Computability Randomness and Apllications

June 20–24, 2016

Verónica Becher: Independence of normal words.

Recall that normality is a elementary form of randomness: an infinite word is normal to a given alphabet if all blocks of symbols of the same length occur in the word with the same asymptotic frequency. We consider a notion of independence on pairs of infinite words formalising that two words are independent if no one helps to compress the other using one-to-one finite transducers with two inputs. As expected, the set of independent pairs has Lebesgue measure 1. We prove that not only the join of two normal words is normal, but, more generally, the shuffling with a finite transducer of two normal independent words is also a normal word. The converse of this theorem fails: we construct a normal word as the join of two normal words that are not independent. We construct a word x such that the symbol at position n is equal to the symbol at position 2n. Thus, x is the join of x itself and the subsequence of odd positions of x. We also show that selection by finite automata acting on pairs of independent words preserves normality. This is a counterpart version of Agafonov's theorem for finite automata with two input tapes.
This is joint work with Olivier Carton (Universitéé Paris Diderot) and Pablo Ariel Heiber (Universidad de Buenos Aires).

Douglas Cenzer: Random numbers as probabilities of machine behaviour.

A fruitful way of obtaining meaningful, possibly concrete, algorithmically random numbers is to consider a potential behavior of a Turing machine and its probability with respect to a measure (or semi-measure) on the input space of binary codes. For example, Chaitin's Omega is a well known ML random number that was obtained by considering the halting probability of a universal prefix-free machine. In the last decade, similar examples have been obtained for higher forms of randomness, i.e. randomness relative to strong oracles. In this work we obtain characterizations of the algorithmically random reals in higher randomness classes, as probabilities of certain events that can happen when an oracle universal machine runs probabilistically on a random oracle. Moreover we apply our analysis to different machine models, including oracle Turing machines, prefix-free machines, and models for infinite online computation. We find that in many cases the arithmetical complexity of a property is directly reflected in the strength of the algorithmic randomness of the probability with which it occurs, on any given universal machine. On the other hand we point to many examples where this does not happen and the probability is a number whose algorithmic randomness is not the maximum possible, with respect to its arithmetical complexity. Finally we find that, unlike the halting probability of a universal machine, the probabilities of more complex properties like totality, cofinality, computability or completeness do not necessarily have the same Turing degree when they are defined with respect to different universal machines.
This is joint work with Christopher Porter.

Peter Cholak: Density-1-bounding and quasiminimality in the generic and coarse degrees.

The primary focus of this talk will be to carefully explain all the terms in the title of this talk. Let $A \subseteq \omega$. Then A is *density-1* if the limit of the densities of its initial segments is 1, or in other words, if $\lim_{n\to\infty} \frac{|A\cap n|}{n} = 1$. A real A is *generically computable* if there exists a partial computable function ϕ such that $dom(\phi)$ is density-1, and ϕ is a partial computation of A. A real A is *coarsely computable* if there exists a total computable function ϕ, whose range is contained in $\{0, 1\}$ such that $\{n \mid \phi(n) = A(n)\}$ is density-1. Analogous to how the notion of computable is used to create the Turing degrees, we can use these notions to create the (uniform and nonuniform) generic and coarse degrees. There is an embedding of the Turing degrees into these degree structures. A set A in one of these degree structures in *quasiminimal* if the only embedded Turing degree below it is 0. We characterize of the level of randomness required to ensure quasiminimality in the uniform and nonuniform coarse and generic degrees. A generic degree $\mathbf{a}$ is *density-1-bounding* if there is a nonzero generic degree $\mathbf{b}$ such that $\mathbf{b} \leq_g \mathbf{a}$ and such that $\mathbf{b}$ is the generic degree of a density-1 real. We consider the open question "Is every nonzero generic degree a density-1-bounding generic degree?" A negative or positive answer provides different structural results about the generic degrees. We show a wide class of assumptions is sufficient to prove density-1-bounding and that a negative example must be quasiminimal. The talk will draw from a number of papers including the two references below:

References

[1] Peter Cholak and Greg Igusa, *Density-1-bounding and quasiminimality in the generic degrees*, Submitted.

