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Fakultät für Informatik und Mathematik

Adaption of tensor product spline spaces to approximation on domains

Nada Sissouno, University of Passau

September 19, 2016

Motivation: domains and B-splines

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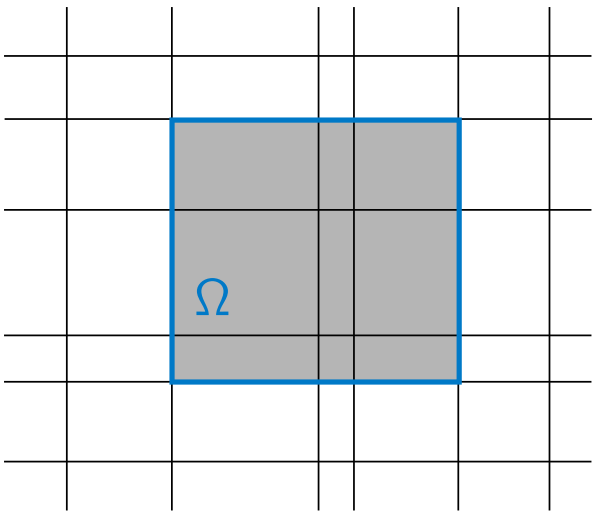

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- grid width $\mathbf{h} = (h_1, h_2)$

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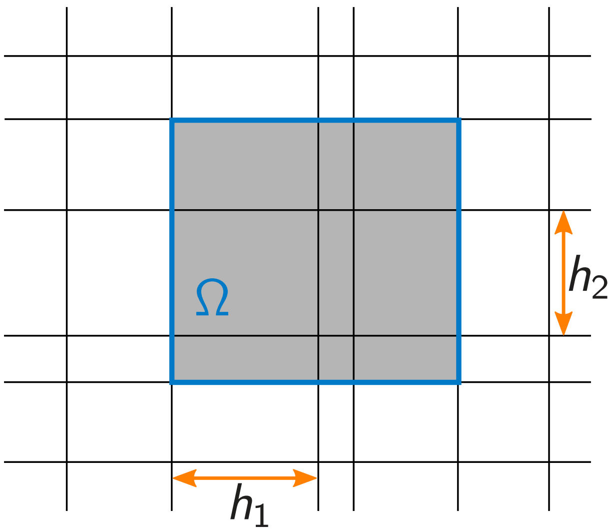

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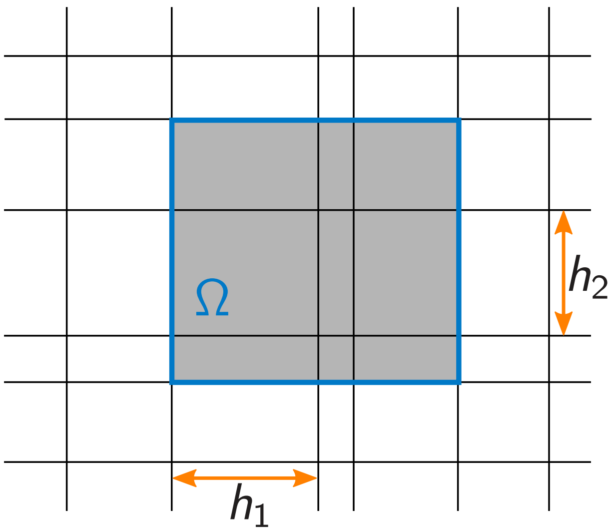

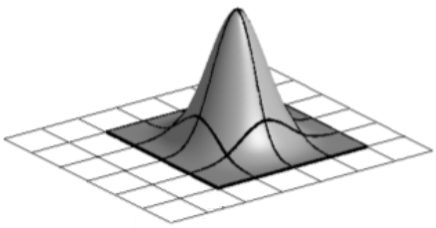

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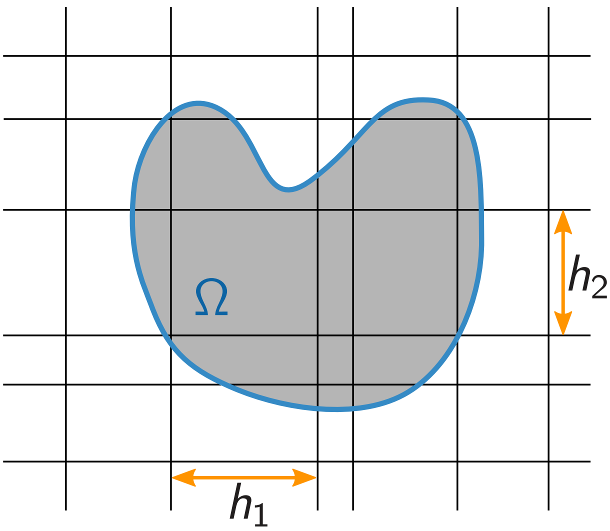

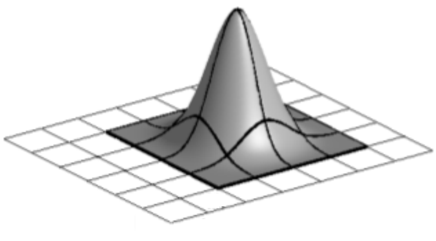

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Theorem (Dahmen, DeVore and Scherer 1980[1])

For appropriate domains $\Omega \subseteq \mathbb{R}^2$ and $f \in W_p^{\mathbf{n}}(\Omega)$ for $1 \leq p \leq \infty$

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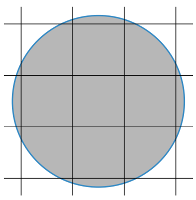

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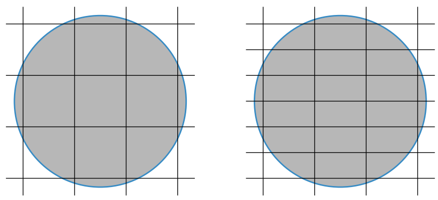

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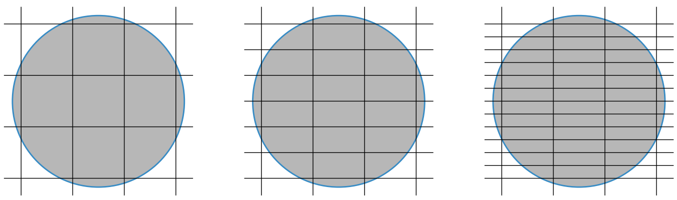

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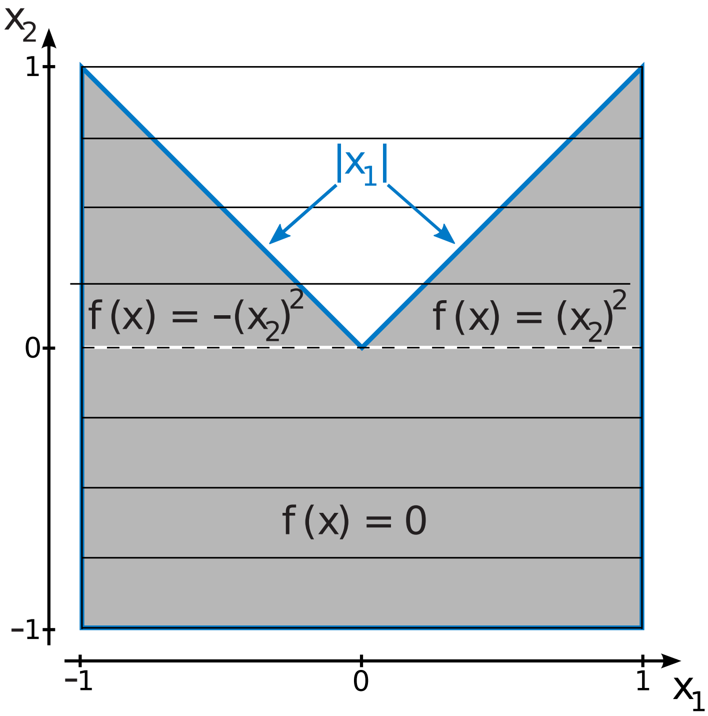
Example: C depends on $\mathbf{h}$

