Electron quantum optics and transport in topological materials

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Spectral theory of novel materials,
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In collaboration with: D. Ferraro, J. Rech, T. Jonckheere

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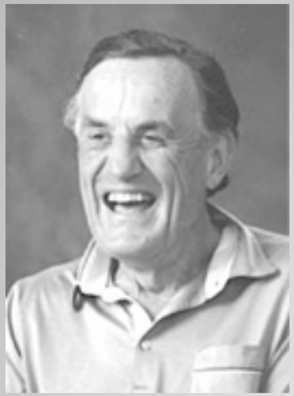
Outline

- The rise of electron quantum optics
- Hanbury-Brown-Twiss and Hong-Ou-Mandel interferometry with individual electrons
- Two-dimensional topological insulators: when Pauli meets topology
- SC/Hall hybrid systems: creation and collision of individual Bogoliubov excitations

Quantum optics: 50 years of history



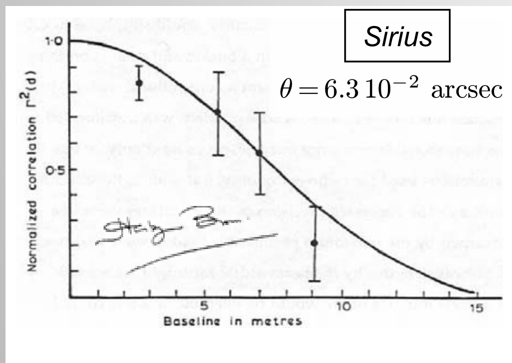
R. Hanbury Brown

The Quantum Theory of Optical Coherence*

ROY J. GLAUBER
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 11 February 1963)

Phys. Rev. **130**, 2529 (1963)

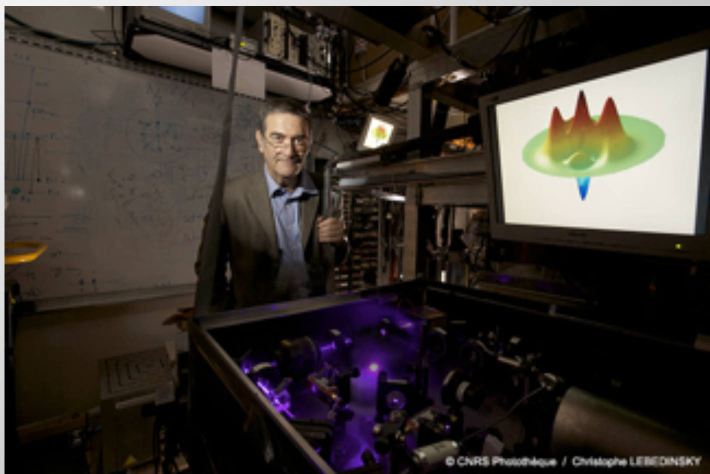
From 1956: stellar interferometry...



$$\Gamma^2(d) = [2J_1(x)/x]^2 \;;\; x = \pi\theta d/\lambda$$

R. Hanbury Brown and R. Q. Twiss, Nature **178**, 1046 (1956)

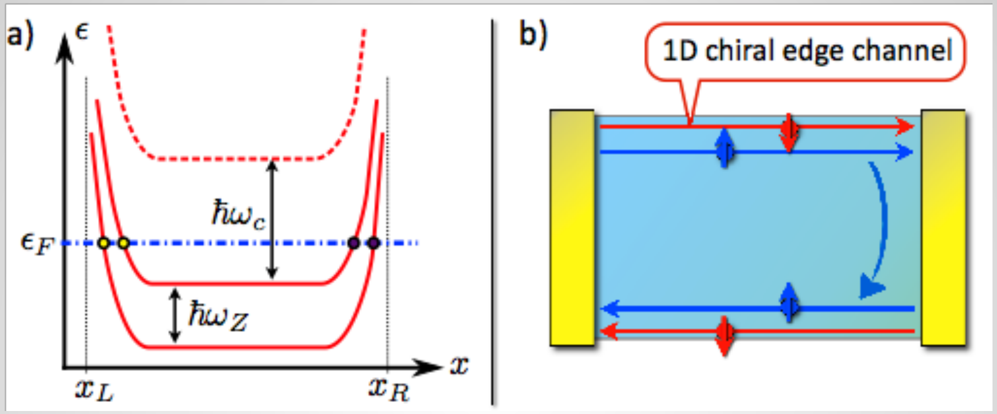
...to 2012: Nobel prize "for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems"



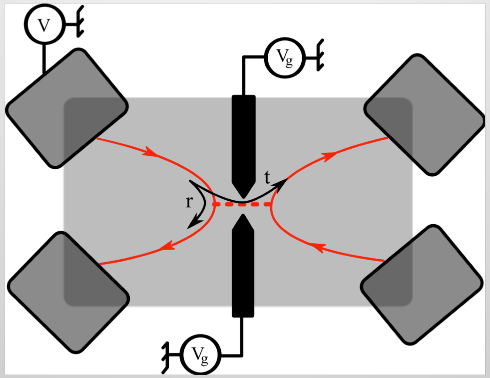
Electron quantum optics

Revising the tools of quantum optics with photons to describe individual electronic wave-packets in mesoscopic systems

E. Bocquillon *et al.*, Ann. Phys. (Berlin) **526**, 1 (2014)



One dimensional electron channels as chiral wave-guides
(several micrometers of elastic mean free path)