[2] Denis Hirschfeldt, Carl Jockusch, Rutger Kuyper, and Paul Schupp, *Coarse reducibility and algorithmic randomness*, J. Symbolic Logic, to appear.

Rod Downey: The Computational Power of Sets of Random Strings.

We consider the computational power of the set of random strings for e.g. plain or prefix-free Kolmogorov complexity, say R_C or R_K. These sets are always *wtt*-completer. In two beautiful papers Kummer [3] proved that R_C is always *tt*-complete regardless of the chosen universal machine, whereas A. A. Muchnik proved that R_{K_U} may or may not be *tt*-complete depending on the choice of universal machine U. Allender and a host of co-authors have proven that if you look at *polynomial time* or other constrained (and hence total) reducibilities, then reducing to sets of random strings can capture complexity classes. For example, Allender, Friedman and Gasarch [1] proved that

$$\Delta^0_1 \cap \cap_U P^{R_{K_U}} \subseteq \text{PSPACE}.$$

In this talk I will review some of this material, and prove various results about sets *tt*-reducible to R_{K_U}'s. For example, it is shown that there are no *tt*-minimal pairs, but if $X \leq R_{K_U}$ for *all* U, then X is computable. This last result shows that the hypothesis "$\Delta^0_1\cap$" can be removed from the Allender, Friedman and Gasarch result, confirming a conjecture of Allender.

These results are joint work with Mingzhong Cai, Rachel Epstein, Steffen Lempp and Joeseph Miller.

References

[1] E. Allender, L. Friedman and W. Gasarch, *Limits on the computational power of random strings*, Information and Computation, 222 (2013), pp. 80-92.

[2] M. Cai, R. Epstein, S. Lempp and J. Miller, *Random strings and truth-table degrees of Turing complete c.e. sets*, Logical Methods in Computer Science, 10(3):15, 24 pp., 2014.

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Willem Fouché: Zero sets and local time of algorithmically random Brownian motion.

We shall discuss the construction of an analogue of Lévy's continuous local time $t \mapsto L(t, \omega, a)$ at each computable point a for each algorithmically random Brownian motion ω (also known as a complex oscillation).

Our construction is based on an effective version of Ito's stochastic integration with respect to the Wiener measure as was recently developed by Mukeru (2015). Other keypoints in the argument are a known explicit layerwise computable construction of a complex oscillation from a Kolmogorov-Chaitin random real number (Davie, Fouché, 2013) and the theory of Martin-Löf randomness of metric spaces as was developed by Hoyrup and Rojas since 2009.

Along these lines new expressions for local times were found, for which the almost sure versions are also new.

References

[1] W.L. Fouché, S. Mukeru, G. Davie, *Fourier spectra of measures associated with algorithmically random Brownian motion.*, ***Logical Methods in Computer Science*** , vol. 10 (3:20), pp. 1–24.

[2] G. Davie, W.L. Fouché, *On the computability of a construction of Brownian motion. Mathematical Structures in Computer Science, 23, pp 1257-1265. Title of article*, ***Mathematical Structures in Computer Science***, vol. 23, pp. 1257-1265.

Johanna N.Y. Franklin: Carleson's Theorem and Schnorr randomness.

Carleson's Theorem states that for $1 < p < \infty$, the Fourier series of a function f in $L^p[-\pi, \pi]$ converges to f almost everywhere. We consider this theorem in the context of computable analysis and show the following two results.

(1) For a computable $p > 1$, if f is a computable vector in $L^p[-\pi, \pi]$ and $t_0 \in [-\pi, \pi]$ is Schnorr random, then the Fourier series for f converges at t_0.

(2) If $t_0 \in [-\pi, \pi]$ is not Schnorr random, then there is a computable function $f : [-\pi, \pi] \to \mathbb{C}$ whose Fourier series diverges at t_0.

This is joint work with Timothy H. McNicholl, and Jason Rute.

Silvère Gangloff: On the algorithmics of entropy of computable metric spaces.

