Spline spaces over planar T-meshes and Extended complete Tchebycheff spaces

Carla Manni

Department of Mathematics
University of Roma "Tor Vergata"

joint work with
C. Bracco, T. Lyche, F. Roman, H. Speleers,

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Extended Tchebycheff spaces

$\mathbb{T}_p(J) \subset C^p(J)$ *extended Tchebycheff space* on J:

- $\dim(\mathbb{T}_p(J)) = p + 1$
- any Hermite interpolation problem with $p + 1$ data on J has a unique solution in $\mathbb{T}_p(J)$

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any non-trivial element in $\mathbb{T}_p(J)$ has at most p roots in J

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Examples

- $\mathbb{P}_p := < 1, x, \dots, x^{p-2}, x^{p-1}, x^p >$,
- $\mathbb{G}^{\exp}_{p,\alpha} := \langle 1, x, \dots, x^{p-2}, \mathbf{e}^{\alpha x}, \mathbf{e}^{-\alpha x} \rangle$,
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Extended Complete Tchebycheff spaces

- $w_0, \ldots, w_p$ positive on J

$$u_0(x) := w_0(x)$$

$$u_1(x) := w_0(x) \int_a^x w_1(t_1)\mathrm{d}t_1$$

$$\vdots$$

$$u_p(x) := w_0(x) \int_a^x w_1(t_1) \ldots \int_a^{t_{p-1}} w_p(t_p)\mathrm{d}t_p \ldots \mathrm{d}t_1$$

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Examples

- $(1, \ldots, 1) \Rightarrow u_i(x) = \frac{(x-a)^i}{i!} \Rightarrow \mathbb{T}_p(J) = \mathbb{P}_p$
- $(1, \ldots, 1, \cos(\alpha x), \frac{1}{\cos^2(\alpha x)} \Rightarrow \mathbb{T}_p(J) = \langle 1, x, \ldots, x^{p-2}, \sin(\alpha x), \cos(\alpha x)) \rangle$

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Tchebycheffian splines

polynomial spline spaces

$$a = x_0 < x_1 < \ldots < x_{n+1} = b$$

$$\{s \in C^r[a,b] : \; s_{|[x_i,x_{i+1})} \in \mathbb{P}_p, \; i = 0, \ldots, n\}$$

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$\mathbb{T}_p(J)$ extended (complete) Tchebycheff space

- spline spaces with sections in $\mathbb{T}_p(J)$: same properties as polynomial splines, including a B-spline like basis
- section spaces to be selected with a problem-dependent strategy
- useful tool in geometric modeling and numerical simulation (IgA)
- straightforward bivariate extension: tensor-product

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Tensor product structures: DRAWBACKS

- ☹tensor-product structure NO efficient local refinements
- Alternatives in modeling and/or simulation: local tensor -product structures
  - T-splines [Sederberg et. al, AMC ToG, 2003], ...
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    Hierarchical splines

    Splines over T-meshes

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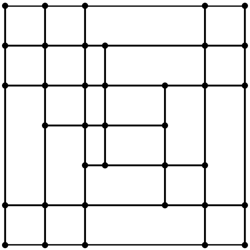
Planar T-meshes

T-mesh: collection of axis-aligned rectangles

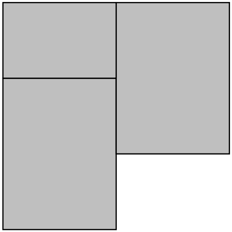
$$\mathcal{T} := (\mathcal{T}_2, \mathcal{T}_1, \mathcal{T}_0)$$



- $\mathcal{T}_2$ is the collection of cells: σ
- $\mathcal{T}_1 = \mathcal{T}_1^h \cup \mathcal{T}_1^v$ *edges*: τ
- $\mathcal{T}_0 := \bigcup_{\tau \in \mathcal{T}_1} \partial\tau$ *vertices*: γ
- $\mathcal{T}_1^o$ *interior edges*,
- $\mathcal{T}_0^o$ *interior vertices*.

T-meshes

- not necessarily rectangular, simply connected, regular



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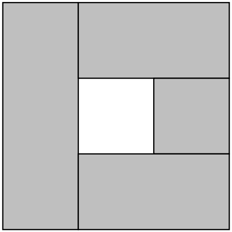

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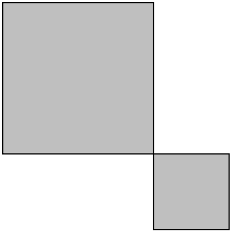

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Spline spaces over T-meshes

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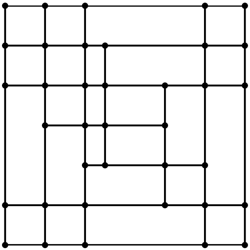

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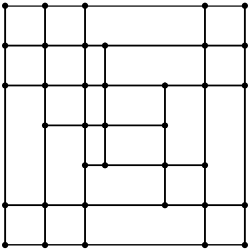

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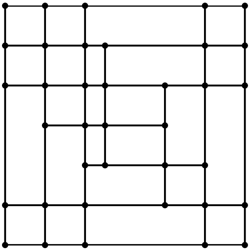

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DIMENSION?