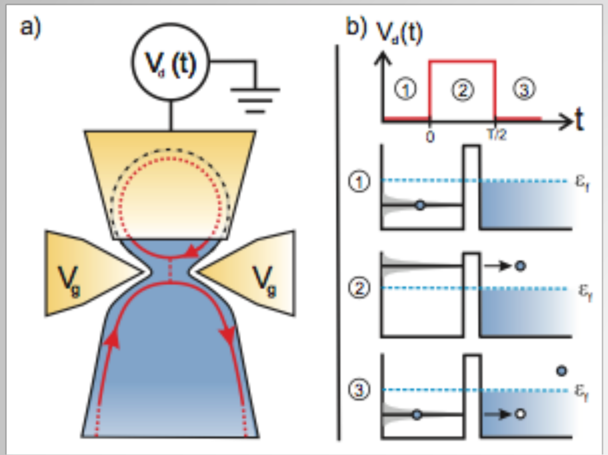


Quantum point contacts as beam splitters

Single electron sources

Driven mesoscopic capacitor in the integer quantum Hall effect

Moskalets *et al.*, Phys. Rev. Lett. **100**, 086601 (2008); G. Fève *et al.*, Science **316**, 1169 (2007)

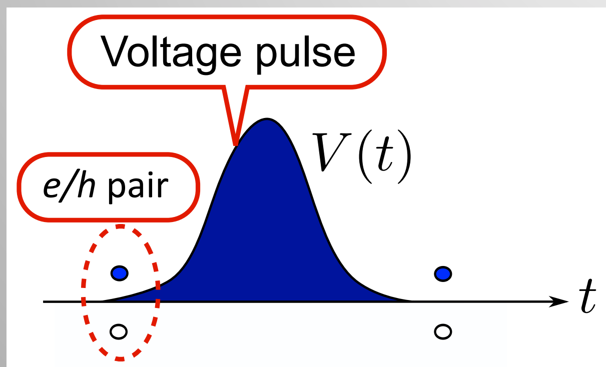


One electron and one hole with exponential wave-packets in time emitted at every period

$$\phi(t) \propto e^{-\frac{\gamma}{2}t} e^{-i\omega_0 t} \Theta(t)$$

Lorentzian voltage pulse in time

L. S. Levitov *et al.*, J. Math. Phys. **37**, 4845 (1996)

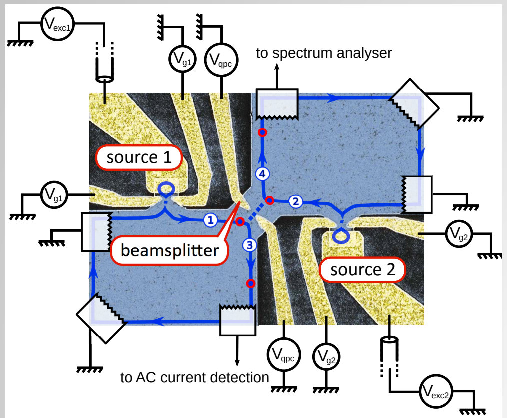


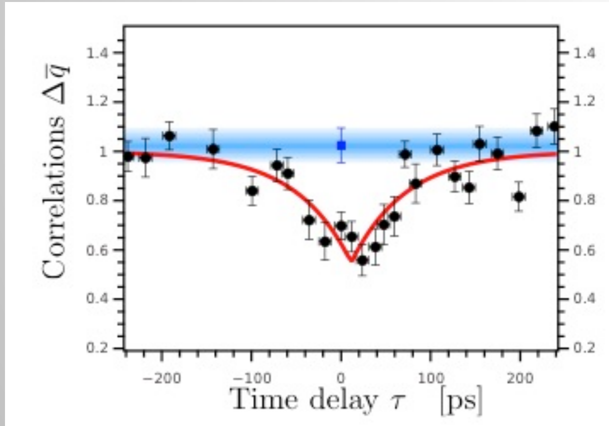
Zero particle-hole contribution at zero temperature

$$V(t) = \frac{n\hbar}{e} \sum_{m=-\infty}^{+\infty} \frac{2\tau_0}{(t - mT)^2 + \tau_0^2}$$

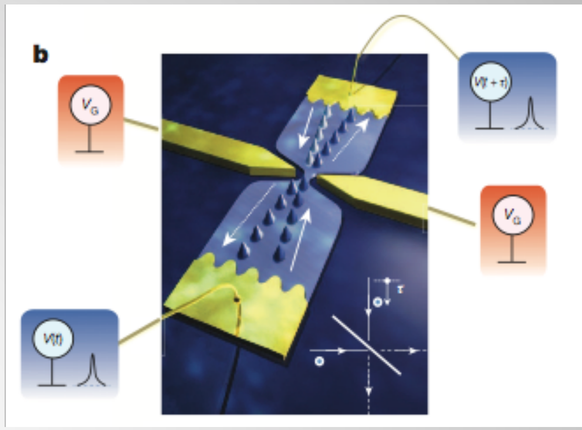
Individual electrons interferometry

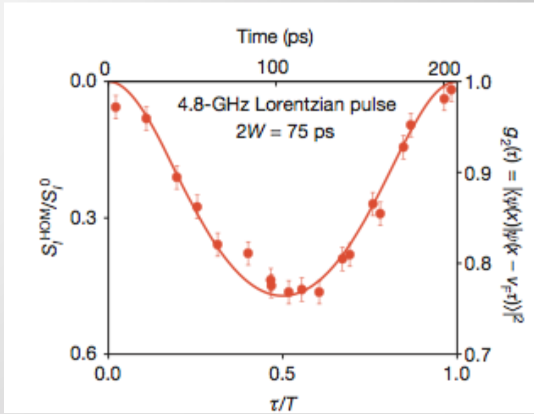
Hanbury-Brown-Twiss (HBT) and Hong-Ou-Mandel (HOM)