The entropy is a topological invariant of dynamical systems measuring their complexity. We will introduce some notion of computable metric space, computable function, and after that present some results about the algorithmic properties of the entropy for various types of computable dynamical systems (which are couples of a computable metric space and a computable function from this space into itself). We will give a general obstruction for the entropies of theses systems which is strengthened for computable systems on the interval, and decidable one- dimensional subshifts with various dynamical restrictions (such as minimality, transitivity). Then, we answer the question of realization (that is to say, given some number α, can we find some computable dynamical system which has entropy α ?) for interval maps, one-dimensional subshifts, maps of Cantor space.
This is joint work with Benjamin Hellouin de Menibus, Mathieu Sablik and Cristobal Rojas.

Jarkko Kari: On the periodicity of multidimensional words of low complexity.

Let $D \subseteq \mathbb{Z}^d$ be finite and let $c \in A^{\mathbb{Z}^d}$ be an infinite d-dimensional word over finite alphabet A. The D-patterns of c are $\tau(c)_{|D} \in A^D$ where τ spans over all translations, i.e., the patterns of shape D that appear in c. We say that c has low D-complexity if there are at most $|D|$ distinct D-patterns in c. We study regularities enforced on c by such low complexity assumption. In particular, we are interested on periodicity, that is, whether there exists a non-trivial translation τ such that $\tau(c) = c$.

We use algebraic geometry to show that if c has low D-complexity for any D then c decomposes into a finite number of periodic components. We apply such decomposition to study Nivat's conjecture: the claim that if a two-dimensional c has low D-complexity for a rectangle D then c is periodic. We prove an asymptotic version that states that any non-periodic c can have low D-complexity only for finitely many distinct rectangles D.

References

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Takayuki Kihara: Borel isomorphism and computability.

Given two spaces are n-th level Borel isomorphic if there exists a bijection between these spaces which preserves the Borel hierarchy above $\mathbf{\Sigma}^0_{n+1}$.

The finite level Borel isomorphism problem asks whether any uncountable Polish space is n-th level Borel isomorphic either to the real line or to the Hilbert cube for some integer n. Jayne's theorem (the Baire class version of the Gel'fand-Kolmogorov theorem) connects this problem with the ring-theoretic (and linear-isometric) classification of Banach algebras of finite class Baire functions on compacta (endowed with the supremum norm and the pointwise ring operation). We solve the finite level Borel isomorphism problem by using notions from computability theory such as degree spectra, Scott ideals (ω-models of weak König's lemma), etc. We also mention the relationship between our solution to the finite level Borel isomorphism problem and Pol's solution to Alexandrov's old problem in infinite dimensional topology.

Joseph S. Miller: A derivation on the field of d.c.e. reals.

Barmpalias and Lewis-Pye [1] recently proved that if α and β are (Martin-Löf) random left-c.e. reals with left-c.e. approximations $\{\alpha_s\}_{s\in\omega}$ and $\{\beta_s\}_{s\in\omega}$, then

$$\frac{\partial\alpha}{\partial\beta} = \lim_{s\to\infty} \frac{\alpha-\alpha_s}{\beta-\beta_s}. \tag{1}$$

converges and is independent of the choice of approximations. Furthermore, they showed that $\partial\alpha/\partial\beta = 1$ if and only if $\alpha-\beta$ is nonrandom; $\partial\alpha/\partial\beta > 1$ if and only if $\alpha-\beta$ is a random left-c.e. real; and $\partial\alpha/\partial\beta < 1$ if and only if $\alpha-\beta$ is a random right-c.e. real.

We extend their results to the d.c.e. reals, which clarifies what is happening. The extension is straightforward. Fix a random left-c.e. real Ω with approximation $\{\Omega_s\}_{s\in\omega}$. If α is a d.c.e. real with d.c.e. approximation $\{\alpha_s\}_{s\in\omega}$, let

$$\partial\alpha = \frac{\partial\alpha}{\partial\Omega} = \lim_{s\to\infty} \frac{\alpha-\alpha_s}{\Omega-\Omega_s}.$$

As above, the limit exists and is independent of the choice of approximations. Now $\partial\alpha = 0$ if and only if α is nonrandom; $\partial\alpha > 0$ if and only if α is a random left-c.e. real; and $\partial\alpha < 0$ if and only if α is a random right-c.e. real.

