Local approximation methods using hierarchical splines

A framework based on the THB-spline basis

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Joint work with Carla Manni

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Motivation

- Splines and B-splines are of interest in a wide range of areas:
  - established tool: geometric modelling, approximation theory
  - recently: isogeometric analysis (paradigm for solving PDEs)
- In higher dimensions usually based on tensor-product topology
  - ☺ computationally efficient, geometrically intuitive
  - ☹ restriction to rectangular meshes
    $\rightarrow$ not well suited for adaptive local refinement
- Alternative spline spaces
  - (Truncated) Hierarchical B-splines
  - T-splines
  - LR-splines
  - B-splines on triangulations

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Hierarchical model

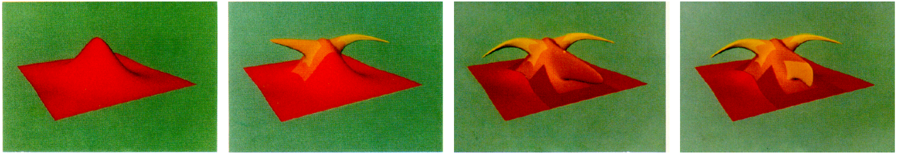
Hierarchical B-splines

[Forsey-Bartels, CG 1988]

[Kraft, 1997]

- framework for surface fitting and modelling

  efficient construction of local features



Hierarchical model

Hierarchical splines

- ✓ local refinement
- ✓ any degree, any smoothness, any dimension

Hierarchical basis

- ✓ linearly independent
- ✓ nonnegative
- ✓ weakly stable
  [Kraft, 1997]

Normalization: truncated basis

- ✓ linearly independent
- ✓ nonnegative
- ✓ partition of unity
- ✓ smaller support
- ✓ improved stability
  [Giannelli-Jüttler-Speleers, CAGD 2012]

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- sequence of n nested (tensor-product) spline spaces on Ω^0

  $\mathbb{V}^0 \subset \mathbb{V}^1 \subset \cdots \subset \mathbb{V}^{n-1}$

  each spline space $\mathbb{V}^\ell$ spanned by normalized B-spline basis

  $\mathcal{B}^\ell = \{B_{i,\ell},\ i = 1, \ldots, N_\ell\}$

- sequence of n sets

  $\Omega^0 \supseteq \Omega^1 \supseteq \cdots \supseteq \Omega^{n-1}$

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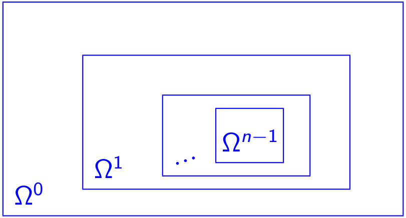
hierarchical set (domain)

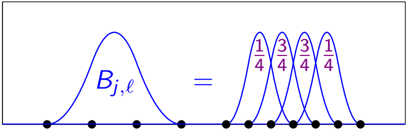


Hierarchical model

From hierarchical basis to truncated basis

- by subdivision, we can write every $B_{j,\ell} \in \mathbb{V}^{\ell} \subset \mathbb{V}^{\ell+1}$

$$B_{j,\ell} = \sum_{i=1}^{N_{\ell+1}} c_{i,\ell+1} \, B_{i,\ell+1}$$



- truncation mechanism:

$$\mathrm{trunc}^{\ell+1}(B_{j,\ell}) = \sum_{i\,:\,\mathrm{supp}^0(B_{i,\ell+1}) \not\subseteq \Omega^{\ell+1}} c_{i,\ell+1} \, B_{i,\ell+1}$$

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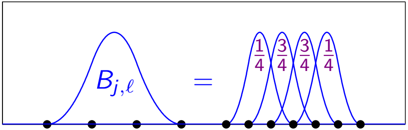

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Hierarchical model

THB-splines: construction [Giannelli-Jüttler-Speleers, CAGD 2012]

recursive definition of $\mathcal{T}$:

