Coherent States and their Applications: A Contemporary Panorama

14 - 18 November, 2016

Paolo Aniello: Square integrable representations, an invaluable tool: from coherent states to quantum mechanics on phase space.

Square integrable representations are not only remarkable mathematical objects but also an invaluable tool in various fields of theoretical and applied physics. We will focus on the role that they play in the definition of families of (generalized) coherent states, in the phase-space formulation of quantum mechanics and the associated star product formalism, and in some applications related to quantum dynamical semigroups.

Isiaka Aremua: $\mathfrak{su}(1,1)$ coherent states for Landau levels: Physical and mathematical description.

This talk addresses a study on the $\mathfrak{su}(1,1)$ Lie algebra and Barut-Girardello coherent states (BGCS) for a physical system governed by a Hamiltonian operator, in a two-dimensional space, of spinless charged particles subject to a perpendicular magnetic field **B**, coupled with a harmonic potential. The underlying physical and mathematical properties are deduced and discussed. A special treatment is devoted to the Berezin–Klauder–Toeplitz quantization, also known as coherent state (or anti-Wick) quantization.
Joint work with Mahouton Norbert Hounkonnou and Ezinvi Balo.

Benjamin Bahr: Renormalization in spin foam quantum gravity with coherent states.

Coherent states have manifold uses in various aspects of (loop) quantum gravity. I will give examples for statements about the semiclassical limit in the canonical, and about renormalization in the covariant version of the theory. For both of these the use of coherent states is crucial. In particular I will discuss the role of diffeomorphism symmetry at the RG fixed point, for which the geometric interpretation of the coherent states is an integral part.

Hervé Bergeron: Affine Coherent State Quantization and Quantum Cosmology.

Coherent State Quantization (CSQ) pertains to a general approach named in [1] integral quantization (see also Chapt. 11 of [2]). In the case of the Weyl-Heisenberg group, CSQ leads to the "standard" Berezin-Klauder-Toeplitz quantization of the plane. If the phase-space is the half-plane $\{(q,p)\,|\,q>0, p\in\mathbb{R}\}$, the natural group is no longer the Weyl-Heisenberg one but the affine one. Integral or CS quantization can be developed following the same lines [1, 3]. I will show how this approach allows to keep the main features of canonical quantization while solving some self-adjointness problem (singularity or boundary problem). I will develop a typical application given by the quantum cosmology framework [4, 5, 6].

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Pierre Bieliavsky: Coherent states and non-commutative surfaces in higher genera.

We construct a version of noncommutative surfaces analogous to the well-known noncommutative torus. More precisely, we define an associative deformed multiplication of the algebra of smooth functions on any compact surface of negative constant curvature. The deformation is non-formal in the sense that the deformed product of any two smooth functions is again a smooth function- rather than a formal power series as in formal star-product theory. The deformation consists in a real one-parameter smooth family of associative products whose infinite jet at the value zero of the parameter defines an associative formal star product directed by the Kaehler two form. For any value of the parameter the deformed algebra admits a natural topology which endows it with the structure of a Fréchet algebra. Each of these noncommutative Fréchet algebras carries a trace defined by the usual integral on the surface. Moreover, these algebras are tracial w.r.t. the trace form, in the sense that the trace of the deformed product of two functions equals the integral of the pointwise multiplication of these functions. The deformed algebra when equipped with the complex conjugation also turns into a star-algebra. In particular they extend to the space of square integrable distributions as an algebra of Hilbert-Schmidt operators. A quantization in the usual sense represent them as subalgebras of bounded operators acting on the projective discrete series representations of SL(2,$\mathbb{R}$). The construction strongly relies on the wavelet theory of the group of affine transformations of the real line.

Guido Chiribella: New applications of coherent states in quantum information theory.

