Heat flow with thermal metamaterials

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- Université de Montpellier2, Université de Lille1

Context

- Success of transformation optics
- Extension to acoustics, mechanics, seismic, hydrodynamics…

Why not thermal?

Could the same techniques be used to manage heat flow?

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INPACT project

(ANR-2014)

- Fresnel (Marseille): optics
- EM2C (Paris): thermal transfer
- LMGC (Montpellier): mathematics
- IEMN (Lille): microtechnologies

I- Analogies optics ⇔ thermal

Propagation versus diffusion

Isotropic, heterogeneous…

- ✓ Optics: $\Delta \mathbf{E} - \varepsilon\mu *_t \partial^2 \mathbf{E}/\partial t^2 = S_{opt}(\mathbf{J},q)$
- ✓ Thermal: $\Delta T - (1/a)\, \partial T/\partial t = - S_0/b = S_{th}$

⇩

$$\Delta u - f(t) *_t u = S$$

⇩

In the Fourier plane:

$$\Delta \breve{u} + k^2(\omega)\, \breve{u} = 0$$

Memory of derivation order is included in the *dispersion law* $k^2(\omega)$

Diffusion ≈ **propagation** en « *specific* » metallic medium

Optics: $k = \omega\sqrt{[\breve{\varepsilon}(\omega)\breve{\mu}(\omega)]}$ dielectric or metallic

Thermal: $k = (1+j)\sqrt{[\omega/2a]}$ "specific metal"

Boundary conditions

Temperature ⇔ Electric field
Heat flux density ⇔ Magnetic field

Optics	Thermal
$\mathbf{n} \wedge \delta\mathbf{E} = 0$	$\delta T = 0$
$\mathbf{n} \wedge \delta\mathbf{H} = \mathrm{sing}(\mathbf{J})$	$\delta[b\partial T/\partial n] = \mathrm{sing}(S)$

Energy balance

- Internal energy: $(1/2)\,[\varepsilon\mathbf{E}^2 + \mu\mathbf{H}^2]$ ⇔ $(\rho C)\,T$
- Absorption ≈ $\int_V$ (temperature) dv

Most optical models and softwares can be used for thermal

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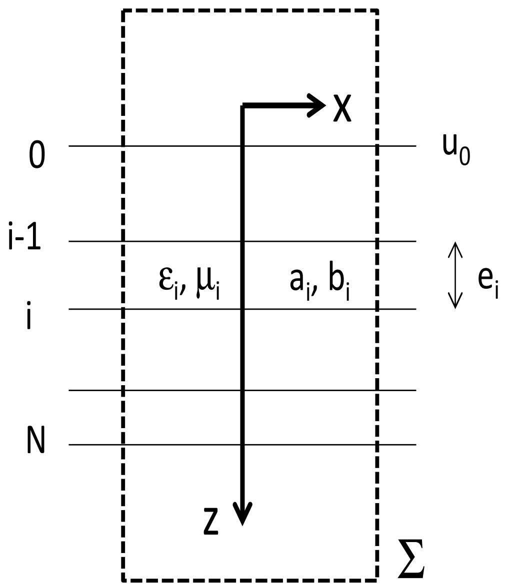
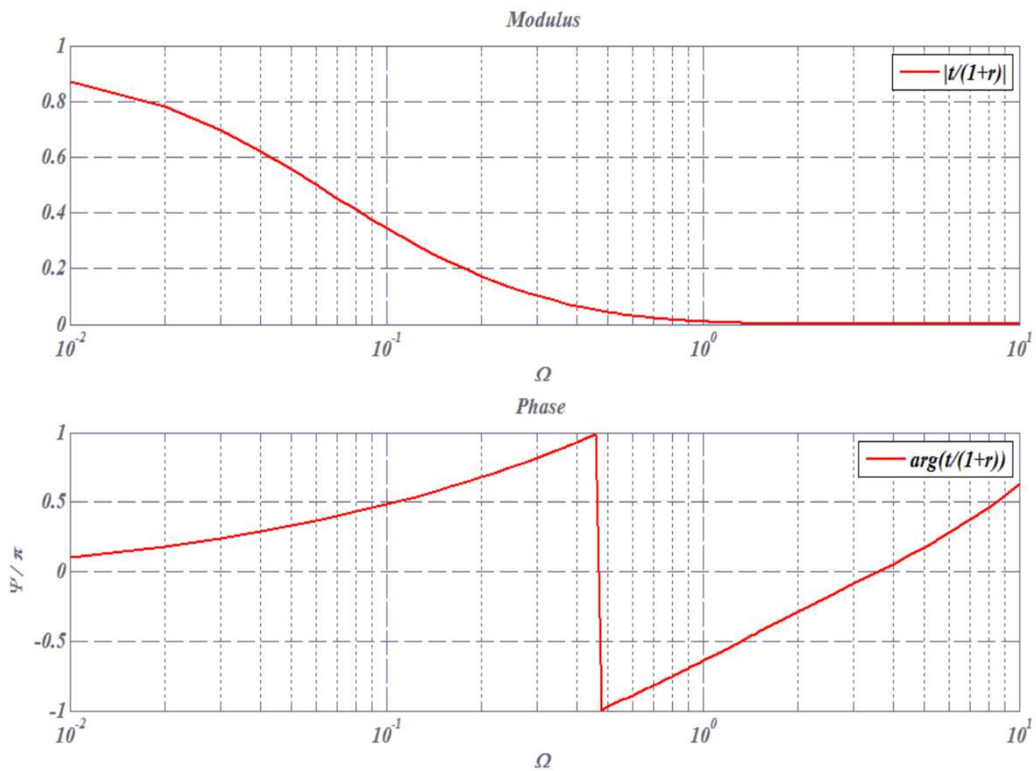
Which optical concepts can be extended to thermal?