(I) initialization: $\mathcal{T}^0 = \{B_{i,0} \in \mathcal{B}^0 : \mathrm{supp}^0(B_{i,0}) \neq \emptyset\}$

(II) recursive case: construct $\mathcal{T}^{\ell+1}$ from $\mathcal{T}^\ell$

$$\mathcal{T}^{\ell+1} = \mathcal{T}_A^{\ell+1} \cup \mathcal{T}_B^{\ell+1}, \qquad \ell = 0, \ldots, n-2,$$

where

$$\mathcal{T}_A^{\ell+1} = \{\mathrm{trunc}^{\ell+1}(B_{i,j}^{\mathcal{T}}) : B_{i,j}^{\mathcal{T}} \in \mathcal{T}^\ell \wedge \mathrm{supp}^0(B_{i,j}^{\mathcal{T}}) \not\subseteq \Omega^{\ell+1}\}$$

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(III) $\mathcal{T} = \mathcal{T}^{n-1}$

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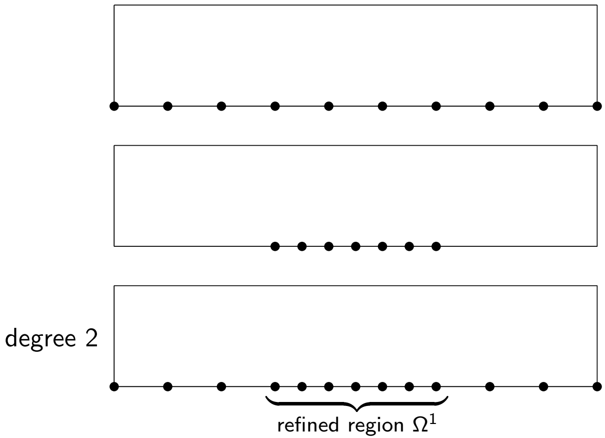
THB-splines: 1D example



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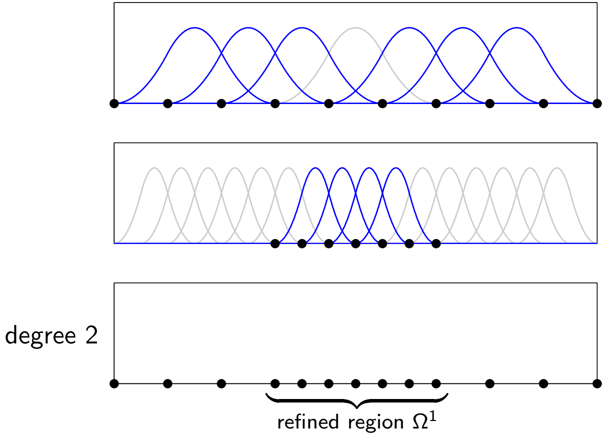

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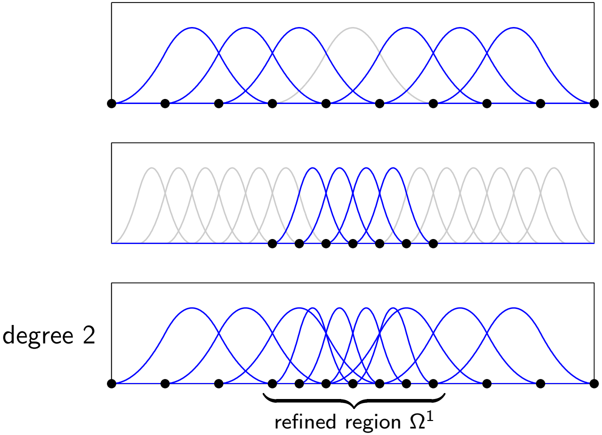
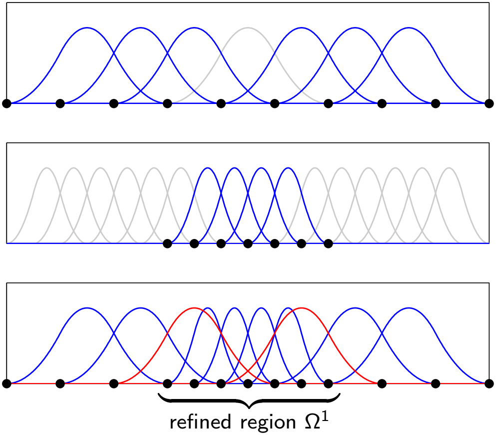