Dimension of a spline space: instability

☺ stable dimension: only depending on degree, smoothness, topology

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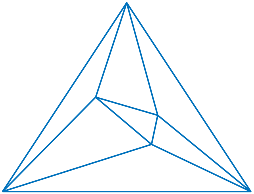
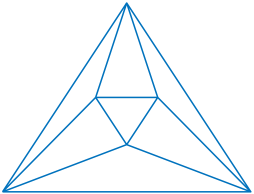
quadratic C^1



dim $= 6$



dim $= 7$

[Morgan Scott, 1974]

Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{r}}(\mathcal{T})$: instability

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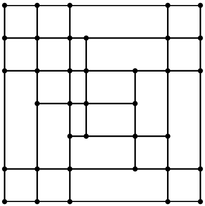
$$\boldsymbol{p} = (2,2),\ \boldsymbol{r} = (1,1)$$



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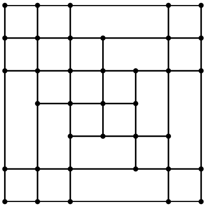

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[Li, Chen, CAGD 2011]

Splines over T-meshes: dimension

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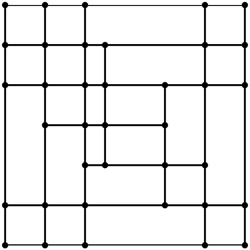

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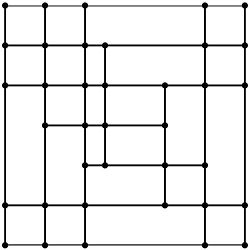

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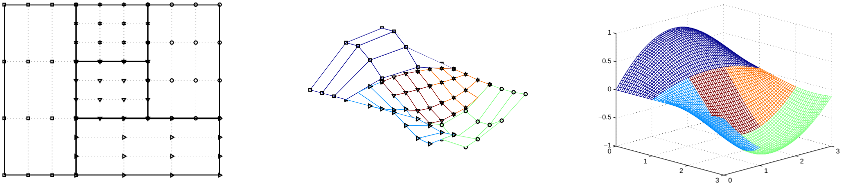
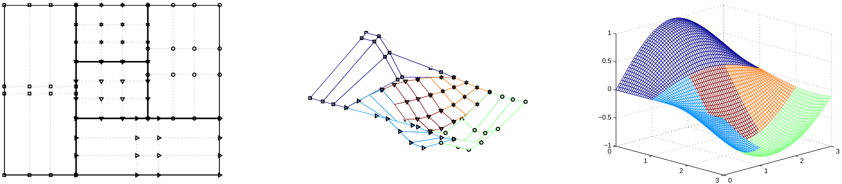


C^1 bi-cubics



$< 1, x, \cos \alpha x, \sin \alpha x >$: C^1 trigonometric (bi-cubics), $\alpha = \frac{2}{5}\pi$, $x \in [0, 1]$

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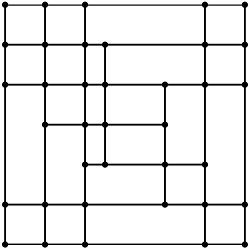

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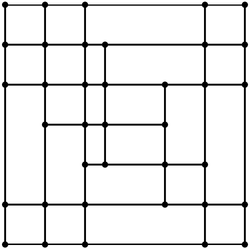

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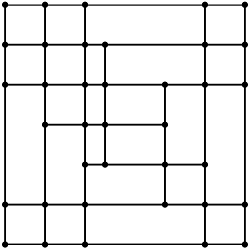
$$\boldsymbol{p} := (p_h, p_v), \quad \boldsymbol{r} := (r_h, r_v) \quad \mathbb{P}_{\boldsymbol{p}}^{\boldsymbol{T}} := \mathbb{T}_{p_h}^{h} \otimes \mathbb{T}_{p_v}^{v}, \quad \boldsymbol{T} := (\mathbb{T}_{p_h}^{h}, \mathbb{T}_{p_v}^{v}).$$

DIMENSION?