- Spectral shaping (dichroic and narrow-band filters…)
- Temporal shaping (chirped mirrors…)
- Spatial shaping (super-resolution)
- Confinement, enhancement (resonances)
- Diffraction and scattering
- Microcavities, modal optics (guided waves)
- **Cloaking**…

Optical admittances in thermal multilayers

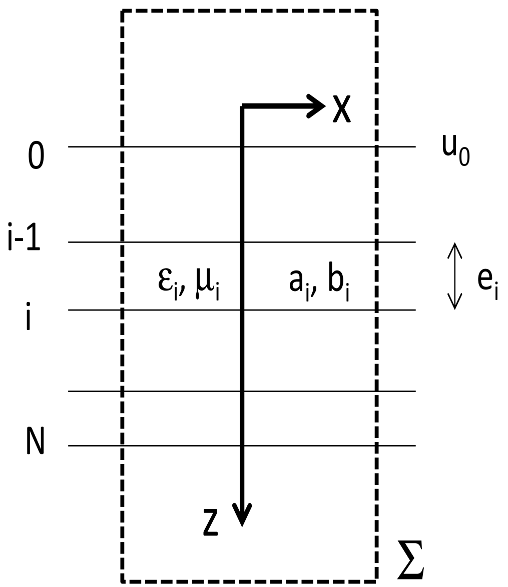


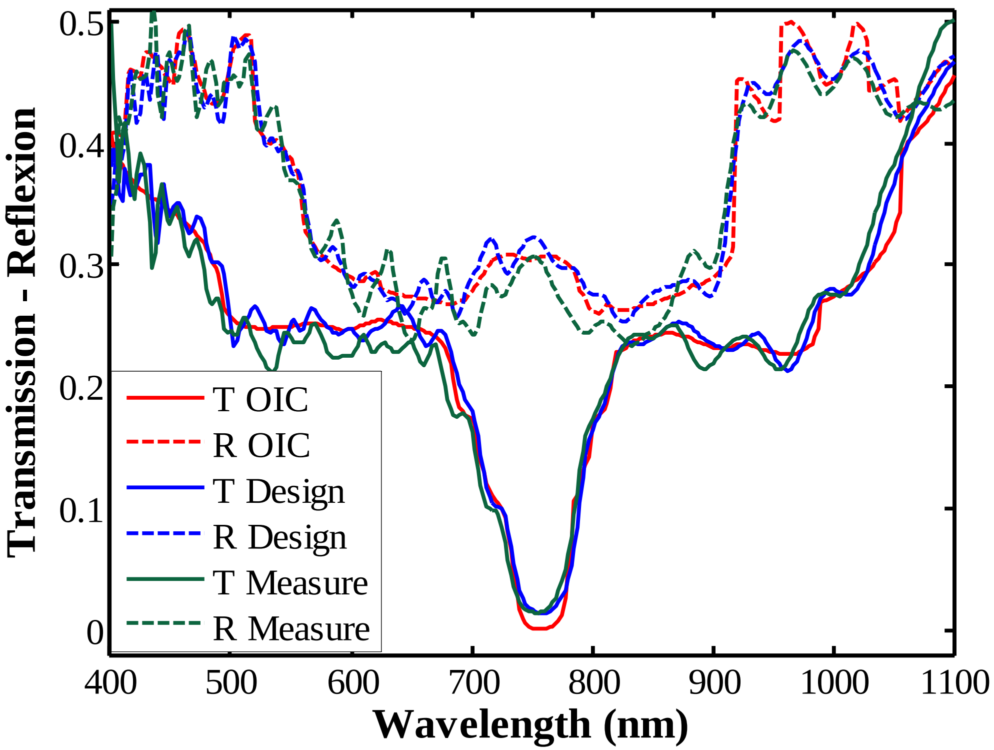


Nine-layer stack- Spectral transmittance and phase
(using optical softwares)

A thin film metal/dielectric multilayer to aproximate a "moose head"

