Nonequilibrium: Physics, Stochastics and Dynamical Systems
January 18 - 22, 2016

Thierry Bodineau: Large time asymptotics of small perturbations of a deterministic dynamics of hard spheres.

We consider a tagged particle in a diluted gas of hard spheres. Starting from the hamiltonian dynamics of particles in the Boltzmann-Grad limit, we will show that the tagged particle follows a Brownian motion after an appropriate rescaling. We will also consider a different type of perturbation and relate it to the linearized Boltzmann equation.

Jean Bricmont: Classical versus quantum equilibrium.

There is a well-known notion of classical equilibrium and a related notion in quantum statistical mechanics. In this talk, I will discuss another notion of equilibrium, the one of quantum equilibrium, which enters in the de Broglie-Bohm theory. This theory can be viewed as a rational completion of ordinary quantum mechanics. Indeed, while the latter limits itself to predicting (very accurately) "results of measurements",the de Broglie-Bohm theory is a deterministic theory of matter in motion that, among other things, explains why the measurements work the way they do, without making them some deus ex machina of the physical theory. But, in order to reproduce the quantum predictions, one has to make assumptions on the initial conditions of the de Broglie-Bohm dynamics and those assumptions is what is called "quantum equilibrium". We will explain what it means and compare its justification with the one of classical equilibrium.

Anna De Masi: Models for evolution and selection

The 'basic model' consists of N Brownian particles which move independently on the line and undergo a birth process and also a death process which involves the right and/or leftmost particles. Different rules for such birth and death processes will be considered which lead, in the macroscopic limit, to free boundary problems. In particular I will present a model in the class introduced by Brunet and Derrida made of independent branching Brownian motions, namely at exponential times each particle duplicates creating a new particle in the same position. The total number of particles is conserved because at each branching the leftmost particle is removed. The biological interpretation is in terms of evolution selection models. Particles are individual cells, their positions describe their state, the larger the position the more fitted is the state. As the individual cells move as brownians there is no bias toward improvement which is instead determined by the rule for which the less fitted cell dies. I will present some preliminary results in collaboration with Pablo Ferrari, Errico Presutti and Nahuel Soprano-Loto concerning the convergence to a free boundary problem for which we prove local existence. I will also discuss variants of this model, results and conjectures.

Wojciech De Roeck: Slow heating and localization effects in quantum dynamics.

We are interested in rigorous aspects of quantum dynamics in extended systems like spin chains. One of the remarkable properties is that there can be a robust phase where the conductivity is exactly zero and the spin chain is stuck forever.This is called Many-Body-Localization and it something like a hot topic nowadays. Rigorous results on this are sparse. But closely related phenomena can be studied rigorously.

These include:
1) Non-perturbative origin of transport, which we call 'asymptotic localization' and which implies that transport is very slow. From the mathematical point of view, these are many-body Nekoroshev estimates.

2) Effective Hamiltonians and slow heating under periodic high-frequency driving.This is similar to the first point in that heating under periodic driving is non-perturbative in the inverse frequency and for exponentially long timescales, the system evolves as if it were described by a time-independent effective Hamiltonian.

The work I will report on includes collaborations with: Francois Huveneers, Alex Bols, Dmitry Abanin, Wen-Wei Ho.

Deepak Dhar: Pattern formation in growing sandpiles.

If we add a large number of particles, say N at the origin, in the abelian sandpile model, starting with a periodic background, and relaxing, we generate very interesting patterns with non-trivial structure. These patterns show proportionate growth: different features of the pattern grow proportionately for large N. This is a very important feature of biological growth in animals, as baby animals grow into adults. I will argue that this is not just an accident, and that the sandpile model captures some important aspects of the actual phenomena. Growth on noisy backgrounds produces quite similar patterns, but is much less understood. I will also discuss patterns formed with anisotropic toppling rules, that give rise to self-affine growth, which produces more complicated growth patterns.

Pablo Ferrari: TASEP Hydrodynamics using microscopic characteristics.

It is known that the rescaled one-dimensional TASEP approaches the solution of the Burgers equation. I will review these results and show a new proof of the limit in the rarefaction fan using microscopic characteristics and coupling. The approach avoids the sub-additive ergodic theorem which is the crucial fact used in the original proof of Hermann Rost.