Hierarchical model

THB-splines: 1D example

degree 2



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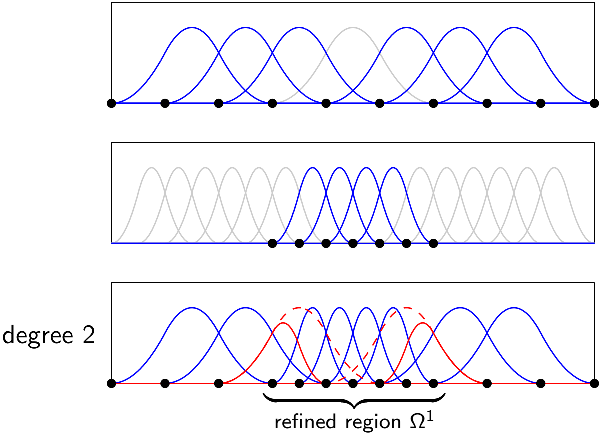

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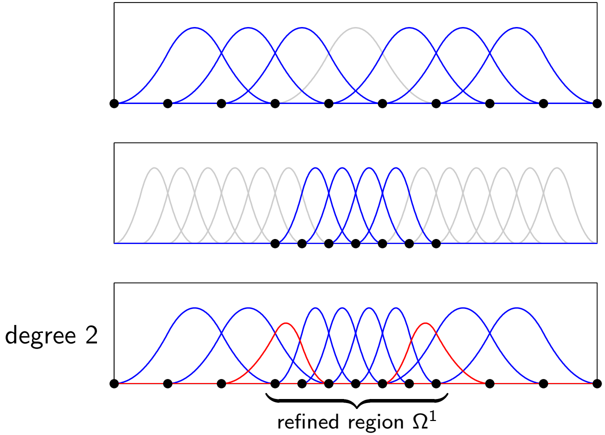

Truncated basis: properties

Truncated basis: another basis for hierarchical spline space

With respect to classical hierarchical basis:

(1) THB-splines are nonnegative and linearly independent

(2) THB-basis is strongly L_∞-stable:
stability constants independent of # levels

(3) reduced support of coarse basis functions;
reduced overlap of basis supports → sparser systems

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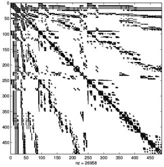
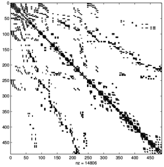
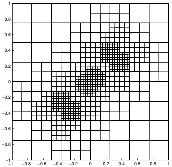
HB: 

THB: 



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(4) preservation of coefficients:

let $\mathcal{I}_\ell := \{i : B_{i,\ell} \in \mathcal{B}^\ell \cap \mathcal{H}\}$, let $D^\ell := \Omega^\ell \setminus \Omega^{\ell+1}$, and let

$$f|_{D^\ell} = \sum_{k=0}^{n-1} \sum_{i \in \mathcal{I}_k} c_{i,k}^{\mathcal{T}} B_{i,k}^{\mathcal{T}}|_{D^\ell} = \sum_{j=1}^{N_\ell} c_{j,\ell} B_{j,\ell}|_{D^\ell}, \quad \forall \ell$$

then $\quad c_{i,\ell}^{\mathcal{T}} = c_{i,\ell}, \quad i \in \mathcal{I}_\ell \quad$ [Giannelli-Jüttler-Speleers, AiCM 2014]

(5) the truncated basis forms a partition of unity on Ω^0

$\Rightarrow$ convex partition of unity

(6) preservation of Greville points

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Quasi-interpolation

Hierarchical quasi-interpolants [Speleers-Manni, NM 2016]