105 layers with 3 materials: $Nb_2O_5/SiO_2/Cr$





Courtesy of the Thin Film Research Team of Institut Fresnel
(Thomas Begou, Fabien Lemarchand and Julien Lumeau)

However:

metal optics ≠ plasmonics (metal/dielectrics)

This reduces the range of thermal devices

that could be derived from optical concepts

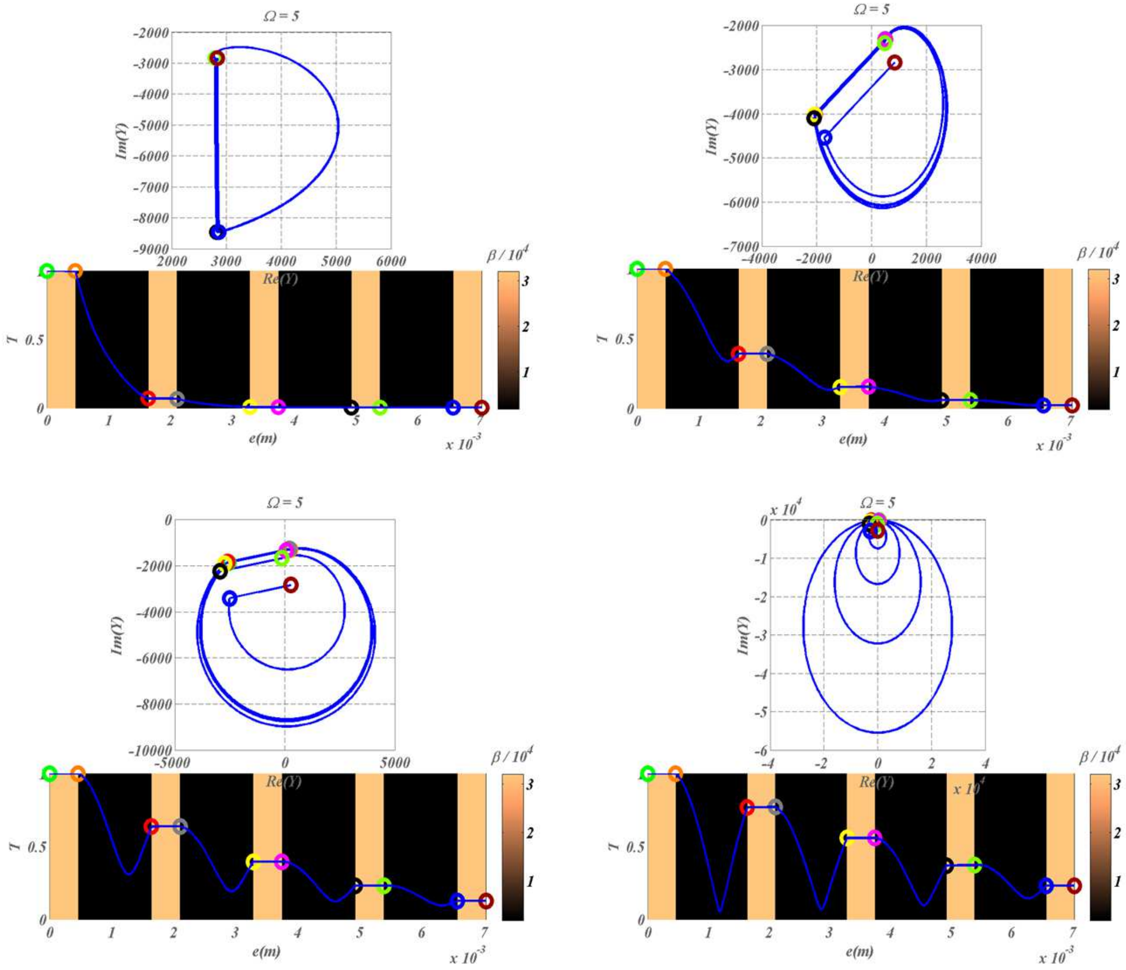
Furthermore: **k' = k''**

(specific thermal metals)

From metal optics to thermal

n = k’ + j k’’ with k’’ = k’ in thermal

k = k’