$h_1 = 2,\ h_2 \ll 1$



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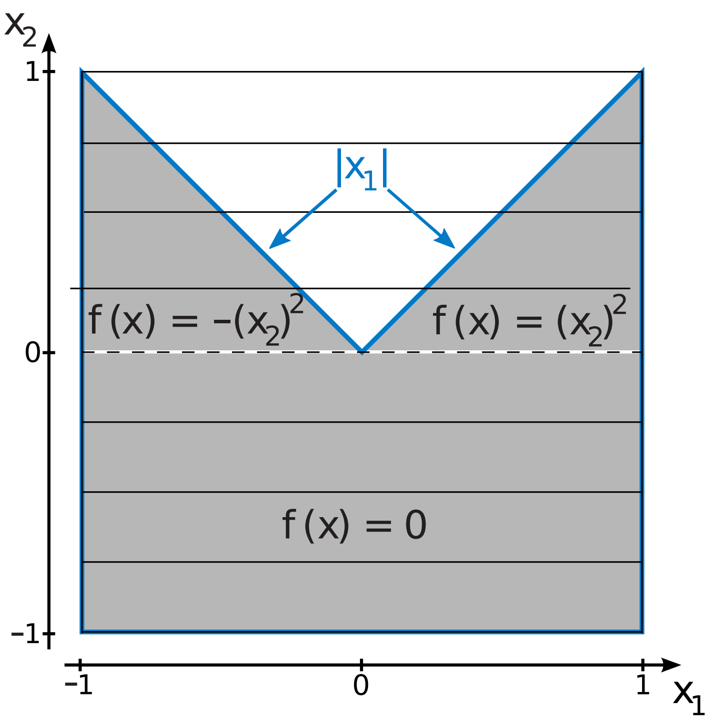

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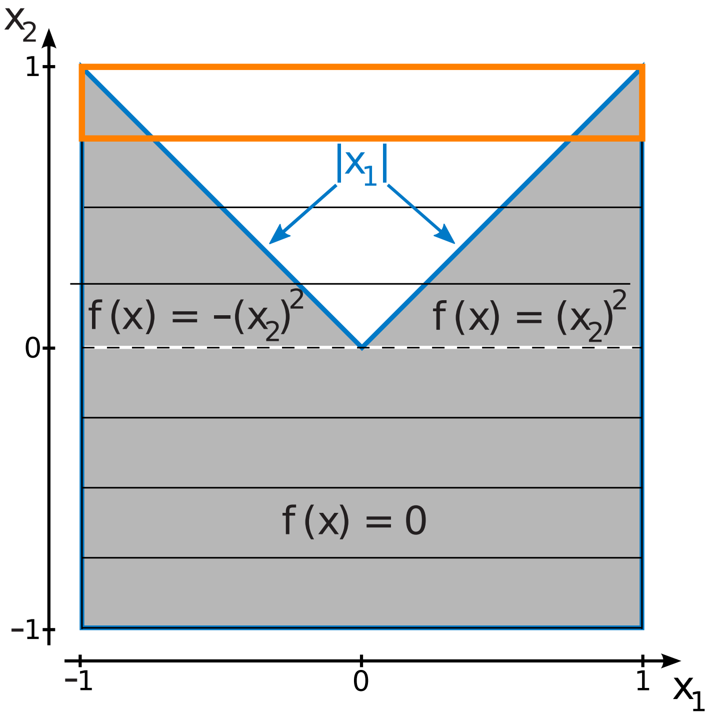

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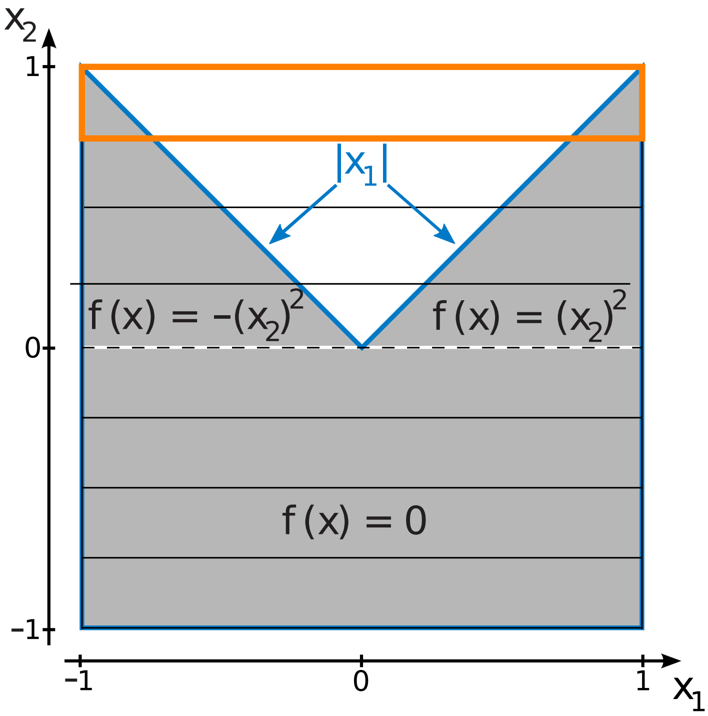

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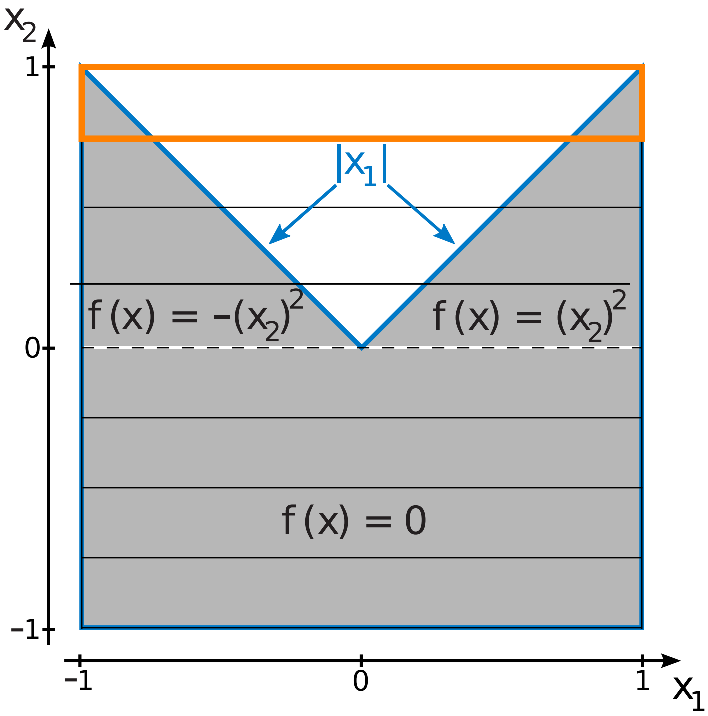