- QI based on truncated hierarchical basis
  - convex partition of unity
  - small support
  - preservation of coefficients

- Consider a sequence of QIs in $\mathbb{V}^\ell$, $\ell = 0, \ldots, n-1$

$$Q^\ell(f) := \sum_{i=1}^{N_\ell} \lambda_{i,\ell}(f)\, B_{i,\ell}$$

Quasi-interpolation

Hierarchical quasi-interpolants [Speleers-Manni, NM 2016]

- QI based on truncated hierarchical basis
  - convex partition of unity → numerical stability
  - small support → local control
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note:

$\lambda_{i,k}$ can have various expressions; e.g, involve function values, derivative values, integrals

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  then $Q(f) := \sum_{k=0}^{n-1} \sum_{i \in \mathcal{I}_k} \lambda_{i,k}(f)\, B_{i,k}^{\mathcal{T}}$

- Polynomial reproduction ($\mathbb{P}_d \subset \mathbb{V}^0$):

  if $Q^\ell(g) = g, \quad \forall g \in \mathbb{P}_d,\ \forall \ell$ then $Q(g) = g, \quad \forall g \in \mathbb{P}_d$

- Spline reproduction (projector):

  if $\begin{cases} Q^\ell(s) = s, \quad \forall s \in \mathbb{V}^\ell,\ \forall \ell \\ \lambda_{i,\ell} \text{ is supported in } \Omega^\ell \setminus \Omega^{\ell+1} \end{cases}$ then $Q(s) = s, \quad \forall s \in \mathbb{S}$

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Hierarchical quasi-interpolants

- Let Υ be a cell, and

$$\Lambda_\Upsilon = \mathrm{conv}\left(\bigcup_{(i,\ell):\mathrm{supp}^0(B^{\mathcal{T}}_{i,\ell}) \bigcap \Upsilon \neq \emptyset} \Lambda_{i,\ell} \cup \Upsilon \right), \quad \Lambda_{i,\ell} : \text{support of } \lambda_{i,\ell}$$

- Local approximation order [Speleers, AiCM 2016]:

if $f \in W_q^{p+1}(\Lambda_\Upsilon)$ and $Q(g) = g,\ \forall g \in \mathbb{P}_p$ then

$$\|f - Q(f)\|_{L_q(\Upsilon)} \leq C\,(\mathrm{diam}(\Lambda_\Upsilon))^{p+1}\,(1 + C_Q)\,|f|_{W_q^{p+1}(\Lambda_\Upsilon)}$$

(similar estimates for higher order derivatives of $f - Q(f)$)

Quasi-interpolation

Hierarchical quasi-interpolants

- Local approximation order: some remarks
  - Q should be bounded: sufficient in infinity norm (bound C_Q)

    ✓ each Q^ℓ is bounded separately
  - $\mathrm{diam}(\Lambda_\Upsilon)$ should be of same order as $\mathrm{diam}(\Upsilon)$

    ✓ each $\lambda_{i,\ell}$ is locally supported

    ✓ bounded number of THB-splines supported on any cell Υ

    $\Rightarrow$ control mesh refinement

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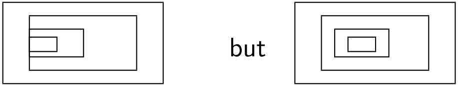

Quasi-interpolation

Numerical example: setup

- C^1 quadratic bivariate tensor-product B-splines
- Building block $\widetilde{Q}^\ell(f) := \sum_{i=1}^{N_\ell} \widetilde{\lambda}_{i,\ell}(f)\, B_{i,\ell}$ at level ℓ:
  - choose a cell $\Upsilon_{i,\ell}$ in support of each $B_{i,\ell}$
  - choose 3×3 points $x_{j,i,\ell} \in \Upsilon_{i,\ell}$, $j = 1, \ldots, 9$
  - solve the system $\sum_{k:\mathrm{supp}(B_{k,\ell}) \bigcap \Upsilon_{i,\ell} \neq \emptyset} c_{k,\ell} B_{k,\ell}(x_{j,i,\ell}) = f(x_{j,i,\ell})$
  - set $\widetilde{\lambda}_{i,\ell} = c_{i,\ell}$
- Set $\widetilde{Q}(f) := \sum_{k=0}^{n-1} \sum_{i \in \mathcal{I}_k} \widetilde{\lambda}_{i,k}(f)\, B^{\mathcal{T}}_{i,k}$