E. Bocquillon *et al.*, Science **339**, 1054 (2013)





J. Dubois *et al.*, Nature **502**, 659 (2013)

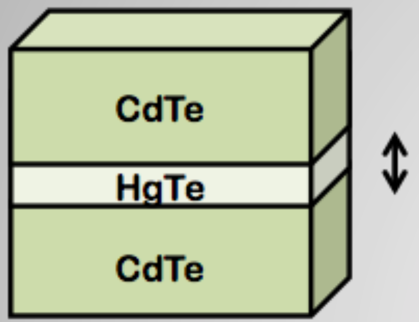
New physics related to Pauli principle and electron-electron interaction

T. Jonckheere, J. Rech, C. Wahl, T. Martin, Phys. Rev. B **86**, 125425 (2012);
C. Wahl, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. Lett. **112**, 046802 (2014)

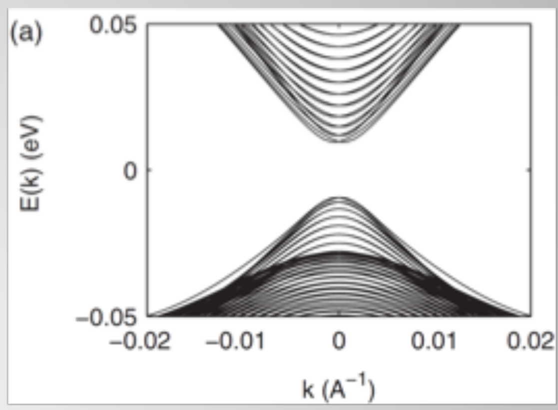
Quantum spin Hall effect (QSHE)

CdTe/HgTe/CdTe quantum wells

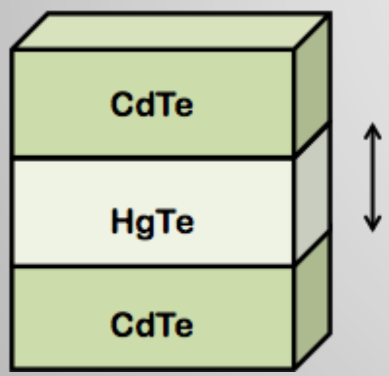
X.-L. Qi and S.-C. Zhang, Rev. Mod. Phys. **83**, 1057 (2011)



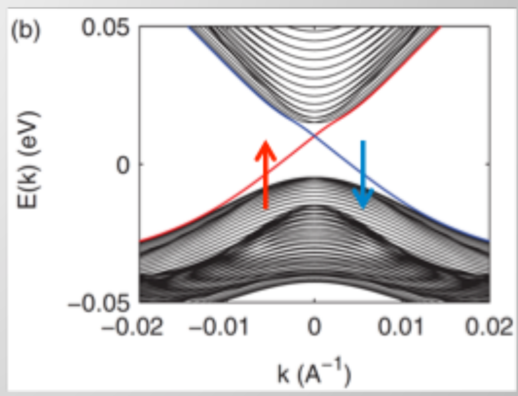
$$d < d_c \approx 6.3nm$$



Trivial insulator



$$d > d_c \approx 6.3nm$$



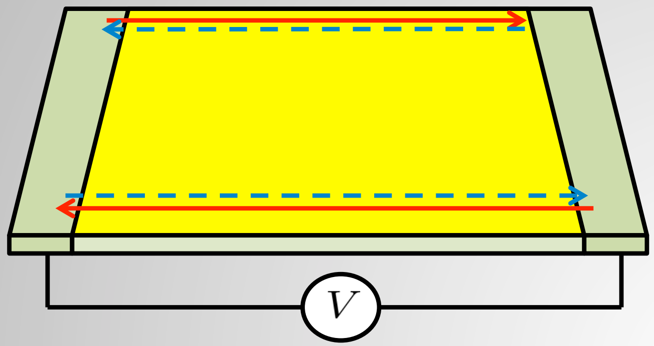
Topological insulator: gapless edge states at zero magnetic field, robust against backscattering

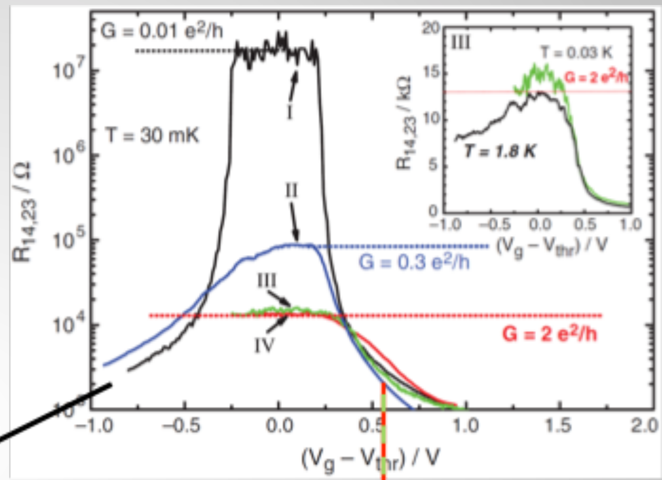
A. B. Bernevig *et al.*, Science **314**, 1757 (2006)