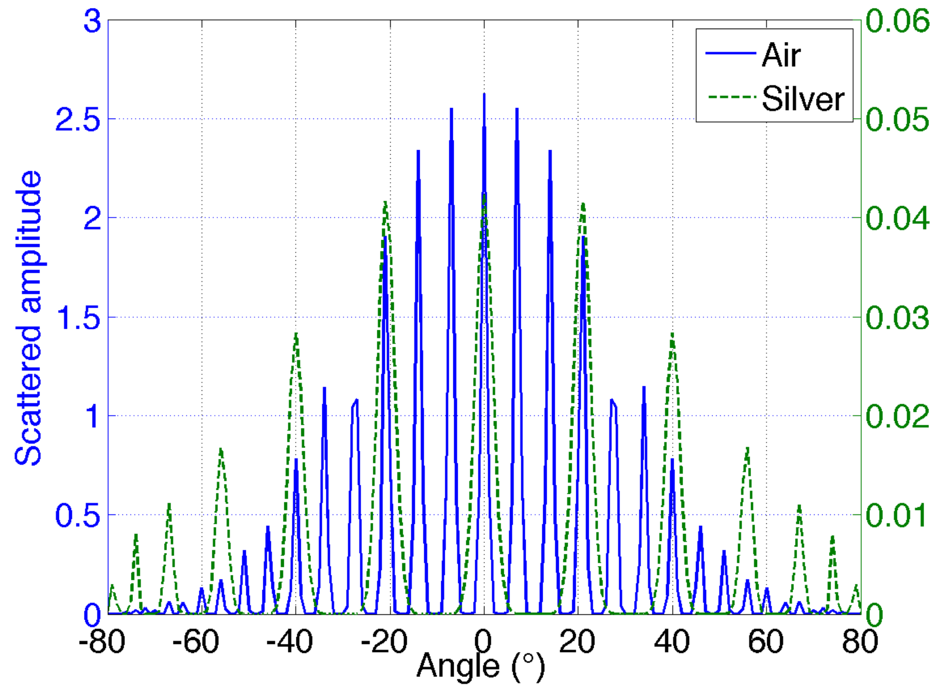
k ≠ k’

Field extrema disappear at k = k’

Diffraction process in heat conduction

(Harmonic regime)

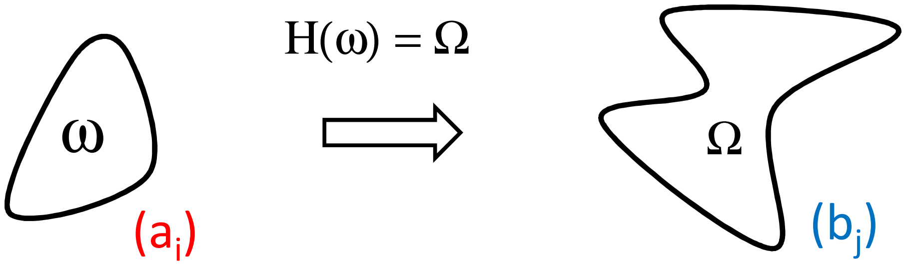
Spatial harmonics of the temperature modulus (grating law)



…but the grating period should be greater than the diffusion length (near field device)

II- Transformation optics in heat conduction

Basic principles



Governing equation : $L_{\omega} \{u, (a_i)\} = 0$

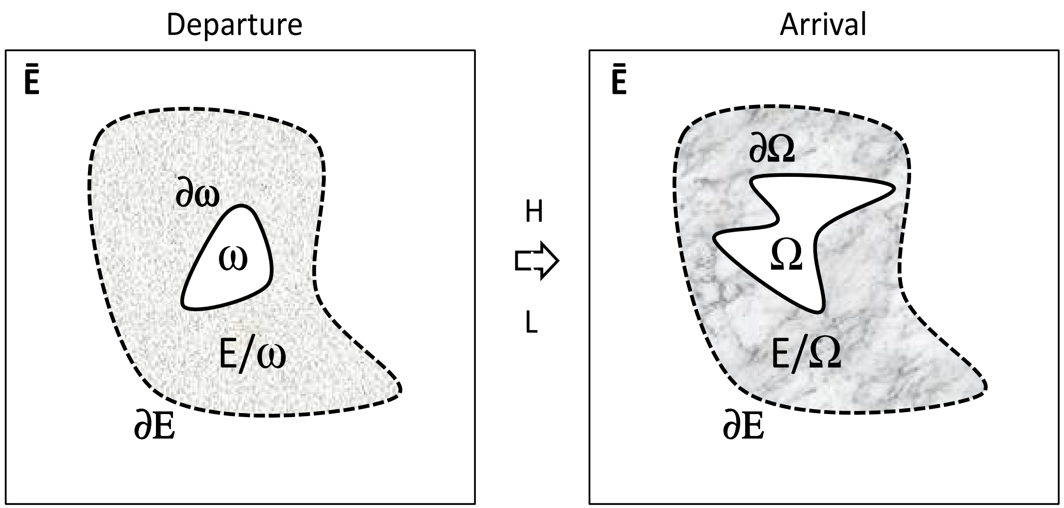
Change of coordinates : $L_{\Omega} \{u, (b_i)\} = 0$

Same governing equation in the arrival space,
with different physical parameters: $b_i = b_i\{a_j\}$

Such technique allows a spatial redistribution of the field values,
provided that we also transform the rest of space.

