limit”

[Rovelli, Smerlak '12, Dittrich '14]

Self-energy-computation

[Perini, Rovelli, Speziale '09, Riello '13]

RG flow in spin foam models:

[BB, Dittrich '09, Dittrich '12]
[Dittrich, Eckert, Martin-Benito '12, BB, Dittrich, Hellmann, Kaminski '13, BB ‘14]

connection to tensor network renormalization

[Dittrich, Martin-Benito, Schnetter '13, Dittrich, Mizera, Steinhaus '14]
[Dittrich, Martin-Benito, Steinhaus '14 Steinhaus '15]

compare: group field theory approach

[e.g. Rivasseau ‘11 & references therein]

compare: CDT & Asymptotic Safety

[Niedermair, Reuter ‘06, Saueressig, Reuter ‘07]
[Ambjorn, Jurkiewicz, Loll, ‘04, Ambjorn, Göhrlich, Jurkiewicz, Kreienbuehl, Loll ‘14]

IV Background-independent renormalization: setup

Basic idea of the RG step: relate path integral of different discretizations of space-time (different lattices, but no lattice constant!)

path-integral amplitude of a

<u>piece of space-time</u> (“vertex”)

(general boundary formalism)

here: hypercuboid (no geometry yet, though!)

vertex amplitude: $\mathcal{A}_v$

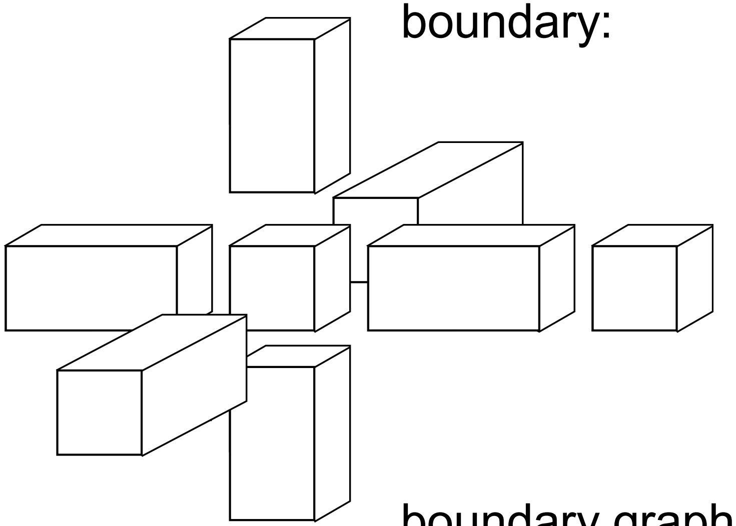
$$Z = \sum_{\vec{j},\vec{\iota}} \prod_f \mathcal{A}_f \prod_e \mathcal{A}_e \prod_v \mathcal{A}_v$$

sum over spins/intertwiners in the <u>bulk</u>

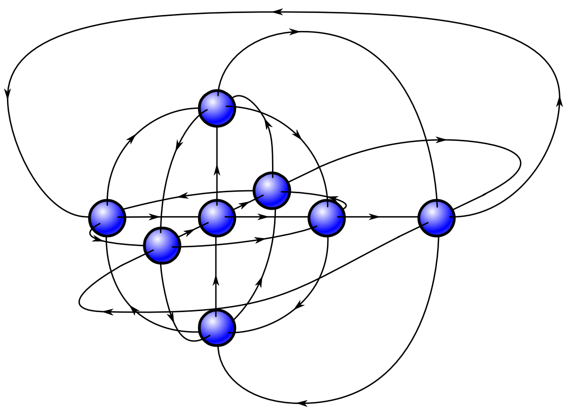
spins/intertwiners on the boundary kept fixed

[Reisenberger '94, Barrett, Crane '99, Oeckl ‘03, Oeckl ‘08, Engle, Pereira, Rovelli, Livine '07, Freidel, Krasnov '07]

boundary:



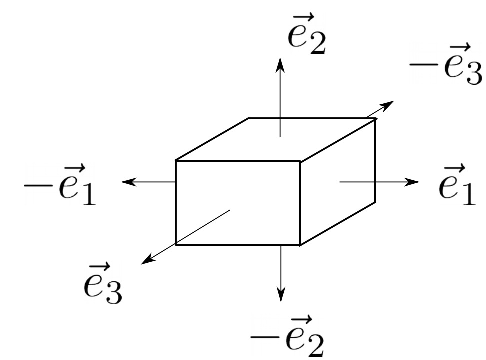
boundary graph:



IV Background-independent renormalization: coarse graining

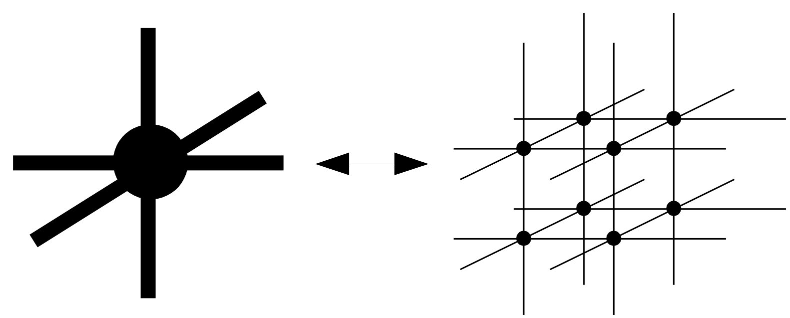
Very crude approximation: restrict to quantum cuboids

$$\iota_{j_1,j_2,j_3} = \int_{SU(2)} dh\; h \triangleright \left[\bigotimes_{i=1}^{3} |j_i\, e_i\rangle |j_i\, -e_i\rangle \right]$$

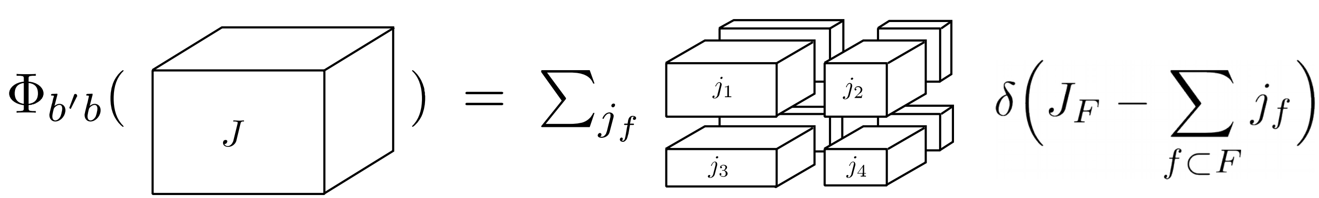


RG step: coarse grain 2 x 2 x 2 x 2 hypercuboids to one → “block spin transformation”

For this one needs to relate coarse and fine graphs:



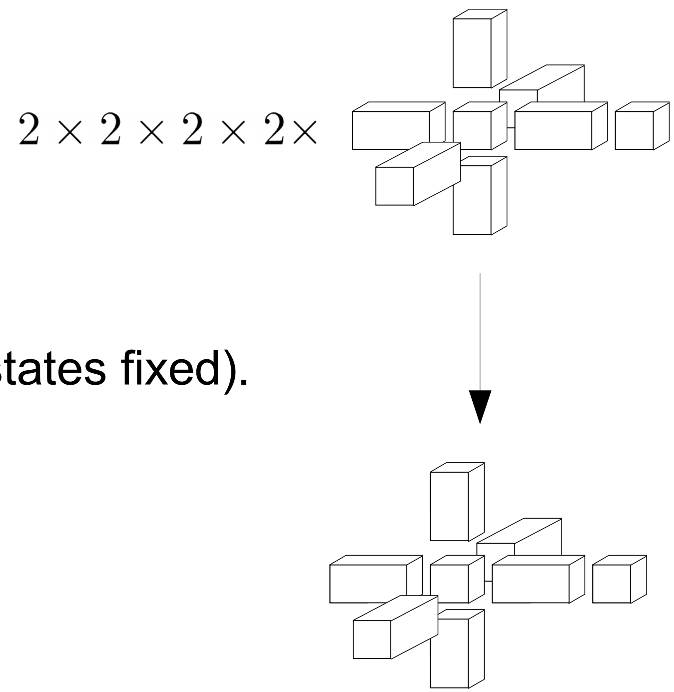
embedding map:



[BB, Steinhaus ‘15, BB, Steinhaus ‘16]

IV Background-independent renormalization: RG step

Path integral (state-sum):



$$Z \;=\; \sum_{\vec{j},\vec{\iota}} \prod_f \mathcal{A}_f^{(\alpha,\beta)} \prod_e \mathcal{A}_e^{(\beta)} \prod_v \mathcal{A}_v^{(\beta,\,\kappa)}$$

sum over states in the bulk (keeping boundary states fixed).