As we have telegraphed by our choice of notation, ∂ is a derivation on the field of d.c.e. reals. In other words, ∂ preserves addition and satisfies the Leibniz law:

$$\partial(\alpha\beta) = \alpha\,\partial\beta + \beta\,\partial\alpha.$$

(However, ∂ maps outside of the d.c.e. reals, so it does not make them a differential field.) We will see how the properties of ∂ encapsulate much of what we know about randomness in the left-c.e. and d.c.e. reals. We also show that if $f\colon \mathbb{R}\to\mathbb{R}$ is a computable function that is differentiable at α, then $\partial f(\alpha) = f'(\alpha)\,\partial\alpha$. This allows us to apply basic identities from calculus, so for example, $\partial\alpha^n = n\alpha^{n-1}\,\partial\alpha$ and $\partial e^\alpha = e^\alpha\,\partial\alpha$. Since $\partial\Omega = 1$, we have $\partial e^\Omega = e^\Omega$.

Given a derivation on a field, the elements that it maps to zero also form a field: the *field of constants*. In our case, these are the nonrandom d.c.e. reals. We show that, in fact, the nonrandom d.c.e. reals form a *real closed field*. Note that it was not even known that the nonrandom d.c.e. reals are closed under addition, and indeed, it is easy to prove the convergence of (1) from this fact. In contrast, it has long been known that the nonrandom left-c.e. reals are closed under addition (Demuth [2] and Downey, Hirschfeldt, and Nies [3]). While also nontrivial, this fact seems to be easier to prove. Towards understanding this difference, we show that the real closure of the nonrandom left-c.e. reals is strictly smaller than the field of nonrandom d.c.e. reals. In particular, there are nonrandom d.c.e. reals that cannot be written as the difference of nonrandom left-c.e. reals; despite being nonrandom, they carry some kind of intrinsic randomness.

References

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Martin Monath: On totally ω-c.e. degrees and complex left-c.e. reals.

We show that a c.e. Turing degree is totally ω-c.e. if and only if every left-c.e. real in the degree is cl-reducible to a complex left-c.e. real. This answers a question proposed by Noam Greenberg.
This is joint work with Klaus Ambos-Spies and Nadine Losert.

Benoit Monin: A resolution of the Gamma question.

Generic-case complexity is a subfield of computational complexity. It started with the observation that some problems that are difficult to solve in full are easy to solve on "most inputs", namely on a set of inputs of asymptotic lower density 1. This notion was introduced by Kapovich, Myasnikov, Schupp and Shpilrain. They showed among other things that for a large class of finitely generated groups, the generic case complexity of the word problem is linear.

This notion has recently been extended by Jockusch and Schupp. The authors identified two notions that can be proved to be incomparable. The first is generic computability, where one must always give the right answer, without having to provide an answer for a small set of input. The second is coarse computability, for which one always have to provide an answer, with the right of being wrong on a small set of inputs. In both cases, a set of inputs is considered small if it is of asymptotic upper density 0.

Then Andrews, Cai, Diamondstone, Jockusch and Lempp assigned in "Asymptotic density, computable traceability and 1-randomness" a value gamma (with lower case 'g') to each set of natural numbers, which indicates how far the set is from being coarse-computable. They used this to assign a value Gamma (with upper case 'G') to each Turing degree, which indicates how far the degree is from being coarse-computable. They proved that the Gamma values of 0, 1/2 and 1 can be realized. They also proved that if a Turing degree has a Gamma value strictly larger than 1/2, then it is the computable degree and its Gamma value in fact equals 1. They asked whether a Turing degree can have a Gamma value strictly in between 0 and 1/2.

Using notions from computability theory, developped by Monin and Nies, together with some techniques from the field of error-correcting codes, we are able to give a negative answer to this question: The only Gamma values that can be realized by a Turing degrees are $0, 1/2$ and 1.