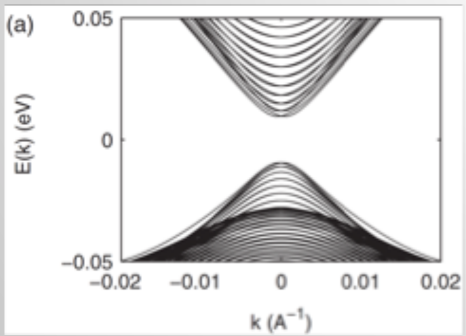
Helical edge states

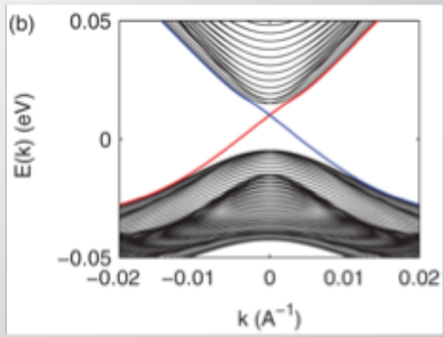
Two terminal measurements

M. König *et al*., Science **318**, 766 (2007)









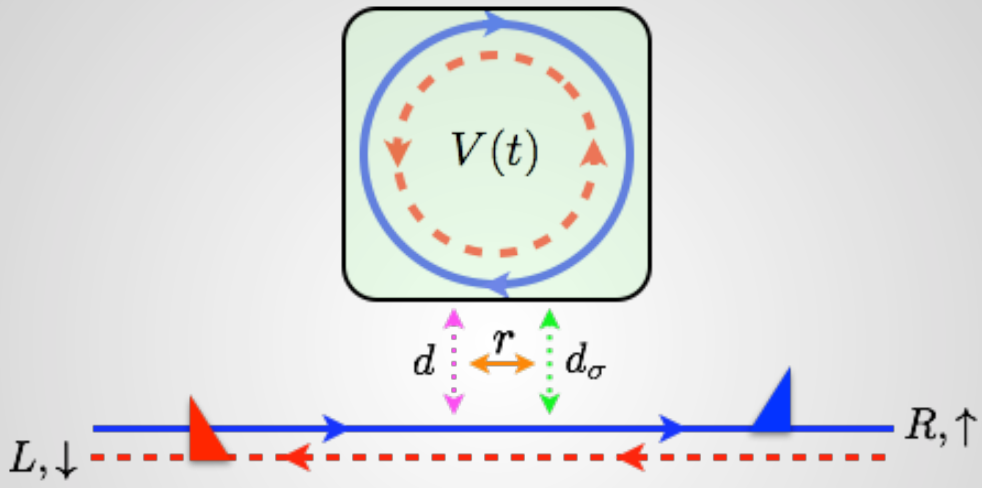
Two counter-propagating edge channels,
conductance quantization consistent with Landauer-Büttiker

M. Büttiker, Science **325**, 278 (2009)

Pair electron source

Driven mesoscopic capacitor coupled to helical edge states

A. Inhofer and D. Bercioux, Phys. Rev. B **88**, 235412 (2013);
P. P. Hofer and M. Buttiker, Phys. Rev. B **88**, 241308(R) (2013)



Spin-preserving and spin-flipping tunneling at the QPC

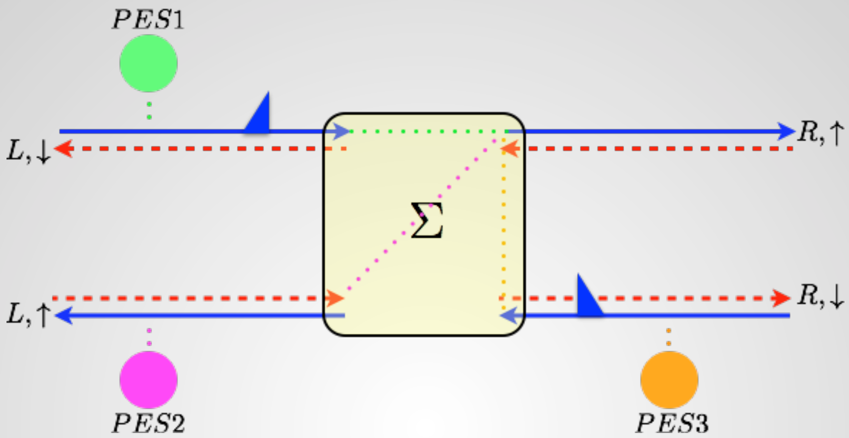
Injection of electrons pairs with opposite spin and propagating in opposite directions

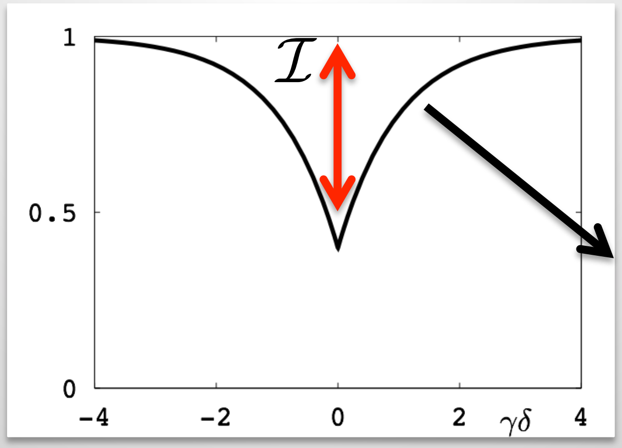
Creation of exponential wave-packets

HOM experiments: two-electrons injection (1)