Davide Gabrielli: Macroscopic fluctuation theory (mini-course).

Lecture 1: Particle systems, scaling limits and large deviations

In this first lecture I will introduce a class of stochastic microscopic models very useful as toy models in non equilibrium statistical mechanics. These are multi-component stochastic particle systems like the exclusion process, the zero range process and the KMP model. I will discuss their scaling limits and the corresponding large deviations principles. Problems of interest are the computation of the current flowing across a system and the understanding of the structure of the stationary non equilibrium states. I will discuss these problems in specific examples and from two different perspectives. The stochastic microscopic and combinatorial point of view and the macroscopic variational approach where the microscopic details of the models are encoded just by the transport coefficients.

Lecture 2: Macroscopic fluctuation theory

In this second lecture I will discuss the basic ideas of the macroscopic fluctuation theory as an effective theory in non equilibrium statistical mechanics. All the theory develops starting from a principal formula that describes the distribution at large deviations scale of the joint fluctuations of the density and the current for a diffusive system. The validity of such a formula can be proved for diffusive stochastic lattice gases. I will discuss an infinite dimensional Hamilton-Jacobi equation for the quasi-potential of stationary non equilibrium states, fluctuation-dissipation relationships, the underlying Hamiltonian structure, a relation with work and Clausius inequality, a large deviations functional for the current flowing through a system.

Lecture 3: Examples and applications

In the last lecture I will apply the macroscopic fluctuation theory to solve specific problems. I will show that several features and behaviors of non equilibrium systems can be deduced within the theory. In particular I will discuss the following issues: the presence of long range correlations in stationary non equilibrium states; the explicit computation of the large deviations rate functional for a few one dimensional stationary non equilibrium states; the existence of dynamical phase transitions in terms of the current flowing across the system, the existence of Lagrangian phase transitions.

Giovanni Gallavotti: Random matrices and rarefied gases properties.

Well known properties of the largest or smallest Lyapunov exponent of a product of real matrices with a “cone property” can be naturally studied via the classical theory of the Mayer series in Statistical Mechanics of rarefied gases.

Cristian Giardinà: Asymmetric dualities in non-equilibrium systems.

Duality theory has proved itself extremely useful in the analysis of non-equilibrium systems. For instance, several one-dimensional systems driven out-of-equilibrium by imposing a density/energy gradient at the boundaries have a dual process with absorbing states, that considerably simplifies the description of the stationary measure. In this talk I will consider the case of systems with an asymmetric dynamics, i.e. processes driven out-of-equilibrium by an external field. I will introduce a general approach to construct asymmetric systems that admit a dual process and, as an example, I will discuss the asymmetric version of the Kipnis-Marchioro-Presutti model. The construction method provides informations on the current statistics, it is algebraic and it has been developed in two recent joint works with Gioia Carinci, Frank Redig and Tomohiro Sasamoto [1,2].

References:

(1) Asymmetric stochastic transport models with $U_q(SU(1,1))$ symmetry, preprint arXiv:1507.0147856.
(2) A generalized Asymmetric Exclusion Process with $U_q(SL_2)$ stochastic duality, preprint arXiv:1407.3367, Probability Theory and Related Fields (2015).

Stefan Grosskinsky: Metastability in a condensing zero-range process in the thermodynamic limit.

Zero-range processes with decreasing jump rates are known to exhibit condensation, where a finite fraction of all particles concentrates on a single lattice site when the total density exceeds a critical value. We study such a process on a one-dimensional lattice with periodic boundary conditions in the thermodynamic limit with fixed, super- critical particle density. We show that the process exhibits metastability with respect to the condensate location, i.e. the suitably accelerated process of the rescaled location converges to a limiting Markov process on the unit torus. This process has stationary, independent increments and the rates are characterized by the scaling limit of capacities of a single random walker on the lattice. Our result extends previous work for fixed lattices and diverging density by Beltran and Landim, and we follow their martingale approach. Besides additional technical difficulties in estimating error bounds for transition rates, the thermodynamic limit requires new estimates for equilibration towards a suitably defined distribution in metastable wells, corresponding to a typical set of configurations with a particular condensate location. The total exit rates from individual wells turn out to diverge in the limit, which requires an intermediate regularization step using the symmetries of the process and the regularity of the limit generator. Another important novel contribution is a coupling construction to provide a uniform bound on the exit rates from metastable wells, which is of a general nature and can be adapted to other models. This is joint work with Ines Armendariz and Michalis Loulakis.