Coherent states have been long known for their applications in quantum optics and atomic physics. In recent years, a number of new applications have emerged in the area of quantum information theory. In this talk I will highlight two such applications. The first is the comparison between classical and quantum strategies to process information. Byproducts of this comparison are benchmarks that can be used to certify quantum advantages in realistic experiments, fundamental relations between quantum copy machines and precision measurements, and theoretical tools for security proofs in quantum cryptography. The second application is the simulation of unitary gates in quantum networks. Here the task is to simulate a given set of unitary gates using gates in another set, a general problem that includes as special cases the simulation of charge conjugate dynamics and the emulation of an unknown unitary gate. The problem turns out to have useful connections with the ultimate precision limits of quantum metrology.

Evaldo M. F. Curado: Construction of linear and nonlinear coherent states using GHA.

The algebraic structure called Generalized Heisenberg Algebra (GHA) was introduced some years ago and in general any physical system having only one quantum number can be written as a GHA. In its simpler form it consists of three operators: a self-adjoint one, essentially the Hamiltonian H; a creation and an annihilation ladder operators; and a characteristic function that relates the $H(n+1)$-th eigenvalue with the n-th eigenvalue. It was shown that this algebra is a useful tool to construct linear and non-linear coherent states. We present some examples of construction of coherent states using GHA and an example where it can be used to construct linear and non-linear coherent states for the same physical system. Also, it allows us to relate the physical system under analysis with another quantum physical system having the same Hilbert space and whose Hamiltonian has the same spectrum as the original one.

Ewa Czuchry: Regularised Bianchi IX potential.

The quantum version of the Bianchi IX potential is presented. It is obtained via integral Gaussian quantisation. The classical escape canyons become regularised and the whole potential is now fully confining. It is considered to be a perturbation of the one associated to integrable systems. Below, the classical Bianchi IX potential (left) and its regularized quantum version (right).

Ingrid Daubechies: Phase retrieval in infinite dimensions.

Retrieving an arbitrary signal from the magnitudes of its inner products with the elements of a frame is not possible in infinite dimensions. Under certain conditions, signals can be retrieved satisfactorily however

Stéphan De Bièvre: Entanglement of quantum circular states of light.

Since entanglement is a resource for quantum computing, it is of interest to produce highly entangled states. We will present in this talk a general approach to calculating or estimating the entanglement of formation for arbitrary superpositions of N two-mode coherent states, placed equidistantly on a circle in the phase space. For small N, such states have been produced experimentally. We provide analytical expressions for their entanglement in the particular case of rotationally-invariant circular. We further analyse

the dependence of the entanglement on the radius of the circle and number of components N in the superposition.
Joint work with D. Horohko (Minsk and Lille), M. Kolobov (Lille) and G. Patera (Lille)

Mariano del Olmo: Covariant integral quantization of the unit disk.

The SU(1,1) covariant integral quantization of functions is implemented on the unit disk (coset space of SU(1,1)), which can be viewed as the phase space of a particle on a 1+1 anti de Sitter space-time. SU(1,1) coherent states quantization is included.

Viktor Dodonov: Coherent and minimum energy states of a charged particle in a uniform magnetic field.

I intend to give a review of various families of coherent and squeezed states for a charged particle in a uniform magnetic field, that have been constructed for almost 50 years. A special attention will be paid to the cases of time-dependent fields in different gauges (circular and Landau ones). Other items include the so called semi-coherent states and minimum energy rotating pure and mixed Gaussian packets.

Miroslav Englis: Hankel operators and the Dixmier trace on the Hardy space.

We give criteria for the membership of Hankel operators on the Hardy space on the disc in the Dixmier class, and establish estimates for their Dixmier trace. In contrast to the situation in the Bergman space setting, it turns out that there exist Dixmier-class Hankel operators which are not measurable (i.e. their Dixmier trace depends on the choice of the underlying Banach limit), as well as Dixmier-class Hankel operators which do not belong to the $(1, \infty)$ Schatten-Lorentz ideal. A related question concerning logarithmic interpolation of Besov spaces is also discussed.
Joint work with G. Zhang, Göteborg.