D. Ferraro, C. Wahl, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **89**, 075407 (2014)

Equal spin injection (analogous to IQH)





$$q^{(2)}_{R\uparrow,L\uparrow}(\delta) = 1 - \mathcal{I} e^{-\gamma|\delta|}$$

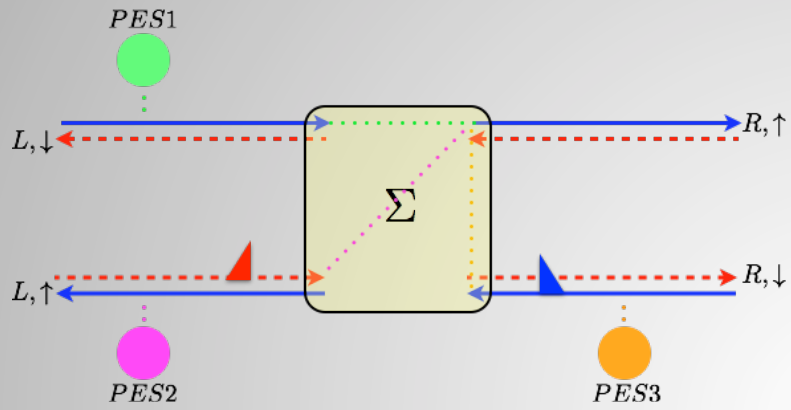
Loss of Pauli dip contrast *without* interaction, only due to additional channels

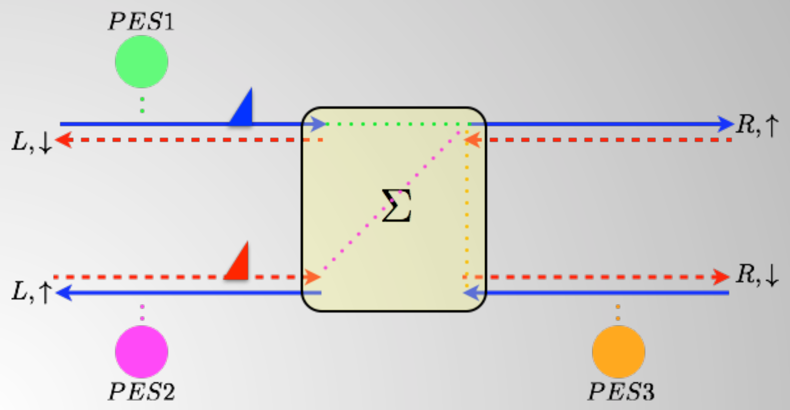
Visibility of the dip depends on QPC properties

HOM experiments: two-electrons injection (2)

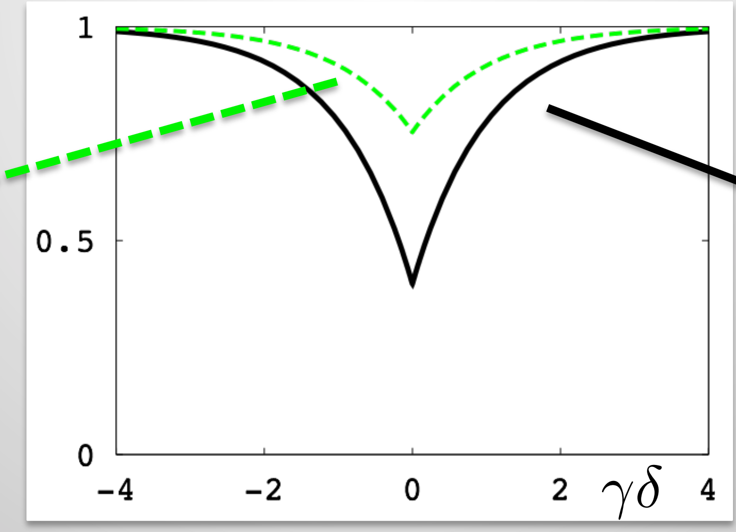
D. Ferraro, C. Wahl, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **89**, 075407 (2014)

Opposite spin injection (no equivalent in IQH)





$$q^{(2)}(\delta)_{R\uparrow,L\downarrow} = 1 - \mathcal{K}e^{-\gamma|\delta|}$$



$$q^{(2)}(\delta)_{R\uparrow,R\downarrow} = 1 - \mathcal{J}e^{-\gamma|\delta|}$$

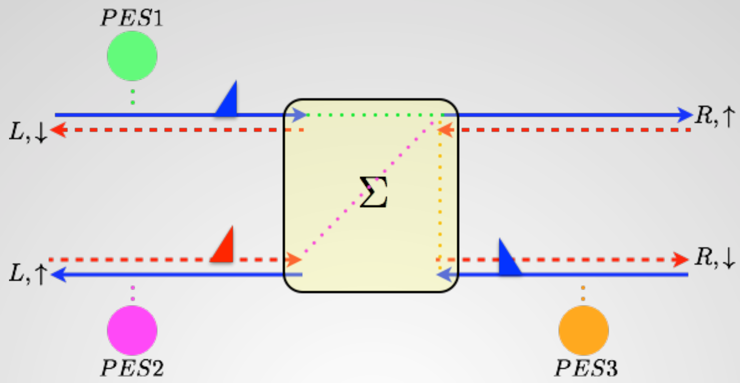
Dip related to the topological structure of the edges