homological approach

Tchebycheffian splines over T-meshes: dimension

$\mathcal{T}_2$ is the collection of cells: σ



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DIMENSION?

homological approach

instability

Tchebycheffian splines over T-meshes: homology

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- for any vertical edge τ belonging to $x = \bar{x}$

$$\mathbb{I}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\tau) := \{q \in \mathbb{P}_{\boldsymbol{p}}^{\boldsymbol{T}} : D_x^i q(\bar{x}, y) \equiv 0, \forall y \in [a_v, b_v], i = 0, \ldots, r(\tau)\},$$

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Tchebycheffian splines over T-meshes: homology

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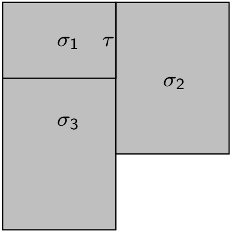

$$D_x^i(s_{\sigma_1} - s_{\sigma_2})(x, y) \equiv 0, i = 0, \ldots, r(\tau) \quad (x, y) \in \tau \subset \sigma_1 \cap \sigma_2$$

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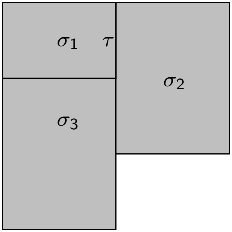

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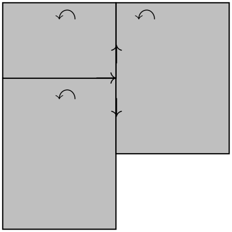
The maps of the complex are induced by the usual boundary maps



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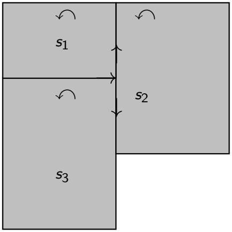

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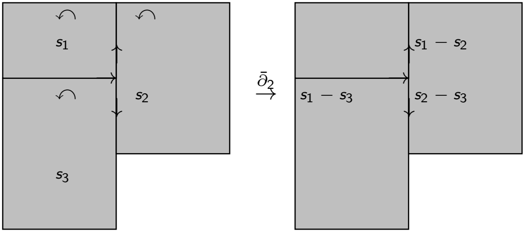

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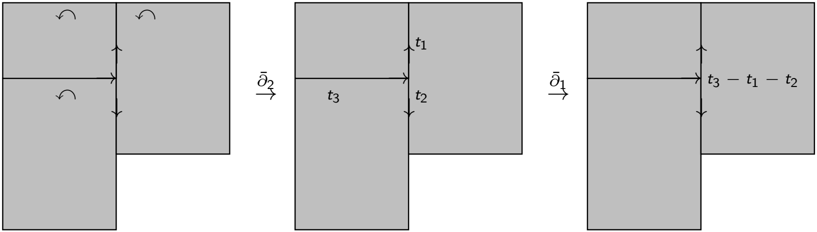

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Tchebycheffian splines over T-meshes: homology

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Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})$

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$$\dim\Big(\bigoplus_{\sigma\in\mathcal{T}_2} \mathbb{P}_{\boldsymbol{p}}^{\boldsymbol{T}}\Big) - \dim\Big(\bigoplus_{\tau\in\mathcal{T}_1^o} \mathbb{P}_{\boldsymbol{p}}^{\boldsymbol{T}}/\mathbb{I}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\tau)\Big) + \dim\Big(\bigoplus_{\gamma\in\mathcal{T}_0^o} \mathbb{P}_{\boldsymbol{p}}^{\boldsymbol{T}}/\mathbb{I}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\gamma)\Big)$$
$$= \dim\big(H_2(\mathfrak{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}})\big) - \dim\big(H_1(\mathfrak{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}})\big) + \dim\big(H_0(\mathfrak{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}})\big).$$

$\downarrow$	$\downarrow$	$\downarrow$
$\dim(\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T}))$	?	0

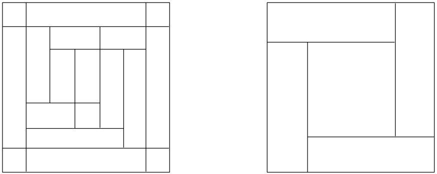
Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})$

$$
\begin{aligned}
\dim(\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})) = & \sum_{\sigma\in\mathcal{T}_2}(p_h+1)(p_v+1) \\
& - \sum_{\tau\in\mathcal{T}_1^{o,h}}(p_h+1)(r(\tau)+1) - \sum_{\tau\in\mathcal{T}_1^{o,v}}(r(\tau)+1)(p_v+1) \\
& + \sum_{\gamma\in\mathcal{T}_0^{o}}(r_h(\gamma)+1)(r_v(\gamma)+1) + \dim(H)
\end{aligned}
$$