Milton Jara: The weak KPZ universality conjecture.

The aim of this series of lectures is to explain what the weak KPZ universality conjecture is, and to present a proof of it in the stationary case.

Lecture 1: The KPZ equation, the KPZ universality class and the weak and strong KPZ universality conjectures.

Lecture 2: The martingale approach and energy solutions of the KPZ equation.

Lecture 3: A proof of the weak KPZ universality conjecture in the stationary case.

Antti Kupiainen: Renormalization group and stochastic PDE's

Stochastic PDE's driven by space time white noise require renormalization of some of their terms to be well posed. I will discuss a renormalization group approach to this problem with the examples of KPZ equation and the Ginzburg-Landau equation.

Claudio Landim: Quantitative analysis of Clausius inequality in driven diffusive systems and corrections to the hydrodynamic limit.

In the context of driven diffusive systems, for thermodynamic transformations over a large but finite time window, we derive an expansion of the energy balance. In particular, we characterize the transformations which minimize the energy dissipation and describe the optimal correction to the quasi-static limit. Surprisingly, in the case of transformations between homogeneous equilibrium states of an ideal gas, the optimal transformation is a sequence of inhomogeneous equilibrium states.

We also consider a one-dimensional, weakly asymmetric, boundary driven exclusion process on the interval $[0, N] \cap \mathbb{Z}$ in the super-diffusive time scale $N^2 \epsilon_N^{-1}$, where $1 \ll \epsilon_N^{-1} \ll N^{1/4}$. We assume that the external field and the chemical potentials, which fix the density at the boundaries, evolve smoothly in the macroscopic time scale. We derive an equation which describes the evolution of the density up to the order ϵ_N.

Alexandre Lazarescu: Hydrodynamic spectrum and dynamical phase transition in one-dimensional bulk-driven particle gases.

Interacting particle gases are often used as toy models to gain insight on more complex systems, especially when out of equilibrium, where a general theoretical framework (such as an equivalent of the free energy) is not available. There are two main ways to approach them analytically: either one can solve them exactly, or one may propose an effective coarse-grained description in the hope that it captures the essential features of the model. The first approach is in principle powerful but very restrictive, and understanding how the second (more versatile) approach can be attempted systematically is a major problem in modern statistical physics. An important step in that direction was achieved through the Macroscopic Fluctuation Theory (MFT), which gives a systematic expression of the large deviation function of hydrodynamic currents and densities for diffusive (boundary-driven or weakly bulk-driven) models. In this talk, we will conjecture that this same formalism can be applied to bulk-driven models through a non-rigorous trick, yielding, for instance, the large deviation function of the current flowing through the system, with a few important differences, which make those models arguably richer than diffusive ones: first, information can be easily obtained on a large family of excited states as well as on the steady state, which is not the case, or at least more difficult, for diffusive systems ; and secondly, one can find regimes where this approach breaks down, leading to dynamical phase transitions which are not present in diffusive systems, and which can be related to the KPZ universality class in some cases. All of these conjectures will be checked against the asymmetric simple exclusion process (ASEP), which is solvable exactly.

Lionel Levine: Threshold state of the abelian sandpile

In the abelian sandpile model of Bak-Tang-Wiesenfeld and Dhar, the microstate of a pile of sand is modeled by an integer-valued function on the vertices of a finite graph. The pile evolves according to local moves called topplings. Some piles stabilize after a finite number of topplings. Others (if there is no way for sand to exit the system) topple forever. For any pile s_0 if we repeatedly add a grain of sand at an independent random vertex, we eventually reach a "threshold state" s_T that topples forever. Poghosyan, Poghosyan, Priezzhev and Ruelle conjectured a precise value for the expected amount of sand in s_T in the limit as s_0 tends to negative infinity. I will outline how this conjecture was proved in `http://arxiv.org/abs/1402.3283` by means of a Markov renewal theorem.