J. M. Edge *et al.*, Phys. Rev. Lett. **110**, 246601 (2013)

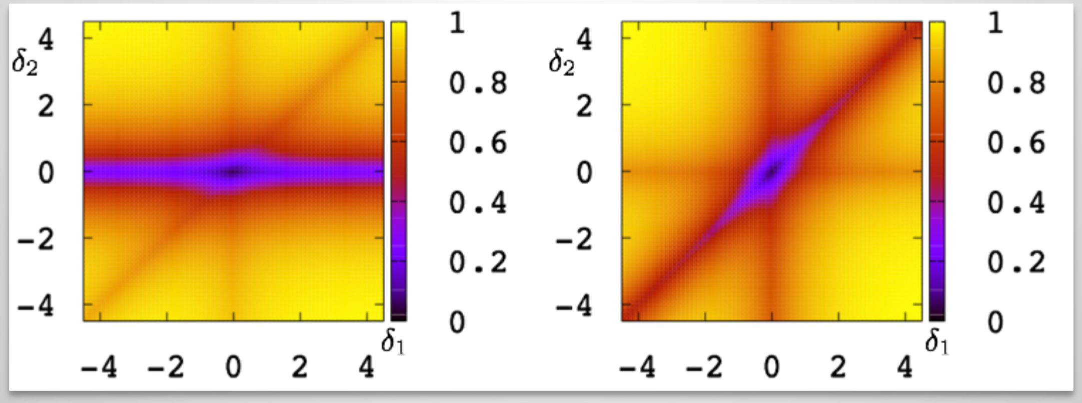
HOM experiments: three-electrons injection

D. Ferraro, C. Wahl, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **89**, 075407 (2014)

Configuration possible only in the QSH case



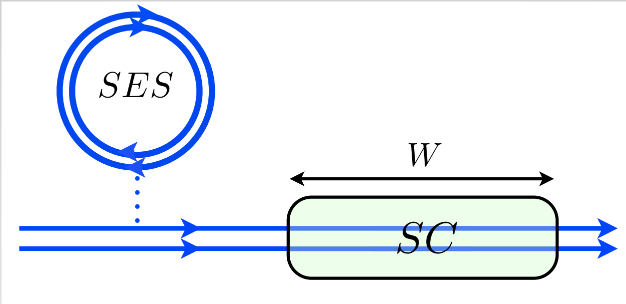
$$q^{(3)}(\delta_1, \delta_2) = 1 - \alpha e^{-\gamma|\delta_1|} - \beta e^{-\gamma|\delta_2|} - (1 - \alpha - \beta) e^{-\gamma|\delta_1 - \delta_2|}$$



Synchronized case: suppression of the noise due to Pauli principle and topology

Not synchronized case: exploring different interference contributions

Source of individual Bogoliubov quasiparticles



Hall edge channels at filling factor 2 degenerate in spin
(neglect Zeeman and interaction) coupled with a SC contact

Action of the transfer matrix on the Nambu spinor

C. W. J. Beenakker, Phys. Rev. Lett. **112**, 070604 (2014)

$$\mathcal{M}(\xi) = e^{i\xi\delta} e^{i\Gamma\tau_z} \mathcal{U}(\tilde{\theta}, \phi, 0) e^{i\Gamma'\tau_z}$$

$$\mathcal{U}(\tilde{\theta}, \phi, 0) = \begin{pmatrix} \cos\tilde{\theta} & 0 & 0 & e^{-i\phi}\sin\tilde{\theta} \\ 0 & \cos\tilde{\theta} & -e^{-i\phi}\sin\tilde{\theta} & 0 \\ 0 & e^{i\phi}\sin\tilde{\theta} & \cos\tilde{\theta} & 0 \\ -e^{i\phi}\sin\tilde{\theta} & 0 & 0 & \cos\tilde{\theta} \end{pmatrix}$$

Consistent with unitarity and particle-hole symmetry

$$|e,\uparrow\rangle \Rightarrow \mathcal{W}_e|e,\uparrow\rangle + \mathcal{W}_h|h,\downarrow\rangle = \cos\tilde{\theta}|e,\uparrow\rangle + \sin\tilde{\theta} e^{i(\phi-2\Gamma)}|h,\downarrow\rangle$$

Electrons emerge as Bogoliubov quasiparticles

“Majorino” *vs* “Majorana”

Zero energy modes of topological p-wave SC

A. Y. Kitaev, Physics-Uspekhi **44**, 131 (2001)

Non-abelian statistical properties with remarkable applications in topological quantum computation

C. Nayak *et al.*, Rev. Mod. Phys. **80**, 1083 (2008)

Various proposals for engineering them in condensed matter systems like: quantum wires, topological insulators, magnetic chains…

J. Alicea, Rep. Prog. Phys. **75**, 076501 (2012)

But…

... “the constraints imposed by fermionic statistics on the symmetries of Bogoliubov-de Gennes Hamiltonians always allow one to bring the Hamiltonian in the Nambu representation to an imaginary form. In turn, Schrödinger’s equation with this imaginary Hamiltonian leads to a real equation of motion for the fields, as in Majorana’s construction”.