André Nies: Randomness connecting to set theory and to reverse mathematics

I will discuss two recent interactions of the field called randomness via algorithmic tests. With Yokoyama and Triplett, I study the reverse mathematical strength of two results of analysis. (1) The Jordan decomposition theorem says that every function of bounded variation is the difference of two nondecreasing functions. This is equivalent to ACA or to WKL, depending on the formalisation. (2) A theorem of Lebesgue states that each function of bounded variation is differentiable almost everywhere. This turns out to be equivalent WWKL (with some fine work left to be done on the amount of induction needed). The Gamma operator maps Turing degrees to real numbers; a smaller value means a higher complexity. This operator has an analog in the field of cardinal characteristics along the lines of the Rupprecht correspondence [4]; also see [1]. Given a real p between 0 and 1/2, d(p) is the least size of a set G so that for each set x of natural numbers, there is a set y in G such that x and y agree on asymptotically more than p of the bits. Clearly, d is monotonic. Based on Monin's recent solution to the Gamma question (see [3] for background, and the post in [2] for a sketch), I will discuss the result with J. Brendle that the cardinal d(p) doesn't depend on p. Remaining open questions in computability (is weakly Schnorr engulfing equivalent to "Gamma = 0"?) nicely match open questions about these cardinal characteristics.

References

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[3] B. Monin and A. Nies. A unifying approach to the Gamma question. In *Proceedings of Logic in Computer Science (LICS)*. IEEE press, 2015.

[4] Nicholas Rupprecht. *Effective correspondents to cardinal characteristics in Cichoń's diagram.* PhD thesis, University of Michigan, 2010.

Jake Pardo: Gambling against some odds.

A restricted value martingale is a martingale whose increments take their magnitude from a given set of positive real numbers S – we call any such martingale an S-martingale. A set B anticipates a set A if every A-martingale is dominated by a countable set of B-martingales, and A evades B if this is not the case. Similarly, B singly anticipates A if every A-martingale is dominated by a single B- martingale, and otherwise A singly evades B. Bavly and Peretz [1] investigated the anticipation/evasion relationships between various sets of real numbers and in the process solved the case where $\sup A$ is bounded and B is bounded away from 0 and the case where B is well-ordered. This left a big question: what happens when 0 is an accumulation point of B?

I will prove that the natural numbers $\mathbb{N}$ singly evade the set $\{x^{-n} : n \in \mathbb{Z}\}$, where x is any positive real number. The proof is based on a betting game between two gamblers, one using an A-martingale as a strategy and the other using a B-martingale, in which a strategy for the A-martingale gambler is produced. The proof has an interesting connection to the n-Fibonacci numbers.

References

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Andrei Romashchenko: On centauric subshifts.

We discuss subshifts of finite type (tilings) that combine virtually opposite properties, being at once very simple and very complex. On the one hand, the combinatorial structure of these subshifts is rather simple: we require that all their configurations are quasiperiodic, or even that all configurations contain exactly the same finite patterns (in the last case a subshift is transitive, i.e., irreducible as a dynamical system). On the other hand, these subshifts are complex in the sense of computability theory: all their configurations are non periodic or even non-computable, or all their finite patterns have high Kolmogorov complexity, the Turing degree spectrum is rather sophisticated, etc.

We start with the simplest example of such centaurisme — with an SFT that is minimal and contains only aperiodic (and quasiperiodic) configurations. Then we discuss how far these heterogeneous properties can be strengthened without getting mutually exclusive.

This is a joint work with Bruno Durand (Univ. de Montpellier).

References

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Noah Schweber: Ultralimits and computability.

In this talk, we will discuss the computability-theoretic properties of ultralimits — specifically, ultralimits as operations on Turing ideals. If $\mathcal{U}$ is an ultrafilter on ω and J is a family of sets of natural numbers, we let $\delta_{\mathcal{U}}(J)$ be the set of limits along $\mathcal{U}$ of sequences in J. Besides being a natural construction, this is a source of a number of combinatorial computability-theoretic questions; we answer some, and raise others.