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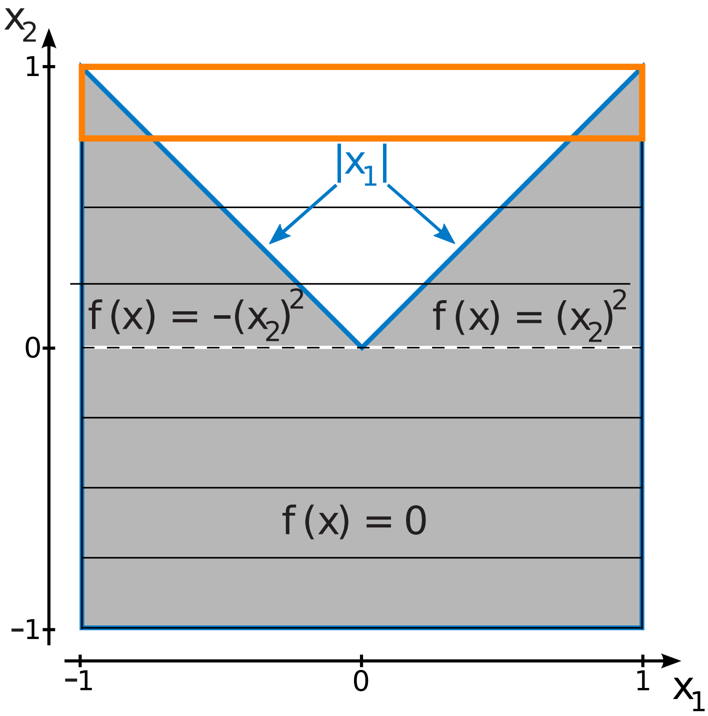

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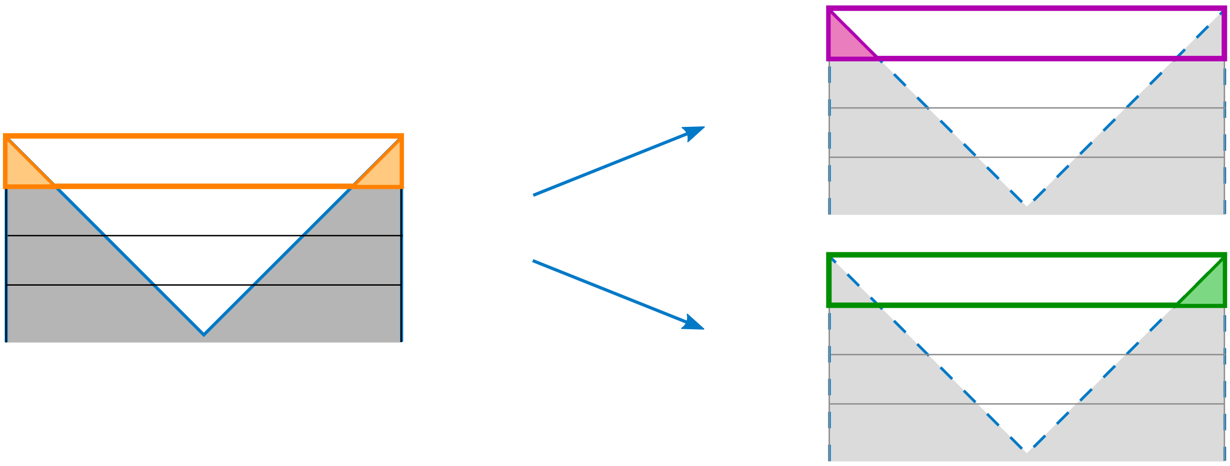
$$\Rightarrow \quad \boxed{C \gtrsim \frac{h_1}{h_2}}$$

Uniform solution: diversified spline space[2]



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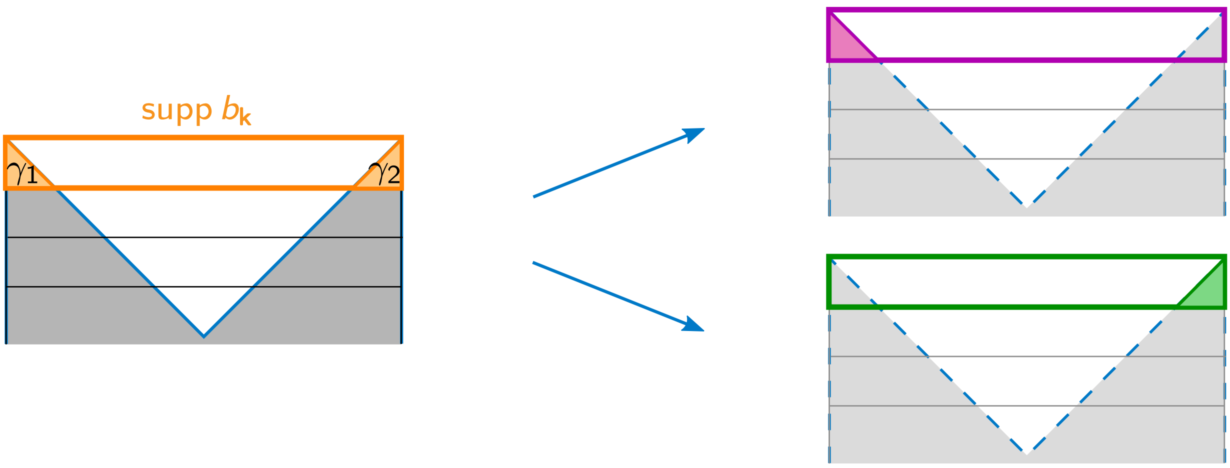

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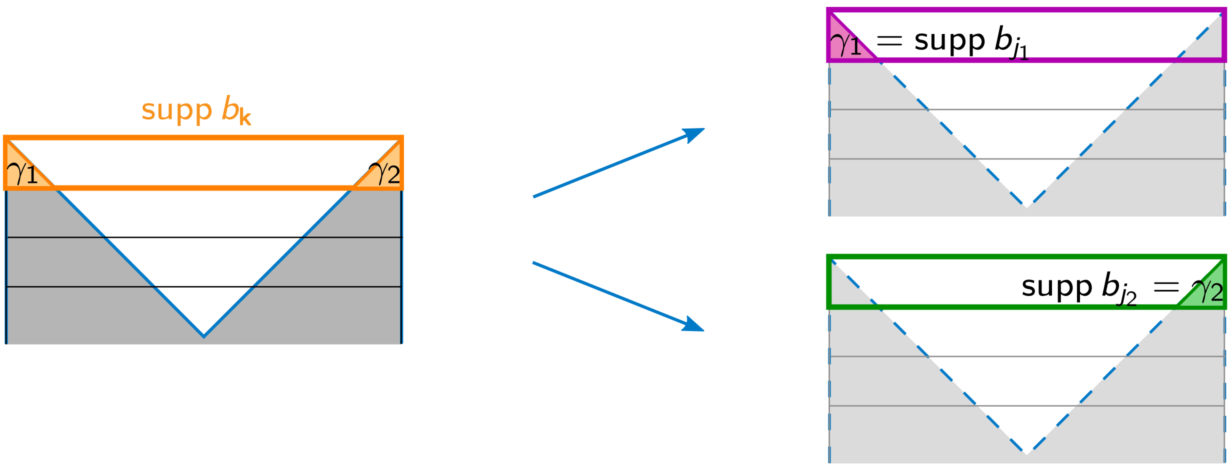