C. Chamon *et al*., Phys, Rev. B **81**, 224515 (2010)

Majorana (real) fermionic field even in absence of zero modes

Relevant physical parameters

$$\delta \approx \frac{W}{v}$$

Time to cross the SC region

$$\phi$$

SC order parameter phase

$$\tilde{\theta}(W, l_s, l_m)$$

l_s induced coherence length

l_m magnetic length

$$l_m \gg l_s$$

Optimal condition: high critical field in the SC

Experiments with Niobium contacts on Graphene: P. Rickhaus *et al*., Nano Lett. **12**, 1942 (2012)

Andreev edge states

H. Hoppe *et al*., Phys. Rev. Lett. **84**, 1804 (2000)

Characterization of the source (1)

D. Ferraro, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **91**, 075406 (2015)

$$|e,\uparrow\rangle = \int_{-\infty}^{+\infty} d\tau \varphi_e(\tau)\Psi^\dagger_\uparrow(\tau)|F\rangle$$

Current and charge

$$\langle e,\uparrow|I(t)|e,\uparrow\rangle = -ev\langle e,\uparrow| : \Psi^\dagger(t)\tau_z\Psi(t) : |e,\uparrow\rangle = -e\cos(2\tilde{\theta})\varphi_e(t-\delta)\varphi_e^*(t-\delta)$$

$$\mathcal{Q} = \int dt\langle e,\uparrow|I(t)|e,\uparrow\rangle = -e\cos(2\tilde{\theta})$$

$$\propto |\mathcal{W}_e|^2 - |\mathcal{W}_h|^2$$

Non conservation of the charge due to Andreev reflections

A. F. Andreev, Sov. Phys. JETP 19, 1228 (1964)

$$\tilde{\theta} = \frac{\pi}{4}$$

Particle and hole contributions compensate: zero outgoing current

Particle density and number

$$\langle e,\uparrow|\rho(t)|e,\uparrow\rangle = v\langle e,\uparrow| : \Psi^\dagger(t)\Psi(t) : |e,\uparrow\rangle = \varphi_e(t-\delta)\varphi_e^*(t-\delta)$$

$$\mathcal{N} = \int dt\langle e,\uparrow|\rho(t)|e,\uparrow\rangle = 1$$

$$\propto |\mathcal{W}_e|^2 + |\mathcal{W}_h|^2$$

Conservation of the particle number

Characterization of the source (2)

D. Ferraro, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **91**, 075406 (2015)

Zero frequency noise at the output of the SC region

$$S_{source} = \int dtdt' \left[\langle e, \uparrow | I(t) I(t') | e, \uparrow \rangle_c \right] = e^2 \sin^2(2\tilde{\theta})$$

$$\propto |\mathcal{W}_e|^2 \times |\mathcal{W}_h|^2$$

Variance of the charge

$$\tilde{\theta} = 0$$

Zero noise: perfect emission of one electron

A. Mahé *et al*., Phys. Rev. B **82**, 201309 (2010)

$$\tilde{\theta} = \frac{\pi}{2}$$

Zero noise: perfect conversion of the electron into an hole

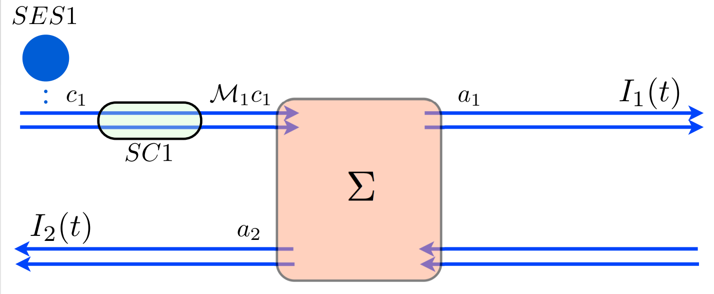
$$\tilde{\theta} = \frac{\pi}{4}$$

Maximum noise associated to zero emitted charge

HBT noise

D. Ferraro, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **91**, 075406 (2015)

$$S = \int_{-\infty}^{+\infty} dt dt' \langle e, \uparrow | I_1(t) I_2(t') | e, \uparrow \rangle_c$$



$$S_1^{HBT} = -e^2 R(1-R)\cos^2(2\tilde{\theta}_1)$$

$$\propto \mathcal{Q}^2$$

Partition noise associated to a non-integer charged wave-packet

$$\tilde{\theta} = 0, \frac{\pi}{2}$$

We recover the standard result for electrons and holes

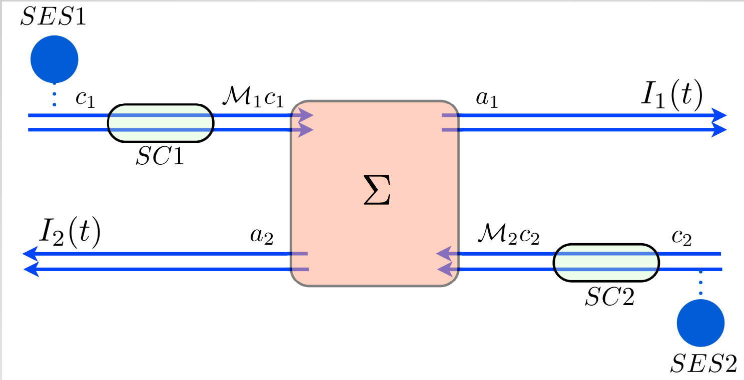
E. Bocquillon *et al*., Phys. Rev. Lett. **108**, 196803 (2012)

$$\tilde{\theta} = \frac{\pi}{4}$$

Zero partition noise associated to a zero emitted charge

HOM noise for two Bogoliubov quasiparticles

D. Ferraro, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **91**, 075406 (2015)



$$S^{HOM} = \Delta S^{HOM} + S_1^{HBT} + S_2^{HBT}$$

$$\Delta S^{HOM} \propto |\mathcal{W}_e^1 \mathcal{W}_e^{2*} - \mathcal{W}_h^1 \mathcal{W}_h^{2*}|^2$$