Sum is weighted by amplitude functions

$$\mathcal{A}_f^{(\alpha,\beta)} \;=\; \Big((|1+\beta|j_f+1)(|1-\beta|j_f-1)\Big)^{\alpha}$$

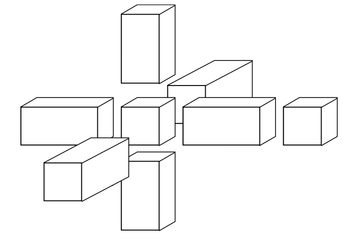
Restriction to quantum cuboids & large- j -limit: only α plays a role

Hypercubic lattice: absorb face- and edge- amplitudes into the vertex amplitude:

$$\hat{\mathcal{A}}_v^{(\alpha)} \;:=\; \mathcal{A}_f^{(\alpha)\,\frac{24}{4}} \mathcal{A}_e^{\frac{8}{2}} \mathcal{A}_v \qquad\qquad Z \;=\; \sum_{\vec{j}} \prod_v \hat{\mathcal{A}}_v^{(\alpha)}$$

IV Background-independent renormalization: RG step

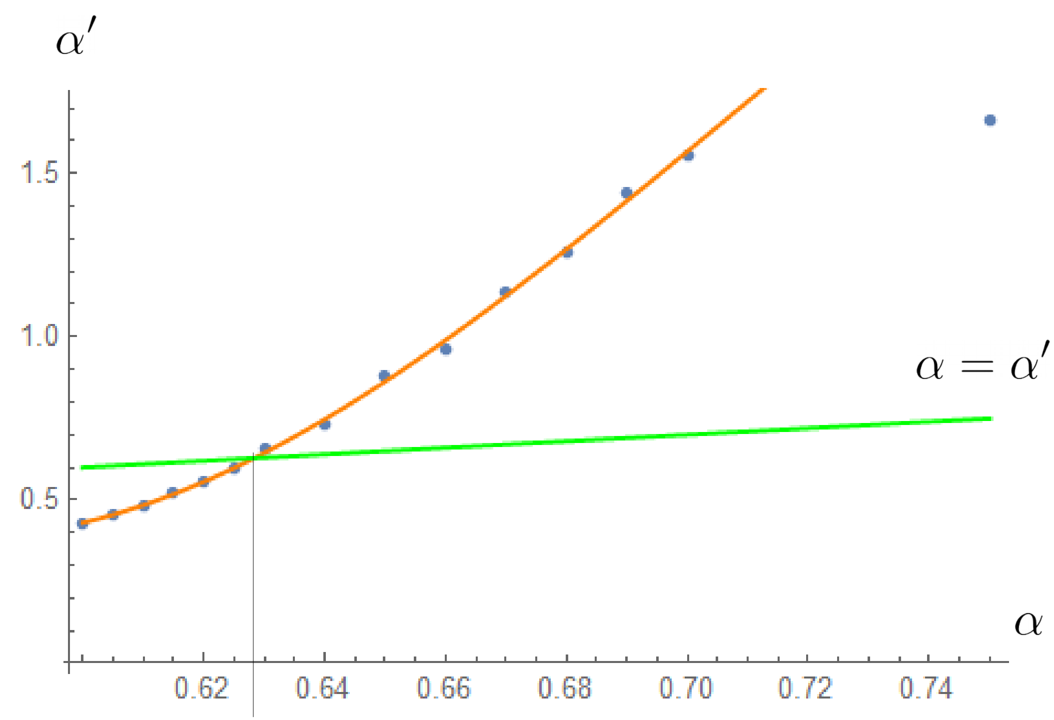
Renormalization group equation:

$2 \times 2 \times 2 \times 2 \times$ 

$$\hat{\mathcal{A}}_v^{(\alpha')}(J_F) \;=\; \frac{1}{N_{\vec{J}}} \int dj_f \; \prod_F \delta\Big(J_F - \sum_{f\subset F} j_f\Big) \prod_{i=1}^{16} \hat{\mathcal{A}}^{(\alpha)}(j_f)$$