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Theorem (Reif, S. 2014[2])

For graph domains $\Omega \subseteq \mathbb{R}^2$, any uniform T with sufficiently small $\mathbf{h}$ and $f \in W_\infty^{\mathbf{n}}(\Omega)$

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Diversified spline space

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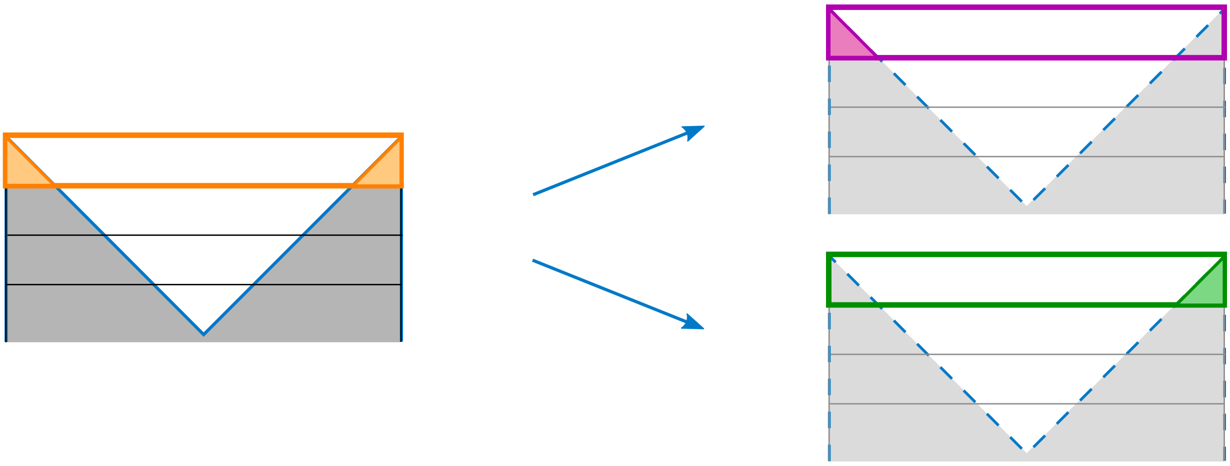

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[3] Sissouno: Anisotropic spline approximation with non-uniform B-splines. Preprint, http://arxiv.org/abs/1601.05275, 2016.

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1. arbitrary grid cell $\Gamma|_\Omega$
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Quasi-interpolant

$$Q : W_p^{\mathbf{n}} \to \mathcal{S}_{\mathbf{n}}^* : \qquad \|Qf\|_{p,\Gamma|_\Omega} \le c_{\mathbf{n},p} \|f\|_{p,\Gamma^*} \qquad \text{and} \qquad Qq = q,\ q \in \mathbb{P}_{\mathbf{n}}$$

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$$Qf := \sum_j b_j \, Q_j f$$

with uniformly bounded $Q_j : W_p^{\mathbf{n}}(\operatorname{supp} b_j) \to \mathbb{R}$

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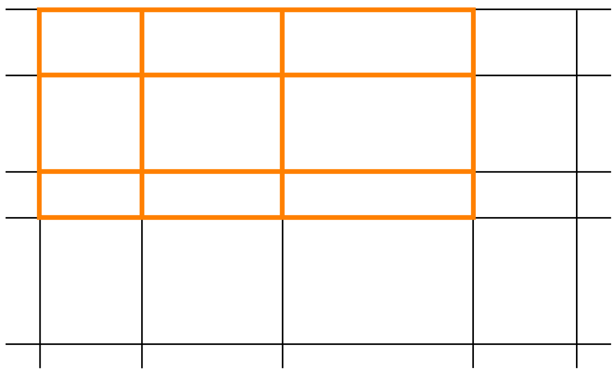
with uniformly bounded $Q_j : W_p^{\mathbf{n}}(\operatorname{supp} b_j) \to \mathbb{R}$



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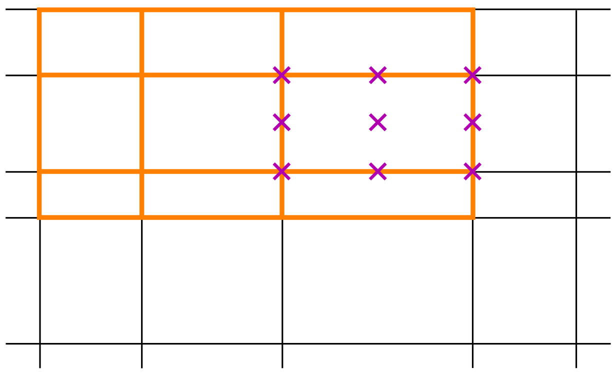

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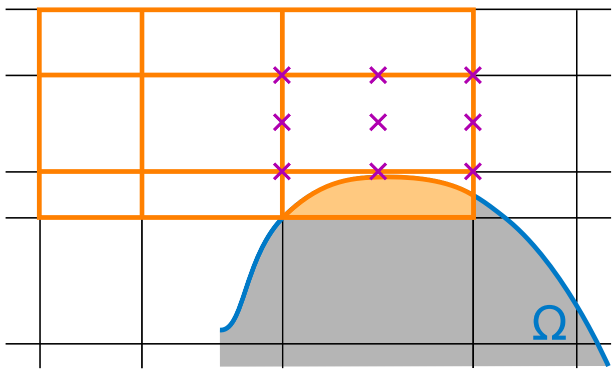

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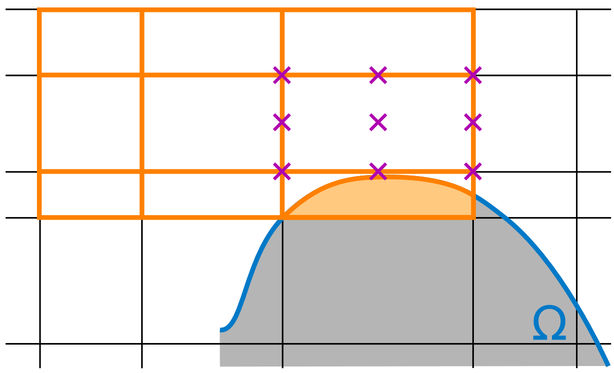

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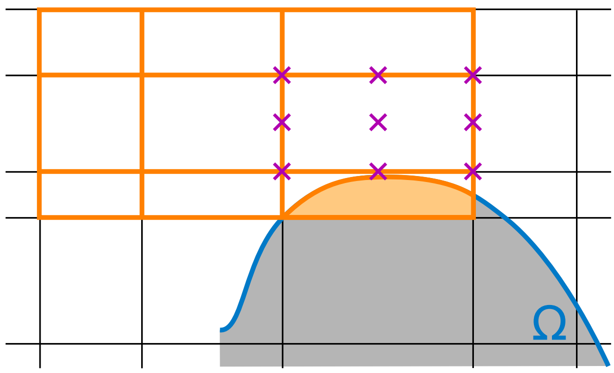

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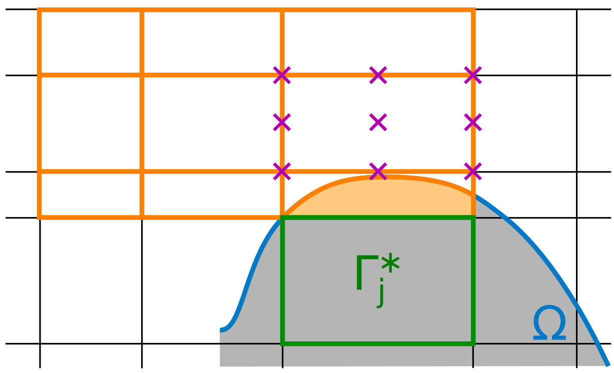