The overcoat transformation

E = working space



$\mathbf{q}_{\Omega}(\bar{E}) = \mathbf{q}_{\omega}(\bar{E})$

Transformation of overcoat: $L(E\backslash\omega) = E\backslash\Omega$

No transformation beyond the coat frontier ∂E

→ **The original field is retrieved beyond the coat in the arrival space**

The « cost » of this mimetism

Anisotropic heterogeneous physical parameters

$k_{\Omega} = (1/\det M_H)\, M_H\, k_{\omega}\, M_H^T$

$k_{E\backslash\Omega} = (1/\det M_L)\, M_L\, k_{E\backslash\omega}\, M_L^T$

$(\rho C)_{\Omega} = (1/\det M_H)\, (\rho C)_{\omega}$

$(\rho C)_{E\backslash\Omega} = (1/\det M_L)\, (\rho C)_{E\backslash\omega}$

(M matrix connected with the H/L transformations)

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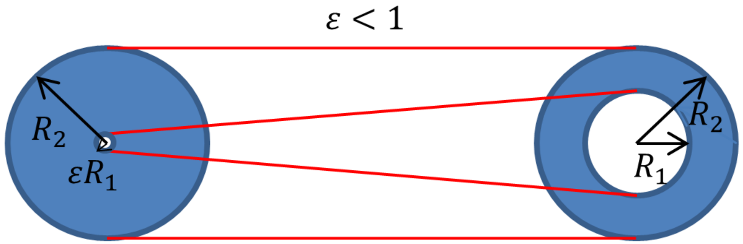
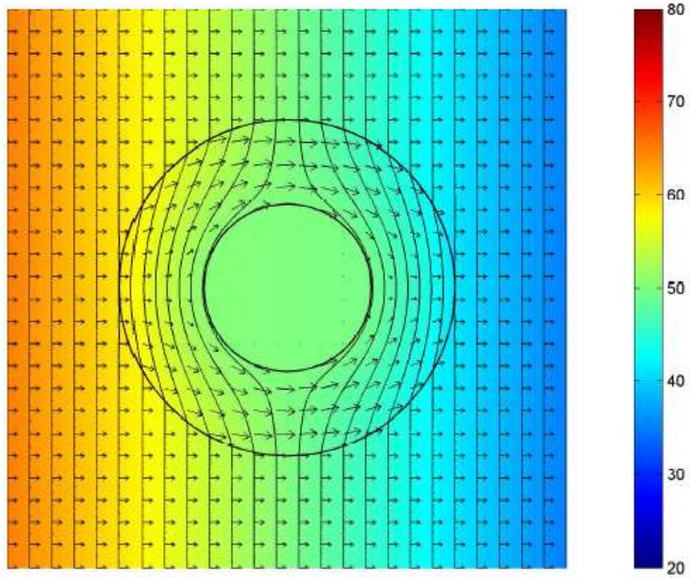
Complexity of materials, Homogeneization techniques,

production constraints (sensitivity, stability…)

The case of cloaking

« The simplest situation : the starting space is homogeneous »