[Bracco, Lyche, Manni, Roman, Speleers, CAGD 2016]

Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})$: bounding $\dim(H)$

- MIS maximal interior segment



- $\mathbb{T}_{p_h}, \mathbb{T}_{p_v}$ complete Tchebycheff spaces

$$0 \le \dim(H) \le \sum_{\rho \in \mathrm{MIS}_h(\mathcal{T})} (p_h + 1 - \omega(\rho))_+ (p_v - r(\rho)) + \sum_{\rho \in \mathrm{MIS}_v(\mathcal{T})} (p_h - r(\rho)) (p_v + 1 - \omega(\rho))_+.$$

$\omega(\rho)$ depends on smoothness, degree, (topology of the) T-mesh

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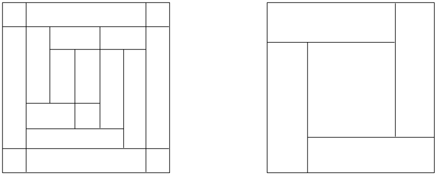

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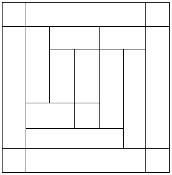

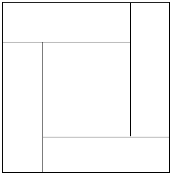

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Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})$: special cases

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- $\mathcal{T}$: T-mesh without cycles of MIS, $\boldsymbol{p} \geq 2\boldsymbol{r}+1$

$$\Rightarrow \quad \omega(\rho) \geq p+1 \quad \Rightarrow \dim(H) = 0$$

- Let $\mathcal{T}$ be a *quasi-cross-cut*
(= every edge is connected to the boundary =no MIS)

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Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})$: special cases

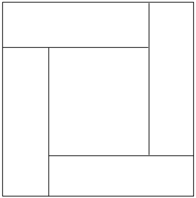
$$0 \leq \dim(H) \leq \sum_{\rho \in \mathrm{MIS}_h(\mathcal{T})} (p_h+1-\omega(\rho))_+ (p_v-r(\rho)) + \sum_{\rho \in \mathrm{MIS}_v(\mathcal{T})} (p_h-r(\rho))(p_v+1-\omega(\rho))_+ .$$

- $\mathcal{T}$: T-mesh without cycles of MIS, $\boldsymbol{p} \geq 2\boldsymbol{r}+1$

$$\Rightarrow \quad \omega(\rho) \geq p+1 \quad \Rightarrow \dim(H) = 0$$

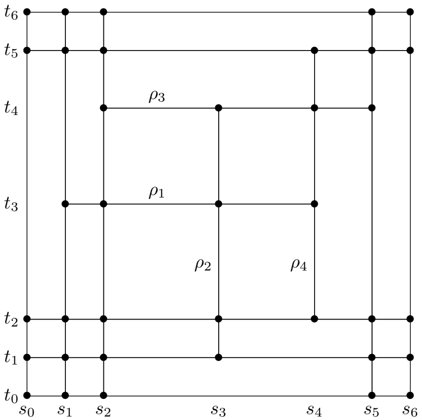
- Let $\mathcal{T}$ be a *quasi-cross-cut*
  (= every edge is connected to the boundary =no MIS)

$$\Rightarrow \quad \dim(H) = 0$$



Dimension of the spline space $\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})$: instability

$$\boldsymbol{p} = (2,2),\ \boldsymbol{r} = (1,1)$$



$$36 \le \dim\left(\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})\right) \le 37, \quad \forall \boldsymbol{T}$$

[Bracco, Lyche, Manni,Speleers, 2016]

Polynomial vs Tchebycheffian spline space: Conjecture

$$\dim\left(\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{T},\boldsymbol{r}}(\mathcal{T})\right) = \dim\left(\mathbb{S}_{\boldsymbol{p}}^{\boldsymbol{r}}(\mathcal{T})\right) \qquad \text{generically}$$

Concluding Message

- (complete) Tchebycheffian splines behave very similar to polynomial splines
- homology techniques can be extended to Tchebycheffian splines (despite the lack of the ring structure)
- we can extend to the Tchebycheff context
  - T-splines
  - LR splines
  - Hierarchical splines

  [Bracco, Cho, CMAME 2014],

  [Manni, Pelosi, Speleers, LNCS 2014],

  [Bracco, Lyche, Manni, Roman, Speleers, App. Math. and Comput. 2016]

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THANKS