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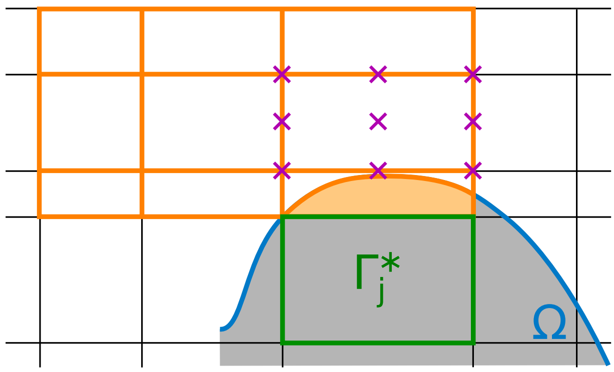

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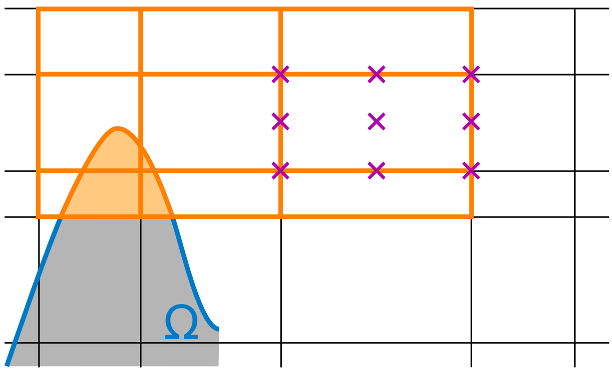

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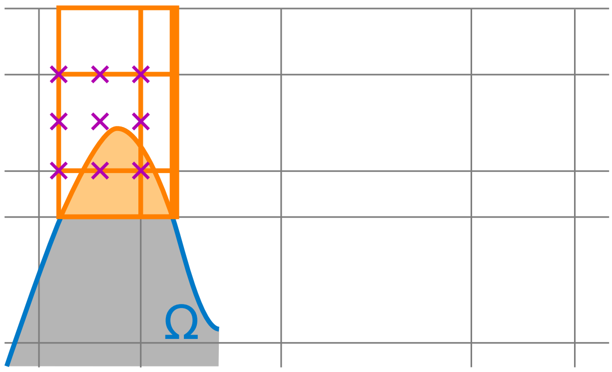

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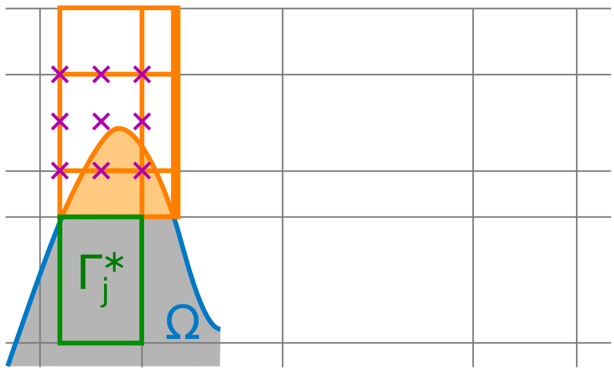

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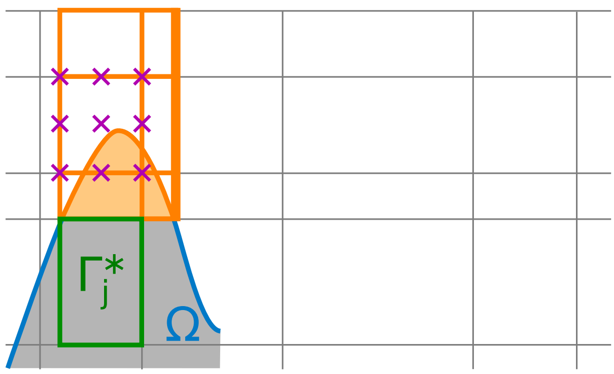

How to adapt tensor product B-splines?

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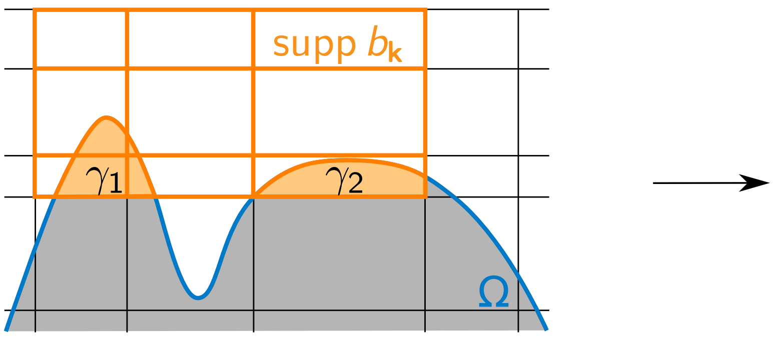
1. Diversification



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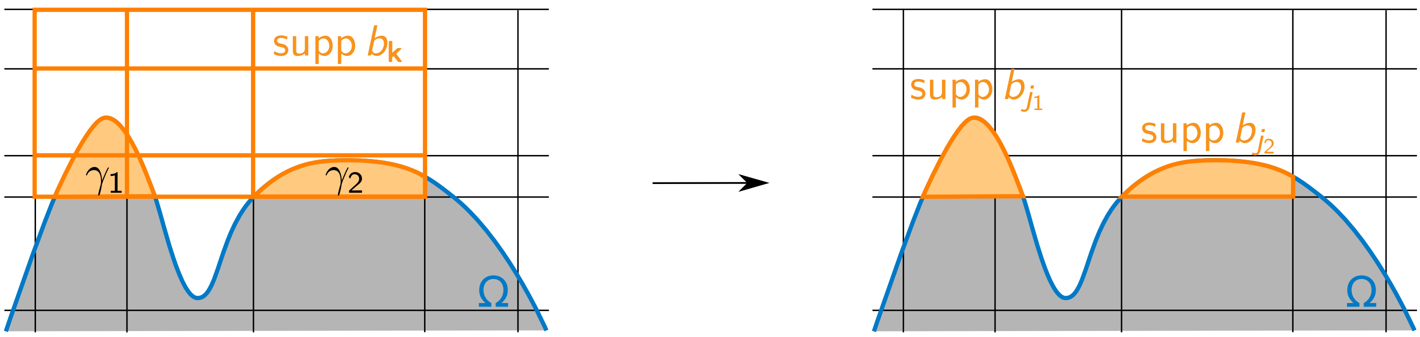

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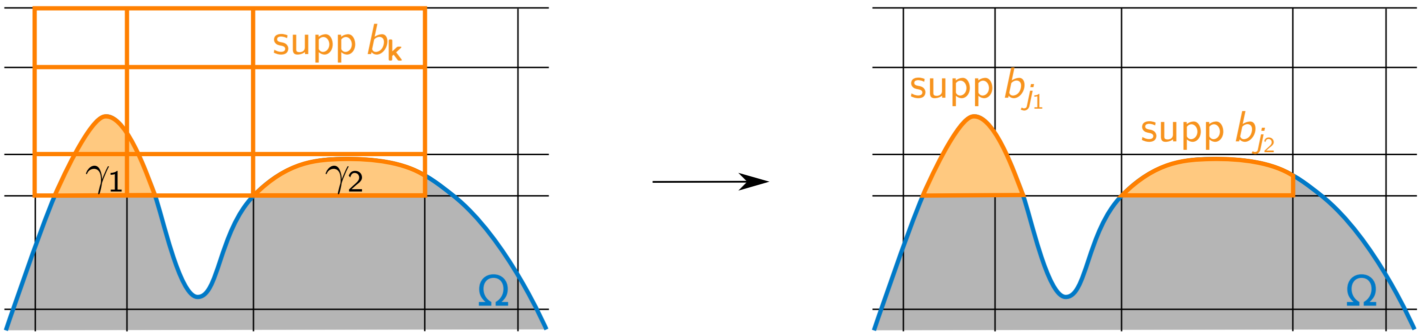

2. Condensation

How to adapt tensor product B-splines?