Synchronized emission through SC differing only on the order parameter phase

$$S_{2SC}^{HOM} = e^2 R(1-R) \sin^2(2\tilde{\theta}) \left[1 - \cos(\phi_1 - \phi_2)\right]$$

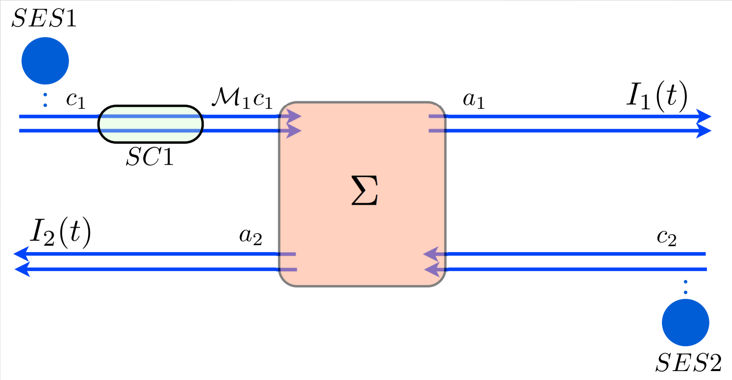
Non local dependence on the order parameter phase

C. W. J. Beenakker, Phys. Rev. Lett. **112**, 070604 (2014)

No dependence on SC phase in the current (first order coherence), oscillatory modulation in the noise (second order coherence)

HOM noise as a spectroscopic tool (1)

D. Ferraro, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **91**, 075406 (2015)



Interference between one Bogoliubov quasiparticle and one electron

$$S_{1SC}^{HOM} = e^2 R(1-R)\left\{\left[1+\cos(2\tilde{\theta})\right] A(\delta_1 - \eta) - \cos^2(2\tilde{\theta}) - 1\right\}$$

Overlap of the wave-packets

$$\mathcal{R}_{1SC} = \frac{S_{1SC}^{HOM}}{(S_1^{HBT} + S_2^{HBT})} = 1 - \frac{1+\cos(2\tilde{\theta})}{1+\cos^2(2\tilde{\theta})} A(\delta_1 - \eta)$$

Ratio usually considered in experiments

E. Bocquillon *et al*., Science **339**, 1054 (2013)

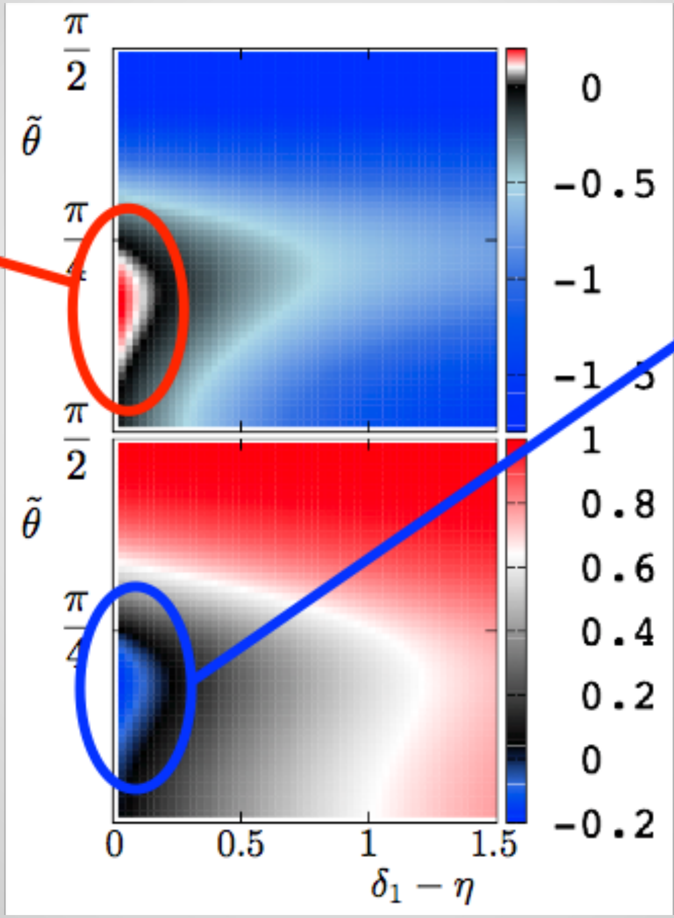
HOM noise as a spectroscopic tool (2)

D. Ferraro, J. Rech, T. Jonckheere, T. Martin, Phys. Rev. B **91**, 075406 (2015)

Emission by a driven mesoscopic capacitor

$S_{1SC}^{HOM} > 0$

$|\mathcal{W}_e|^2 > |\mathcal{W}_h|^2$



$\mathcal{R}_{1SC} < 0$

forbidden in HOM with electrons due to charge conservation

Spectroscopy of Bogoliubov excitations

Conclusions

- HBT and HOM interferometers as milestones in electron quantum optics
- Two and three electron interferometry in quantum spin Hall systems
- Interferometry and spectroscopy of individual Bogoliubov quasiparticles

Further perspectives

- Robustness of the discussed results in presence of electron-electron interaction