Michael Fanuel: Coherent states, Support Vector Machines and function estimation.

There exist analogies between certain function estimation methods, especially Support Vector Machines, and coherent states. In particular, these function estimation methods rely on a primal and dual optimization problem involving a Reproducing Kernel Hilbert Space. A major practical question in this framework concerns the choice of kernel and its bandwidth. The relevance of objects such as coherent states for the design of kernels will be discussed, as well as possible applications of graph wavelets to function estimation on graphs. This is a work in progress.

Hartmut Führ: Wavelet Approximation Theory in Higher Dimensions.

The success of wavelets in applications such as signal and image processing is to a large degree based on their approximation theoretic properties. The central mathematical statement that summarizes these properties is the characterization of smoothness spaces such as Besov spaces in terms of summability conditions on the wavelet coefficients. In more modern terminology, the elements of a Besov space can be understood as the sparse signals with respect to the wavelet systems.

In higher dimensions, an increasing variety of generalized wavelet systems becomes available, each of which induces its own spaces of sparse signals. In this talk, the methodology of *coorbit spaces*, developed by Feichtinger and Gröchenig, is introduced as a very general and flexible means of describing spaces of sparse signals over generalized wavelet systems. The method provides very general consistency and discretization results for the wavelet systems under review. Here consistency refers to the important property that, for any two analysing wavelets within a well-understood class of functions, the spaces of sparse signals coincide.

Thus wavelet coorbit space theory provides a comprehensive and unified scheme that comprises many known examples such as isotropic wavelets and Besov spaces in higher dimensions, shearlets and their coorbit spaces in arbitrary dimensions, and many more.

Andreas Fring: Coherent states for unitary quantum evolution for time-dependent quasi-Hermitian systems with non-observable Hamiltonians.

We give a brief overview of PT-symmetric non-Hermitian quantum mechanics for a time-independent setting. Subsequently we demonstrate how one can maintain unitary time-evolution even for time-dependent non-Hermitian Hamiltonians when the metric operator is explicitly time-dependent. We demonstrate here that the time-dependent Dyson equation and the time-dependent quasi-Hermiticity relation can be solved consistently in such a scenario for a time-dependent Dyson map and time-dependent metric operator, respectively. The solutions are obtained at the cost of rendering the non-Hermitian Hamiltonian to be a non-observable operator as it ceases to be quasi-Hermitian when the metric becomes time-dependent. We discuss how coherent states are constructed in these scenarios.

Katarzyna Górska: Hermite polynomials in two complex variables Mathematical properties.

I will present algebraic and analytic features of Hermite polynomials in two complex variables. Their algebraic properties are analogous to those obeyed by the standard Hermite polynomials. A generalization of non-rotational orthogonality invented by van Einjdhoven and Meyers is used to investigate analytic properties and to show that the Hermite polynomials in two complex variables give rise to orthonormal bases in Bargmann-like Hilbert spaces of entire functions.

Julio Guerrero: Non-Hermitian coherent states for finite-dimensional systems.

Since the introduction of non-Hermitian Hamiltonians [1] in Quantum Mechanics, only a few papers have been devoted to coherent states (CS) for non-Hermitian systems (see, for instance, [2], where Gazeau-Klauder CS are constructed using the definition of scalar product in terms of the CPT norm, or [3]). Here we shall introduce Gilmore-Perelomov CS for finite-dimensional non-unitary representations of non-compact groups, and discuss the main similitudes and differences with respect to ordinary Gilmore-Perelomov CS.

The example of CS for the non-unitary finite dimensional representations of SO(2,1) is considered and use them to describe the propagation of light in coupled PT-symmetric optical devices [4,5].

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Brian Hall: Coherent state transforms for compact groups, and their large-N limits.