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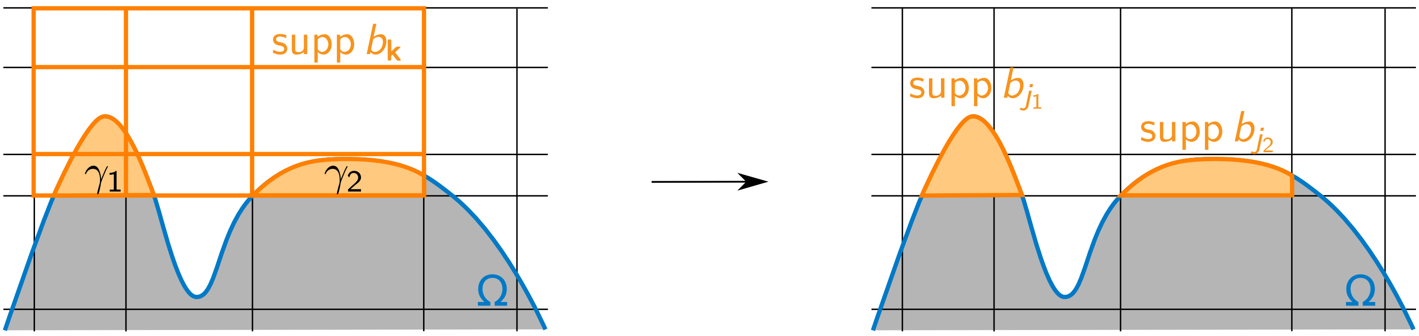

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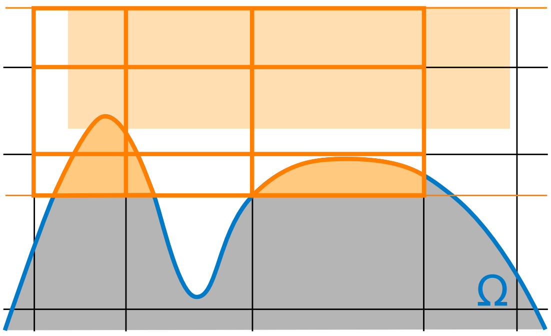

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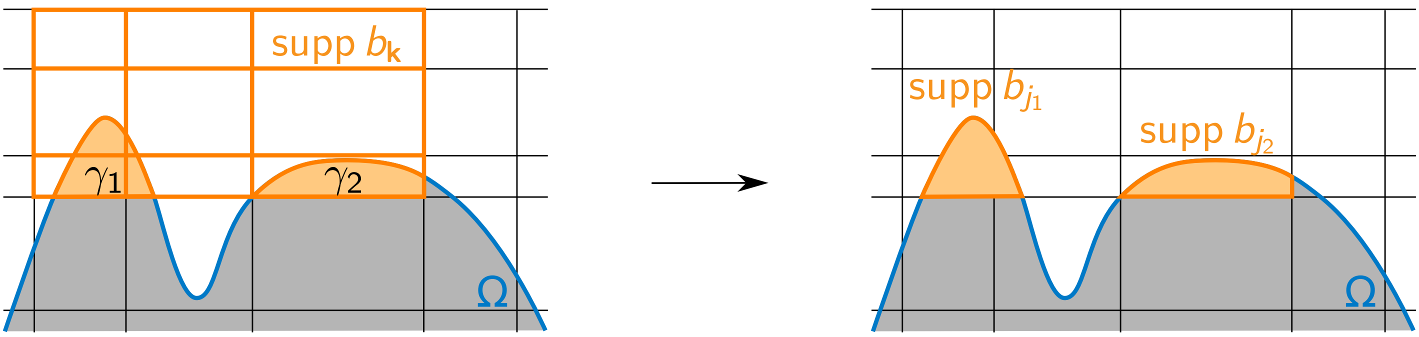

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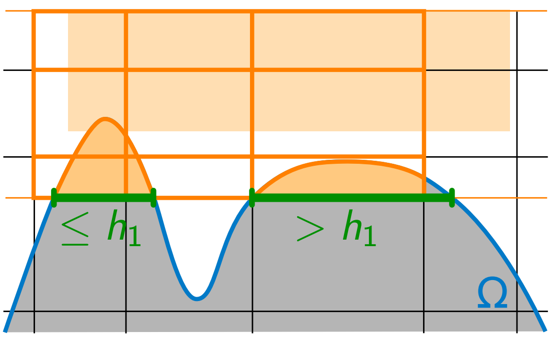

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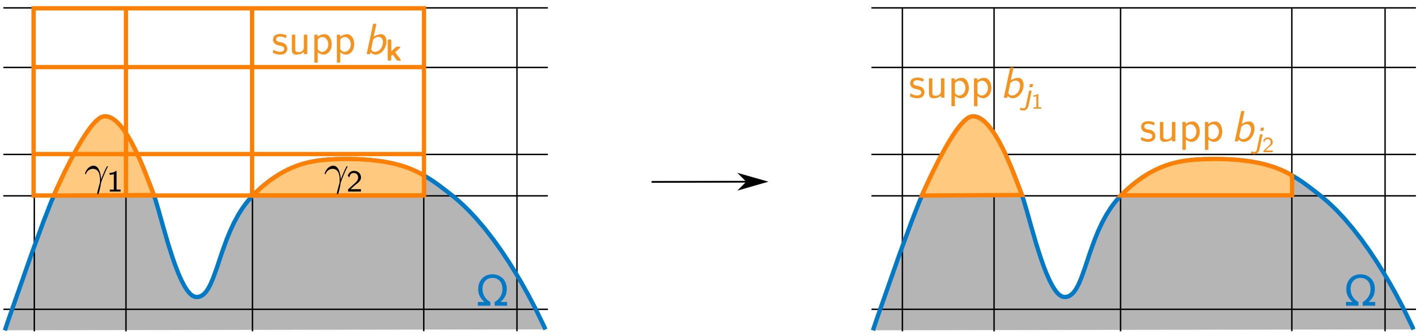