Christian Maes: Nonequilibrium physics (Mini-cours)

We review some major challenges of today's nonequilibrium physics. In particular we will focus on response and fluctuation theory to understand the nature and the role of statistical forces induced by nonequilibrium media. We discuss how these forces are generically non-gradient and non-additive, how they are connected with steady state thermodynamics and still carry kinetic information, and how they can possible stabilize what under contact with equilibrium media remains unstable.
Lecture 1: What is nonequilibrium, questions and models.
Lecture 2: Statistical forces from nonequilibrium media I.
Lecture 3: Statistical forces from nonequilibrium media II.

Karel Netočný: Nonequilibrium generalization of the Nernst's heat theorem.

Steady states of (small) nonequilibrium stochastic systems can at low temperatures be conveniently described in terms of dominant configurations and attractors exhibiting the largest dynamical activity. After a brief account of this approach I will focus on the problem of heat exchange along slow transformations between steady states, to show how its low-temperature asymptotics derives from dominant excitation and relaxation paths associated with the lowest excitations. As a main result we obtain a nonequilibrium generalization of the Nernst's principle stating that under suitable conditions the reversible component of heat vanishes in the zero-temperature limit. Its general (non) validity will be discussed.

Errico Presutti: Latent heat and the Fourier law.

I consider the Kawasaki dynamics for a $d = 1$ lattice gas with interactions given by an attractive Kac potential with range γ^{-1}, $\gamma > 0$ the Kac scaling parameter. The temperature is fixed in a range where the system has a phase transition in the limit $\gamma \to 0$. The particles are confined in an interval of $\mathbb{Z}$ of length $\gamma^{-1}\ell$. At the endpoints we add birth-death processes so that the density at the left and right endpoints are kept at the values ρ_- and $\rho_+ > \rho_-$. The most interesting case is when ρ_+ is metastable and $\rho_- = 1 - \rho_+$ is (by symmetry) also metastable. In such a case we prove that after the limit $\gamma \to 0$ there is a stationary profile (for the $\gamma \to 0$ limit macroscopic equation) which for ℓ large has a non zero current which goes from the smaller to the larger density, i.e. the current goes in the direction of the gradient against what stated by the Fourier law. The phenomenon can be interpreted in terms of the latent heat associated to the phase transition. There is an apparent contradiction with the law of thermodynamics because using this result one could theoretically construct stationary motions with a non zero current without external forces, this is due to the fact that the theory describes what happens after the limit $\gamma \to 0$ and therefore does not take into account the large deviations which will eventually occur.

The result therefore does not describe the behavior of the system for infinitely long times when $\gamma > 0$. I will present some computer simulations of the system which give more details on the phenomenology.
The talk is based on a paper in preparation that I have in collaboration with Matteo Colangeli and Anna De Masi.

David Ruelle: Generalized detailed balance and biological applications.

A detailed balance relation is an identity based on time reversal symmetry of time evolution equations. This identity, which is part of statistical mechanics, relates the probability of an evolution $j \to k$ and the probability of the reverse evolution $k \to j$. Ambitious applications of detailed balance to biological replication have been proposed by J. England. Instead of using a stochastic dynamics approximation we look here seriously at the deterministic time evolution, obtaining a detailed balance relation different from that used by England, and physically more realistic. This does however not seriously affect England's biological discussion.

Tomohiro Sasamoto: Determinantal structures for 1D KPZ equation and related models.

The Kardar-Parisi-Zhang (KPZ) equation is a simple stochastic nonlinear PDE describing a growing interface motion. In the last few years, its one-dimensional version has attracted much attention and many results have been obtained. For instance our understanding of its definition has deepened. Its universality has turned out to be much stronger than previously thought. These developments are to some extent related to the fact that the 1D KPZ equation is exactly solvable; some explicit formulas for important quantities such as the height distribution and two-point correlation function have been obtained.

In this presentation we first review recent developments on the 1D KPZ equations and related models. Then we discuss the mechanism of the solvability of these models, in particular from the point of view of connection to random matrix theory and its generalizations.
This is based on collaborations with T. Imamura.