I will review results about the generalized Segal-Bargmann transform for compact Lie groups. The coherent states for this transform are defined by replacing the Gaussian function on Euclidean space with the heat kernel on a compact group. I will briefly describe connections to geometric quantization, quantum gravity, and the quantization of 1+1-dimensional Yang-Mills theory. I will then describe recent results with Bruce Driver and Todd Kemp about the large-N limit of the transform for the unitary group $U(N)$. These results connect to the concept of the master field in quantum field theory and to ideas of free probability in the context of random matrix theory.

Andrzej Horzela: Hermite polynomials in two complex variables: Coherent states.

Coherent states constructed using various Hermite-like polynomials of a complex variable have been proposed in physical literature for a few years. I am going to present a construction of two particle coherent states based on a recently developed theory of Hermite polynomials in two complex variables. Properties of such coherent states, especially those implied by their non-factorial character and related to entanglement, will be discussed.

Véronique Hussin: Coherent states for supersymmetric partners of solvable systems.

Supersymmetric methods have been very fruitful for constructing partners of exactly solvable systems in nonrelativistic quantum mechanics. Starting from an initial Hamiltonian, a partner is obtained through an intertwining relation including the introduction of a differential operator, called supercharge. The Hilbert space span by the new energy eigenfunctions is a direct sum of two sets. One set constitutes a subset almost isospectral (same energy eigenvalues) with respect to the one of the initial Hamiltonian. The second set is new and its dimension is always finite.

The construction of coherent states for such partners is an important non-trivial question. Indeed, we know already that there exist many definitions of such states for a long list of solvable systems. Some of them rely on the existence of ladder operators acting on the eigenfunctions. The presence of two distinct sets of eigenfunctions for the supersymmetric partner makes the construction of those states more complicated. We present such a construction for two systems, in particular, the infinite well and the truncated oscillator. We study also the properties of those states with respect to uncertainty relations and entanglements.

Alain Joye: Representations of CCR describing infinite coherent states.

We investigate the infinite volume limit of quantized photon fields in multimode coherent states. We show that for states containing a continuum of coherent modes, it is natural to consider their phases to be random and identically distributed. The infinite volume states give rise to Hilbert space representations of the canonical commutation relations which are random as well and can be expressed with the help of Itoö stochastic integrals. We analyze the dynamics of the infinite coherent state alone and that of open systems consisting of small quantum systems coupled to the infinite coherent state. Under the free field dynamics, the initial phase distribution is shown to be driven to the uniform distribution, and coherences in small quantum systems interacting with the infinite coherent state, are shown to exhibit Gaussian time decay, instead of the exponential decay caused by infinite thermal states.
Joint work with Marco Merkli.

John Klauder: Enhanced Quantization: The *Right* Way to Quantize *Everything.*

Canonical quantization promotes phase space variables that are "Cartesian" to canonical operators even though a classical phase space contains no metric. Enhanced quantization offers a different connection between quantum and classical variables that (i) yields the same result as canonical quantization for systems when canonical quantization works, and (ii) offers an acceptable result when canonical quantization fails (e.g., scalar field theory in spacetime dimensions greater than 4). With $\hbar > 0$ throughout, the key idea is that the action functional for enhanced classical systems is obtained from the action functional for quantum systems when the set of vectors that may be varied is limited to a suitable set of coherent states. Two idealized but basic examples with nontrivial classical behavior and trivial quantum behavior when canonically quantized are shown to have nontrivial, acceptable quantum behavior when quantized the new way. Although not presented, similar improvements also hold for covariant scalar fields in all spacetime dimensions, Einstein quantum gravity, and other examples.

A.B. Klimov and I. J.L. Romero: Wigner-like function for variable spin systems: semiclassical limit and asymptotic quantization.

We analyze the semiclassical limit of an SU(2) covariant Wigner-like map for variable-spin systems. It follows from the analysis of the exact star-product that the semiclassical dynamics of the corresponding Wigner function is described by classical trajectories on the cotangent bundle T^*S_2. This opens a possibility for an asymptotic quantization of a particle on the two-dimensional sphere.

Przemysław Małkiewicz: Coherent states in a study of time problem.