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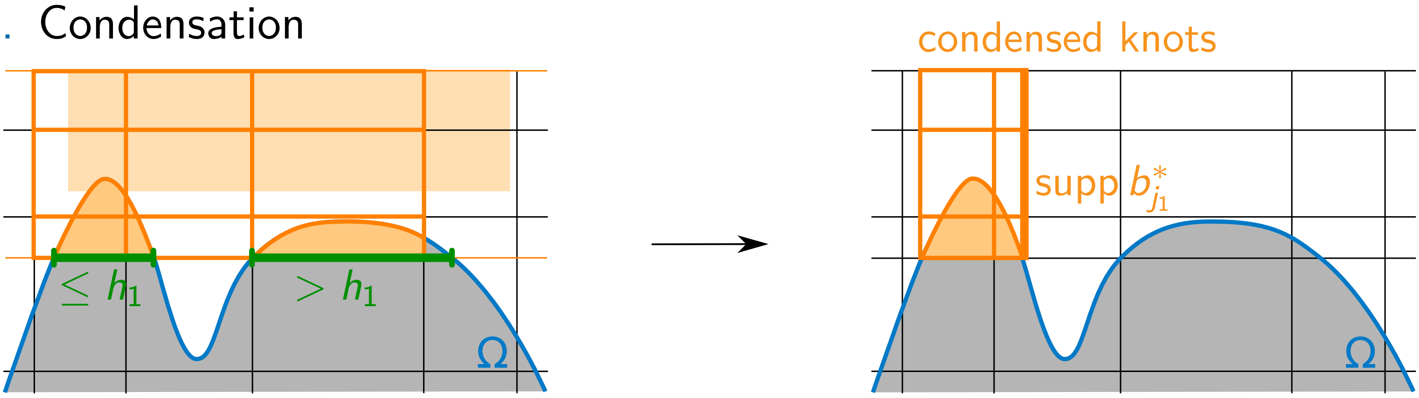

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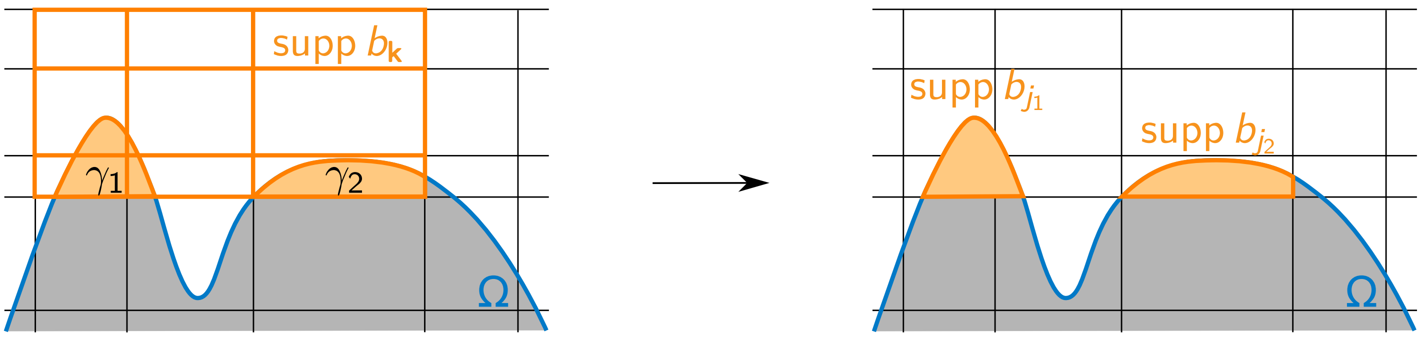
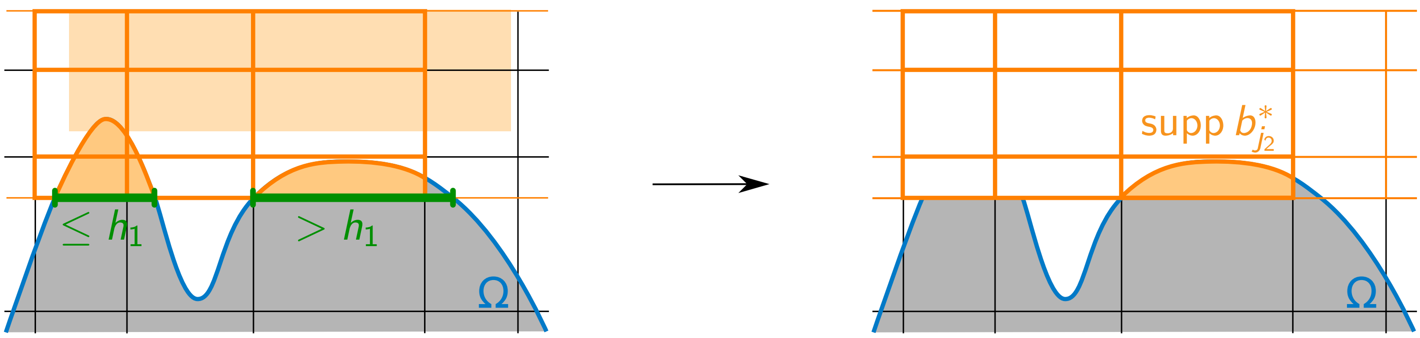


2. Condensation

'condensed' knots



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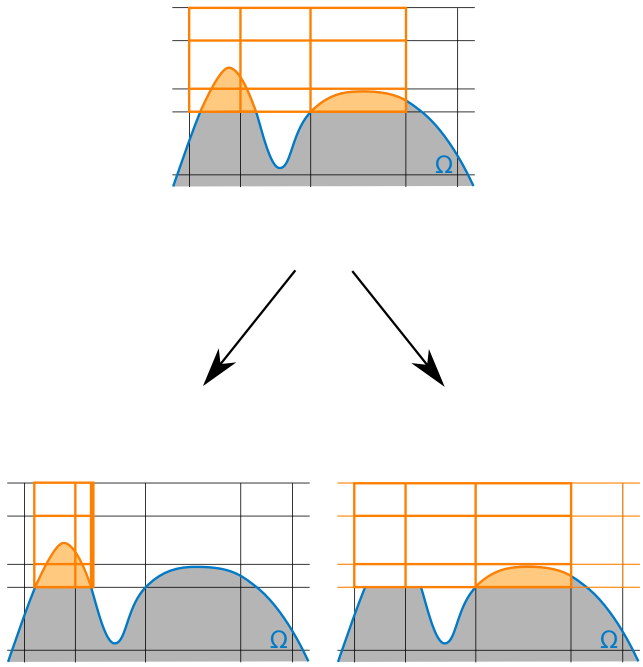
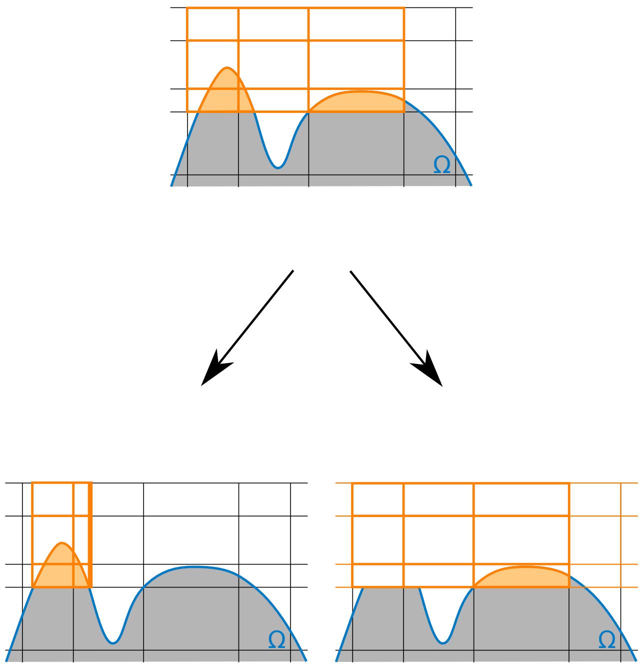