RG step: $\alpha \;\longrightarrow\; \alpha'$

Fixed point at $\alpha_c \;\approx\; 0.63$



$\alpha_c \;\approx\; 0.63$

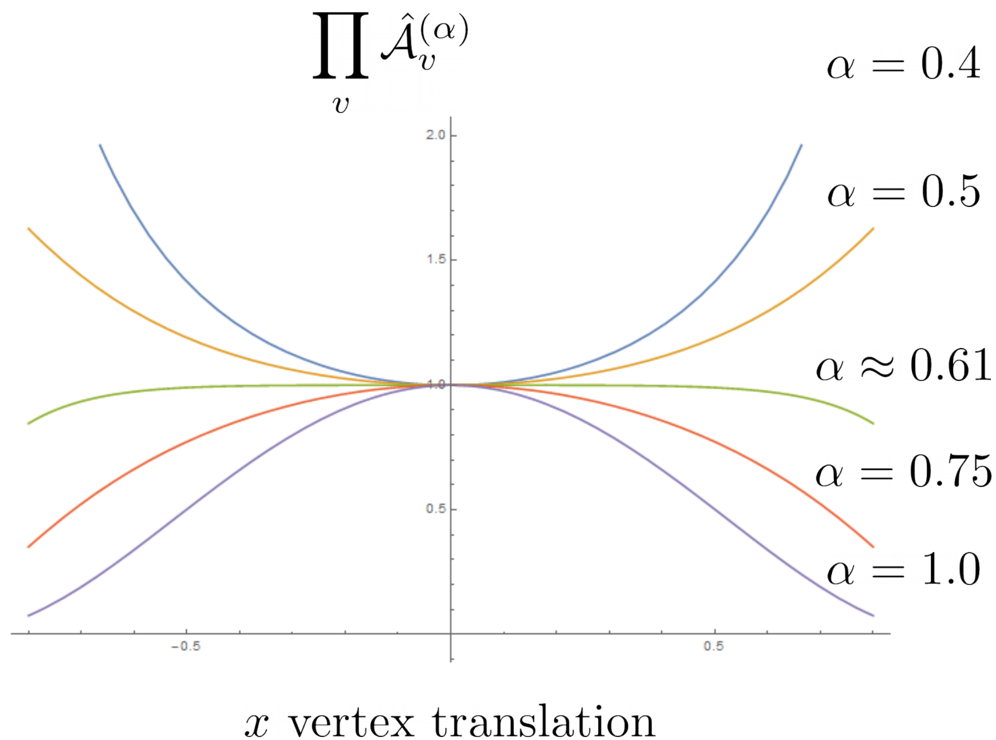
IV Background-independent renormalization: properties of the fixed point

Encouraging:

* There is a fixed point of the RG flow: all characteristics of non-Gaussian fixed point: interacting, UV-attractive

* At the fixed point, the path integral measure becomes invariant under vertex-translations: discrete analogue of diffeomorphisms

* Value of α_c varies only marginally for different boundary values



[Livine, Speziale ‘06]

[Dittrich ‘08, BB, Dittrich ‘09, BB, Dittrich, Steinhaus ‘11, Rovelli ‘11]

[Niedermair, Reuter ‘06, Saueressig, Reuter ‘07]

V: Summary & Outlook

V Summary & outlook: CCS

Coherent states play a crucial role in (loop) quantum gravity calculations

complexifier coherent states:

* built on Hall's group coherent states
* approximate phase space point in LQG
  (peakedness, Ehrenfest theorem, etc.)
* exist in gauge-variant and -invariant form.
* used for testing the semiclassical limit of the quantization
* used for semiclassical perturbation theory

open questions:

* Gauss constraints satisfied by gauge-invariant CCS. How about
  spatial diffeomorphisms and Hamilton constraints?
  → search for coherent states stable under "time evolution"
  → need states on several graphs simultaneously

V Summary & outlook: LS states

Coherent states play a crucial role in (loop) quantum gravity calculations

Livine-Speziale states:

* built on Gilmore-Perelomov's group coherent states
* sharp in areas, maximally uncertain in extrinsic curvature
* coherent polyhedra: wave packets on space of shapes

Many uses (see talk by Rovelli!)

* testing the semiclassical limit of the spin foam path integral
* black hole / white hole transition
* used for calculations in renormalization of the path integral

V Summary & outlook: renormalization

LS coherent states can be used to compute the Wilsonian RG flow of the path integral!

* First crude approximation: restrict “sum over all states” to “sum over quantum cuboids”. Disadvantage: no curvature degrees are being summed over. But: excellent to investigate vertex translation symmetry.

* Still nontrivial result: nontrivial RG flow of coupling constant α

* UV fixed point!

* Vertex translations seems to only be a symmetry at fixed point.

* Critical value of α separates two regions “very regular space-times” and “very irregular space-times”: phase transition?

* Need to incorporate more degrees of freedom → geometric interpretation due to the nature of coherent states