In general relativity there is no preferred notion of time. For quantizations of general relativistic models one often employs an internal degree of freedom to play a role of clock with respect to which quantum dynamics of the remaining degrees of freedom is formulated. I will show that the internal clock fixes the canonical structure in the Hamiltonian formalism. Since the canonical structure is a starting point for quantizations, one expects the nature of induced quantum dynamical effects to be tied to the choice of clock. Next, I will discuss a basic model of singularity resolution in a Friedmann-Robertson-Walker (FRW) universe derived from (affine) coherent state quantization. I will make use of a consistent semiclassical framework to study physical dissimilarities between the basic models of singularity resolution, which were obtained in different clocks. I will show you the 'clock effect'.

Ugo Moschella: Two dimensional de Sitter spinors and their $SL(2,\mathbb{R})$ covariance.

I will present a construction related to Dirac fields on the two-dimensional de Sitter manifold. The steps of the construction include a description and a novel interpretation of the coset space $SL(2,\mathbb{R})/A$ (where A is the abelian subgroup in a Iwasawa decomposition) as the double covering of the two-dimensional de Sitter manifold. This part of my talk may have connection with the construction of new families of de Sitter coherent states (not covered in my talk)

Zouhaïr Mouayn: Orthogonal polynomials attached to coherent states for the symmetric Pöschl-Teller oscillator.

We consider a one-parameter family of nonlinear coherent states by replacing the factorial in coefficients $z^n/\sqrt{n!}$ of the canonical coherent states by a specific generalized factorial $x_n^\gamma!, \gamma \geq 0$. These states are superposition of eigenstates of the Hamiltonian with a symmetric Pöschl-Teller potential depending on a parameter $\nu > 1$. The associated Bargmann-type transform is defined for $\gamma = \nu$. Some results on the infinite square well potential are also derived. For some different values of γ, we discuss two sets of orthogonal polynomials that are naturally attached to these coherent states.

Anatol Odzijewicz: Classical and quantum Kummer shape algebras.

We present a family of integrable systems of nonlinear coupled harmonic oscillators on the classical and quantum levels. We show that the integrability of these systems follows from their symmetry characterized by algebras, called by us Kummer shape algebras. Using Berezin covariant symbols defined by Glauber coherent states we show that the quantum Kummer shape algebra corresponds to the classical Kummer shape algebra in the limit $\hbar \to 0$. Next, combining the classical reduction procedure with the quantum one we show that one can obtain the coherent states of quantum Kummer shape algebra by the reduction of Glauber coherent states. We also show, that passing to the classical limit $\hbar \to 0$ intertwines classical and quantum reduction procedures.

As a by-product of our investigation we find the explicit formula for a wide class of reproducing kernels.

The quantum Kummer shape algebras were investigated in a systematic way in [2], [3], where the authors had in mind their applications to nonlinear phenomena in quantum optics. We believe it was V.P. Karassiov, see [4], who first applied these structures to quantum optics problems. They were also discussed in numerous papers by different authors as so-called deformed bosonic oscillator algebras.

The connection of these algebras with the coherent state method of quantization and the theory of q-special functions was investigated in [7].

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Robert Oeckl: Fermionic coherent states in infinite dimensions.

While coherent states for bosons have played an important role in the description of quantum systems for a long time, this is much less true for fermionic coherent states in spite of the abundance of fermions in nature. There are basically two approaches in the existing literature. One mimics the structure and features of the bosonic coherent states at the price of introducing anti-commuting parameters. This means that the constructed objects are not states in the Hilbert space, but live in a super-extension. The other is a more geometric approach in the sense of Gilmore and Perelomov, promising for its potential to shed light on the semiclassical aspects of fermions. So far the literature on these coherent states is scarce and almost exclusively limited to systems with finitely many degrees of freedom. We present new developments on these coherent states and the associated mathematical structures in infinite dimensions, as required for quantum field theory. In particular, we shall see that the space of fermionic coherent states can be understood as an infinite-dimensional reductive homogeneous space which is a Kaehler manifold. Moreover, the fermionic Fock space becomes a reproducing kernel Hilbert space. Infinite dimensional Lie groups as well as quantum group symmetries play important roles.