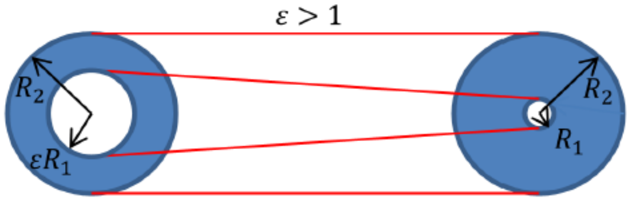


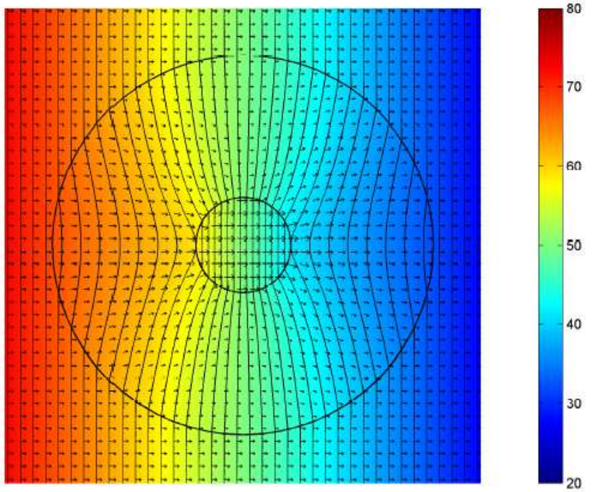


Isotherm correction beyond the cloak

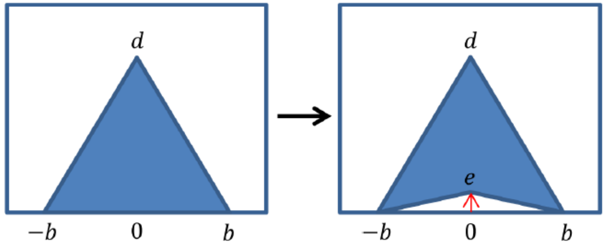
Other examples

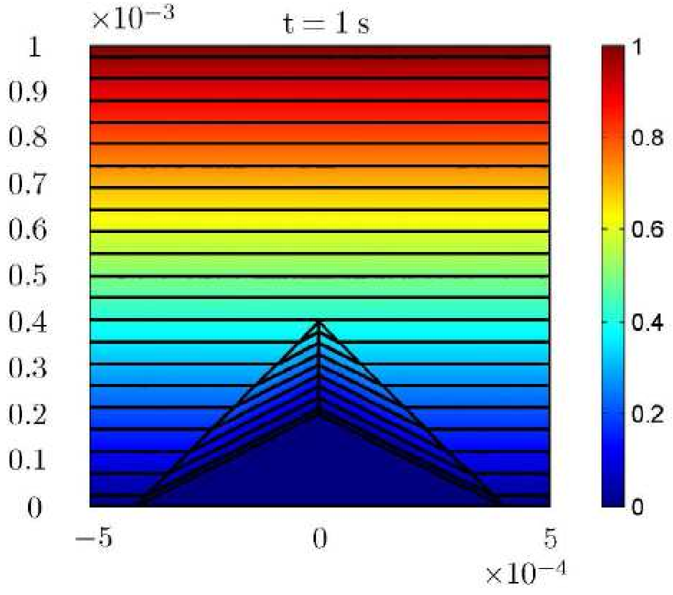
Concentrator



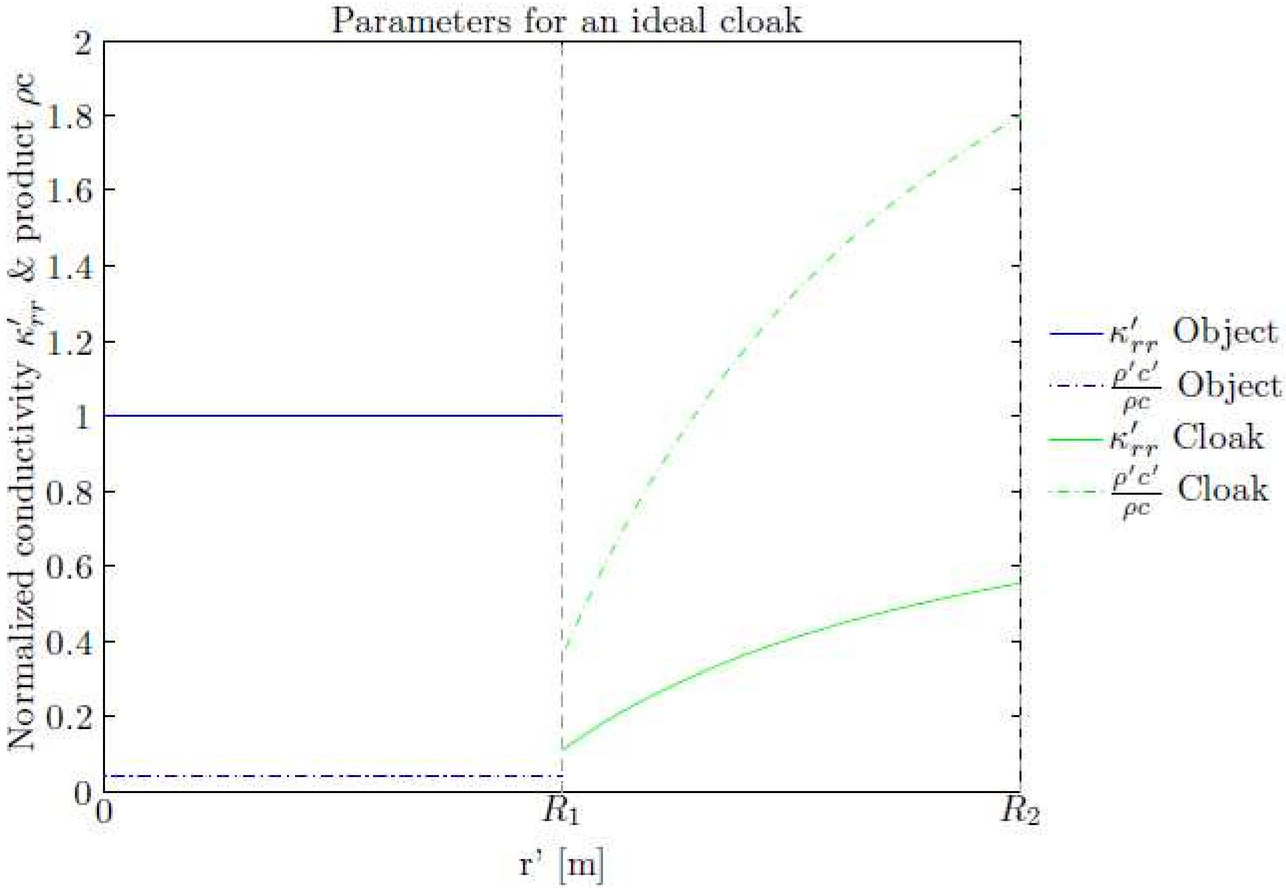


Carpet

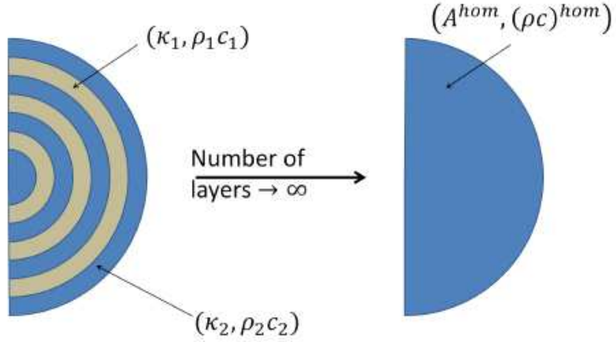




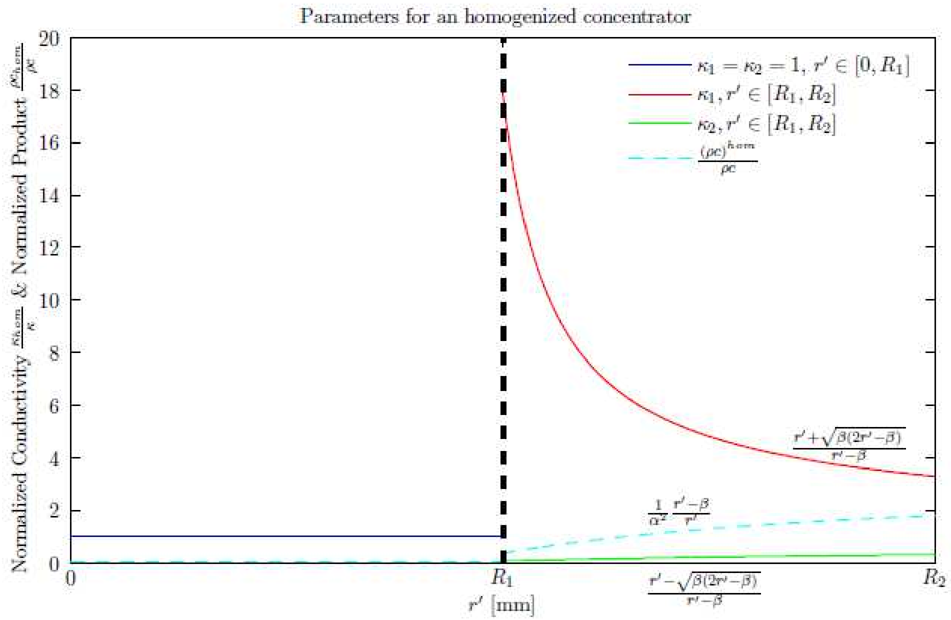
Physical parameters



Homogeneisation with 2 conductivity gradients

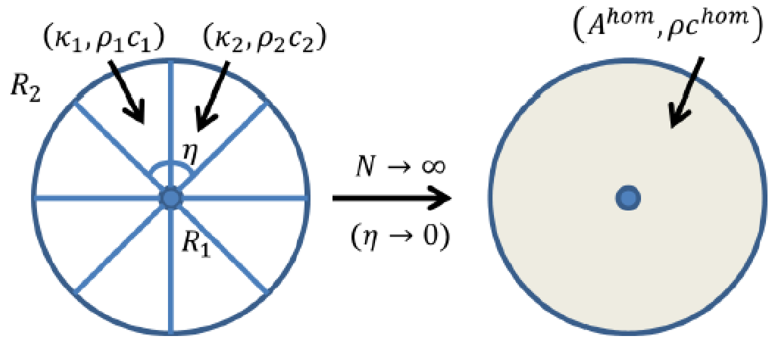


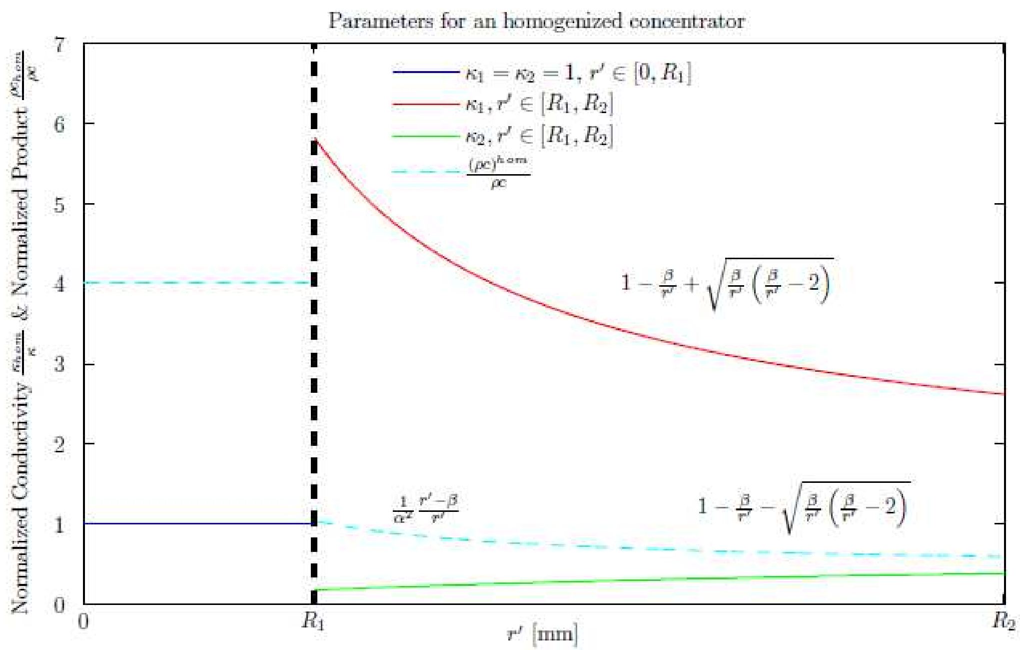