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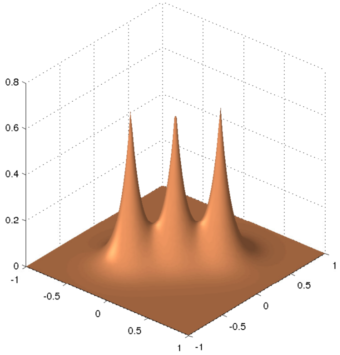
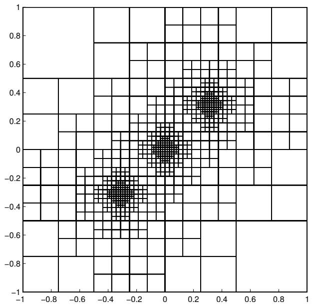
Quasi-interpolation

Numerical example

$$f(x,y) = \frac{2}{3\exp(\sqrt{(10x-3)^2+(10y-3)^2})} + \frac{2}{3\exp(\sqrt{(10x+3)^2+(10y+3)^2})} + \frac{2}{3\exp(\sqrt{(10x)^2+(10y)^2})}$$





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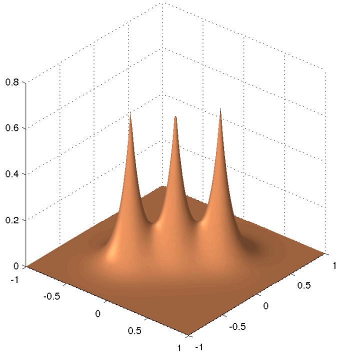

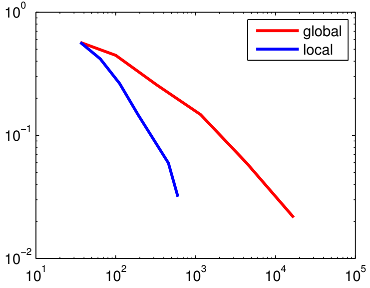

max-error versus dimension

Concluding message

Truncated B-splines

- the hierarchical model: local refinement
- the truncation mechanism:
  construction of a basis for a hierarchical space with properties
  - ✓ linear independence
  - ✓ nonnegativity
  - ✓ small support
  - ✓ partition of unity
  - ✓ preservation of coefficients
  - ✓ strong L_∞-stability
  - ✓ quasi-interpolants
- not confined to tensor-product splines, e.g., also box splines

References

- C. Giannelli, B. Jüttler and H. Speleers. *THB-splines: The truncated basis for hierarchical splines*. Comput. Aided Geom. Design 29, 485–498. 2012
- C. Giannelli, B. Jüttler and H. Speleers. *Strongly stable bases for adaptively refined multilevel spline spaces*. Adv. in Comput. Math. 40, 459–490. 2014
- H. Speleers and C. Manni. *Effortless quasi-interpolation in hierarchical spaces*. Numer. Math. 132, 155–184. 2016
- H. Speleers. *Hierarchical spline spaces: quasi-interpolants and local approximation estimates*. Adv. in Comput. Math. 2016

Thank you for your attention

Announcement

CIME Summer School

“Splines and PDEs: Recent Advances from Approximation Theory to Structured Numerical Linear Algebra”

Organizers:

Tom Lyche, Carla Manni, Hendrik Speleers

Date:

July 2–8, 2017

Place:

Hotel S. Michele, Cetraro, Italy