Carlo Rovelli: The essential role of coherent states in quantum gravity.

Conventional quantum field theory techniques do not work for extracting physical information from a background-independent quantum theory of gravity. A technique that works is Oeckl's boundary formalism, with semiclassical coherent states on the boundary. I illustrate how this technique has allowed us to compute the lifetime of a black hole in loop quantum gravity. This is an astrophysical relevant quantity that could have observational consequences.

Barry Sanders: Spacetime replication of continuous-variable quantum information.

Combining the relativistic speed limit on transmitting information with linearity and unitarity of quantum mechanics leads to a relativistic extension of the no-cloning principle called spacetime replication of quantum information. We introduce continuous-variable spacetime-replication protocols, expressed in a Gaussian-state basis, that build on novel homologically constructed continuous-variable quantum error correcting codes. Compared to qubit encoding, our continuous-variable solution requires half as many shares per encoded system. We show an explicit construction for the five-mode case and how it can be implemented experimentally. As well we analyze the ramifications of finite squeezing on the protocol.
Reference: Patrick Hayden, Sepehr Nezami, Grant Salton and Barry C. Sanders, *New Journal of Physics*(accepted) arXiv:1601.02544.

S. Sanjib Dey: Higher order squeezing of noncommutative q-photon-added coherent states.

Nonclassical states play a key role in quantum information processing. We construct a nonclassical state, namely, the photon-added coherent state for a q-deformed oscillator in a noncommutative space associated with the generalised uncertainty principle. We explore several nonclassical properties of the constructed states by providing generic expressions for the higher order squeezing coefficients, which not only help us to understand the behaviour of our system, but also they can be utilised to realise any kind of q-deformed quantum systems.

Michael Speckbacher:Reproducing pairs and Gabor systems at critical density

A reproducing pair consists of two families in a Hilbert space that give rise to a bounded and invertible analysis/synthesis process. We use this concept to study Gabor systems at critical density. First, we present a generalization of the Balian-Low theorem to the reproducing pairs setting. Then, we prove that there is no reproducing partner for the Gabor system of integer time-frequency shifts of the Gaussian. In other words, the coefficients of the expansion of $f \in L^2(\mathbb{R})$ in terms of the canonical coherent states sampled on the von Neumann lattice cannot be calculated using inner products with an arbitrary family in $L^2(\mathbb{R})$. This is possibly the last open question for this system.

Mauro Spera: Geometric aspects of coherent states.

In this talk we review some geometric properties of coherent states together with their use in several contexts. After a brief outline of the geometry of coherent states based on [S-93],[S-00], we discuss a recent joint paper with A. Galasso ([GS-16]) devoted to the geometric quantization approach to Landau levels, where, in particular, we propose an interpretation of (magnetic) translational symmetry breaking in terms of coherent states and index theory, stressing the relationship with the Fourier-Mukai-Nahm transform, also making contact with the geometric approach to the Riemann surface braid group developed in [S-15]. Subsequently, we turn to the moment map approach to the Schroedinger and Pauli theories developed in [S-16], wherein a sort of "tautological" coherent state representation of the group of volume preserving diffeomorphisms in $\mathbb{R}^3$ was introduced. Finally, time permitting, we briefly review the applications of coherent state techniques to KP theory given in [PS-11].

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Simone Speziale: Coherent states in Loop Quantum Gravity and phase spaces of shapes of polyhedra.

Loop quantum gravity is a background-independent approach to the quantization of general relativity, where the fundamental quanta are excitations of spacetime itself, describing fuzzy geometries with discrete spectra. Coherent states play a crucial role in studying the semiclassical limit of the theory. In this talk I will give an overview of the mathematics of the coherent states and their geometric interpretation, based on the relation between SU(2) invariant tensors and Kapovich-Millson's phase space of shapes of polyhedra.