$$A^{hom} = \begin{pmatrix} \frac{2\kappa_1\kappa_2}{\kappa_1+\kappa_2} & 0 \\ 0 & \frac{1}{2}(\kappa_1+\kappa_2) \end{pmatrix}$$



Homogeneisation of concentrator

A polar gradient must be added to the concentric multilayers ⇨



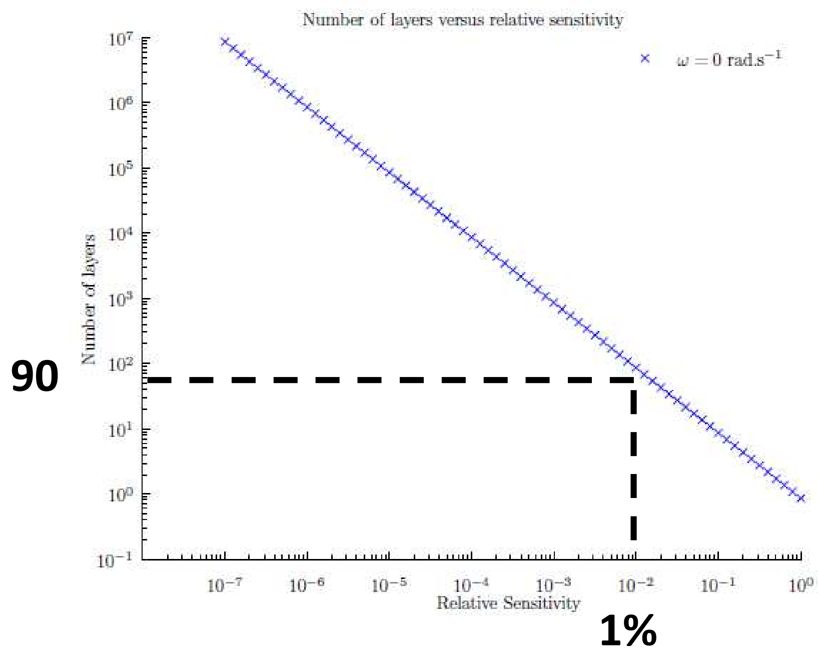


Not really easy to produce

Efficiency of cloaking

$$||u-u_\varepsilon||_{L^2(\Omega)} = \sqrt{\int_\Omega |u-u_\varepsilon|^2 \mathrm{d}\Omega} \leq C\varepsilon = \frac{C'}{N}$$

Distance between ideal cloak and N-homogeneized cloak



Allows to choose the mesh versus the required performance