We begin by examining the closure properties of $\delta_{\mathcal{U}}(J)$ As long as J is reasonably closed — much less than being a Turing ideal is required here — the set $\delta_{\mathcal{U}}(J)$ is a Scott set containing the jump of each element of J. Moreover, so long as J is countable the converse is true, and this leads to a new proof that WKL_0 is strictly weaker than ACA_0; meanwhile, it is consistent with $ZFC + PD$ that the converse *fails* for uncountable J in a very strong way.

We then turn to lowness notions associated with the class of maps defined above. Say that a real X is *ultrafilter-low* if for some nonprincipal ultrafilter $\mathcal{U}$, $\delta_{\mathcal{U}}(deg(X)) = \delta_{\mathcal{U}}(REC)$. We show that all reals bounded by a 2-generic, or by a computably traceable, are ultrafilter-low; the question of an exact characterization remains open, as do the situation with respect to measure and the existence of a Δ^0_2 ultrafilter-low real. It is open even whether every noncomputable real X satisfies $\delta_{\mathcal{U}}(X) \neq \delta_{\mathcal{U}}(REC)$ for *some* $\mathcal{U}$; however, a counterexample would have to be Δ^0_2.

Finally, we look at the structure on the set of all ultrafilters on ω induced by the construction $\mathcal{U} \mapsto \delta_{\mathcal{U}}$. The ultrafilters carry a natural partially ordered monoid structure, and the ordering is refined by the Rudin-Keisler order; there is also a lightface version of the ordering, which is refined by the lightface Rudin-Keisler ordering. Unfortunately, the monoid structure is topologically badly behaved: in particular, there are no idempontents.

This is joint work with Uri Andrews, Mingzhong Cai, and David Diamondstone [1].

Frank Stephan: On Block Pumpable Languages.

Ehrenfeucht, Parikh and Rozenberg gave an interesting characterisation of the regular languages called the block pumping property. When requiring this property only with respect to members of the language but not with respect to nonmembers, one gets the notion of block pumpable languages. It is shown that these block pumpable are a more general concept than regular languages and that they are an interesting notion of their own: they are closed under intersection, union and homomorphism by transducers; they admit multiple pumping; they have either polynomial or exponential growth.
This is joint work with Christopher Chak, Rusins Freivalds and Henrietta Tan.

Pascal Vanier: Turing degree spectra of minimal subshifts.

Subshifts are shift invariant closed subsets of $\mathbb{Z}^d$, minimal subshifts are subshifts in which all points contain the same patterns, they are fundamental to the study of subshifts since any subshift contains a minimal subshift. We prove (work done with Emmanuel Jeandel) that the degree spectra of non-periodic minimal subshifts is upward closed: it contains the cone of degrees above any of its degrees. Furthermore, we construct (work done with Mike Hochman) that there exists subshifts whose spectrum consists of an uncountable number of cones with disjoint base.

Linda Brown Westrick: Seas of squares with sizes from a Π^0_1 set.

For each Π^0_1 set $S \subseteq \mathbb{N}$, consider a two-dimensional subshift X_S on the alphabet {black, white} whose elements consist of black squares of various sizes on a white background, where the side length of each square is in S. There is a natural infinite c.e. set of forbidden patterns generating such X_S. Building on the self-similar Turing machine tiling construction of Durand, Romashchenko and Shen (2010), we show that each X_S is sofic. That is, there is a shift of finite type Y_S on an alphabet with finitely many distinct kinds of black symbol and white symbol, and X_S is the image of Y_S under the map that forgets the distinctions.

Liang Yu: On higher Friedman's conjecture.

Friedman's conjecture says that every uncountable hyperarithmetic set of reals contains an upper cone of hyper degrees. The conjecture was confirmed by Martin. Kechris, Martin and Solovay asked whether this is true for uncountable Δ^1_{2n+1} sets respect to Δ^1_{2n+1}-degrees under projective determinacy. We first confirm the conjecture for $n = 1$ by working on Martin-Solovay tree presentation of Δ^1_{2n+1}-sets. Then by recent work of Yizheng Zhu, we answer the question forall levels. This is joint work with Yizheng Zhu.