F.H. Szafraniec: The anatomy of coherent states.

I intend to expose the fairly general construction of coherent states proposed by Horzela and myself. It is the most economical way of looking at the matters, which preserves their essential features as well as covers the majority of the instances appearing in the physical and mathematical literature so far.

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K. Thirulogasanthar: Coherent state quantization and the Heisenberg uncertainty relation in the quaternionic setting.

Coherent states and coherent state quantization are introduced in a right quaternionic Hilbert space. Through the quantization procedure annihilation, creation and position operators are obtained. Due to the non-commutativity of quaternions a self-adjoint momentum operator cannot be defined with right scalar multiple of oper- ators. In quaternionic quantum mechanics, defining a momentum operator analogous to the complex momentum operator is a long standing open problem. However, by introducing a basis dependent left scalar multiplication of right linear operators we shall define a self-adjoint momentum operator in a quaternionic Hilbert space. Then using the so-defined operators, in the quaternionic setting, we shall investigate the Heisenberg uncertainty principle.

Felix Voigtlaender (on behalf of Gitta Kutyniok): Shearlets: Theory, Applications and Generalizations.

Coherent states form a common framework for the construction of frames, in particular for (continuous) Gabor, wavelet and shearlet systems. In this talk, we will concentrate on shearlets, which have had tremendous success in recent years, both in theory and practice.

For applications, the crucial property of shearlets is that they can optimally sparsely approximate cartoon-like functions, which serve as a model class for functions governed by curvilinear features. Due to this sparse approximation property, shearlets are a natural choice as a regularizer for many inverse problems such as inpainting, image separation and MRI reconstruction.

On the theoretical side, the existence theorem for compactly supported shearlet frames on $\mathbb{R}^2$ was recently extended to a construction of shearlet systems on bounded domains.

Again, due to the optimal sparse approximation property, these systems are well-adapted for the discretization of PDEs, given that the solution is governed by curvilinear features.

As a final topic, we will touch on the connection of shearlets and *decomposition spaces*: While it is well-known that the approximation spaces associated to a wavelet frame coincide with certain Besov spaces, almost nothing seems to be known about shearlet approximation spaces. Only recently, these spaces were identified with certain decomposition spaces. For these spaces—which also appear elsewhere in the study of coherent states, e.g. when studying the associated coorbit spaces—a comprehensive theory of embeddings, Banach frames and atomic decompositions is available and will be presented.

A. Vourdas: Coherent spaces, Boolean rings and their applications.

Coherent spaces spanned by a finite number of coherent states, are introduced. Their coherence properties are studied, using the Dirac contour representation. It is shown that the corresponding projectors resolve the identity, and that they transform into projectors of the same type, under displacement transformations, and also under time evolution. The set of these spaces, with the logical OR and AND operations is a distributive lattice, and with the logical XOR and AND operations is a Boolean ring (Stone's formalism). Applications of this Boolean ring into classical and quantum gates with coherent states, are discussed.

Karol Życzkowski: Finite dimensional Hilbert space: Spin Coherent, Basis Coherent and Anti-coherent states

Among the set of all pure states living in a finite dimensional Hilbert space $\mathcal{H}_N$one distinguishes subsets of states satisfying some natural condition. One basis independent choice, consist in selecting the spin coherent states, corresponding to the $SU(2)$ group, or generalized, $SU(K)$ coherent states. Another often studied example is basis dependent, as states coherent with respect to a given basis are distinguished by the fact that the moduli of their off-diagonal elements (called 'coherences') are as large as possible. It is natural to define 'anti-coherent' states, which are maximally distant to the set of coherent states and to quantify the degree of coherence of a given state can by its distance to the set of anti-coherent states. For instance, the separable states of a system composed of two subsystems with N levels are coherent with respect to the composite group $SU(N) \times SU(N)$, while in this setup, the anti-coherent states are maximally entangled.