Summary

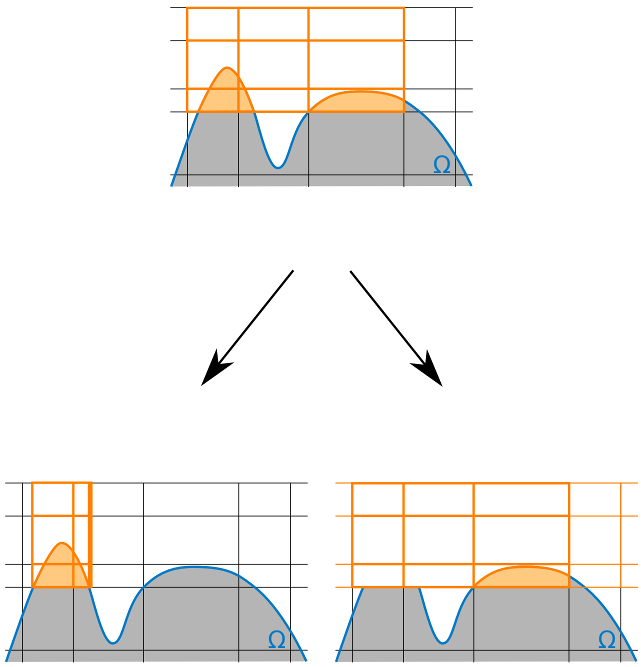
Diversified condensed B-splines



Summary

Diversified condensed B-splines

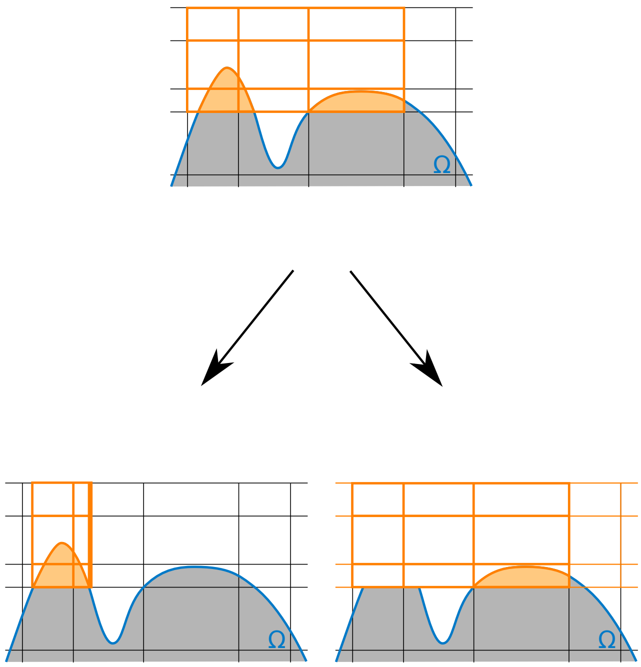
- diversification $\Rightarrow$ spline space



Summary

Diversified condensed B-splines

- diversification ⇒ spline space
- condensation ⇒ basis



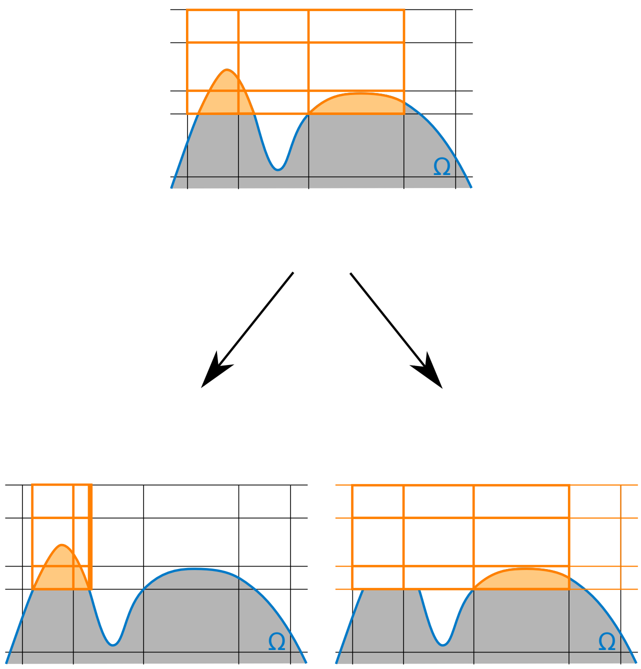
Summary

Diversified condensed B-splines

- diversification $\Rightarrow$ spline space
- condensation $\Rightarrow$ basis

We get:

New anisotropic error estimate



Summary

Diversified condensed B-splines

- diversification $\Rightarrow$ spline space
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We get:

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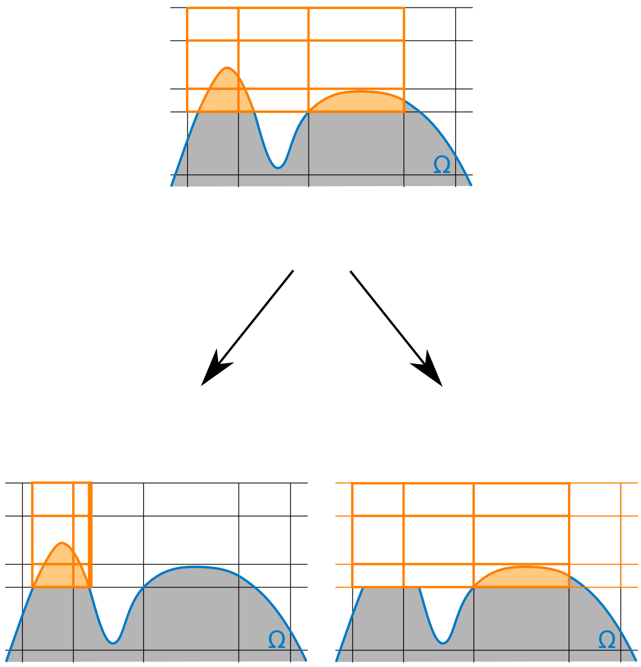
1. for graph domains in $\mathbb{R}^2$



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- diversification $\Rightarrow$ spline space
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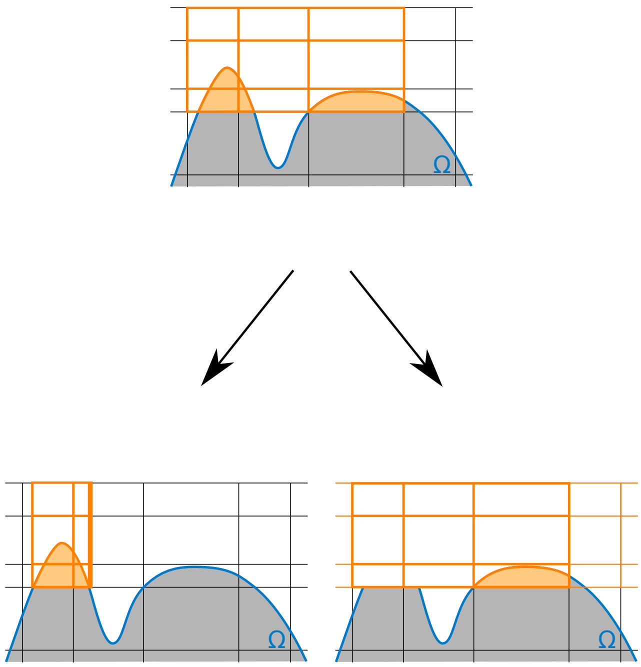

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1. for graph domains in $\mathbb{R}^2$
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Thank you for your attention!