… **and sensitivity remains to check**

III- Inverse engineering

Until now the objects to cloak or mimik could not be

chosen since they were forced by the tensor:

$k_{\Omega} = (1/\det M_H)\ M_H\ k_{\omega}\ M_H^T$ $\qquad$ $(\rho C)_{\Omega} = (1/\det M_H)\ (\rho C)_{\omega}$

$k_{E\backslash\Omega} = (1/\det M_L)\ M_L\ k_{E\backslash\omega}\ M_L^T$ $\qquad$ $(\rho C)_{E\backslash\Omega} = (1/\det M_L)\ (\rho C)_{E\backslash\omega}$

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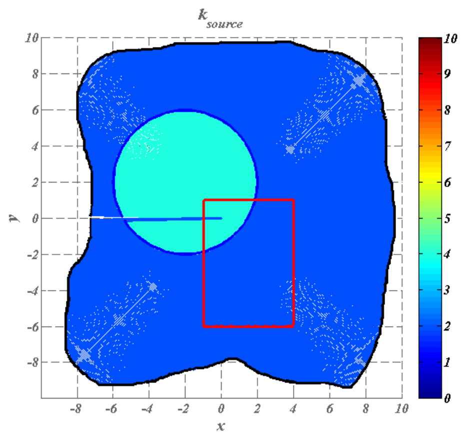
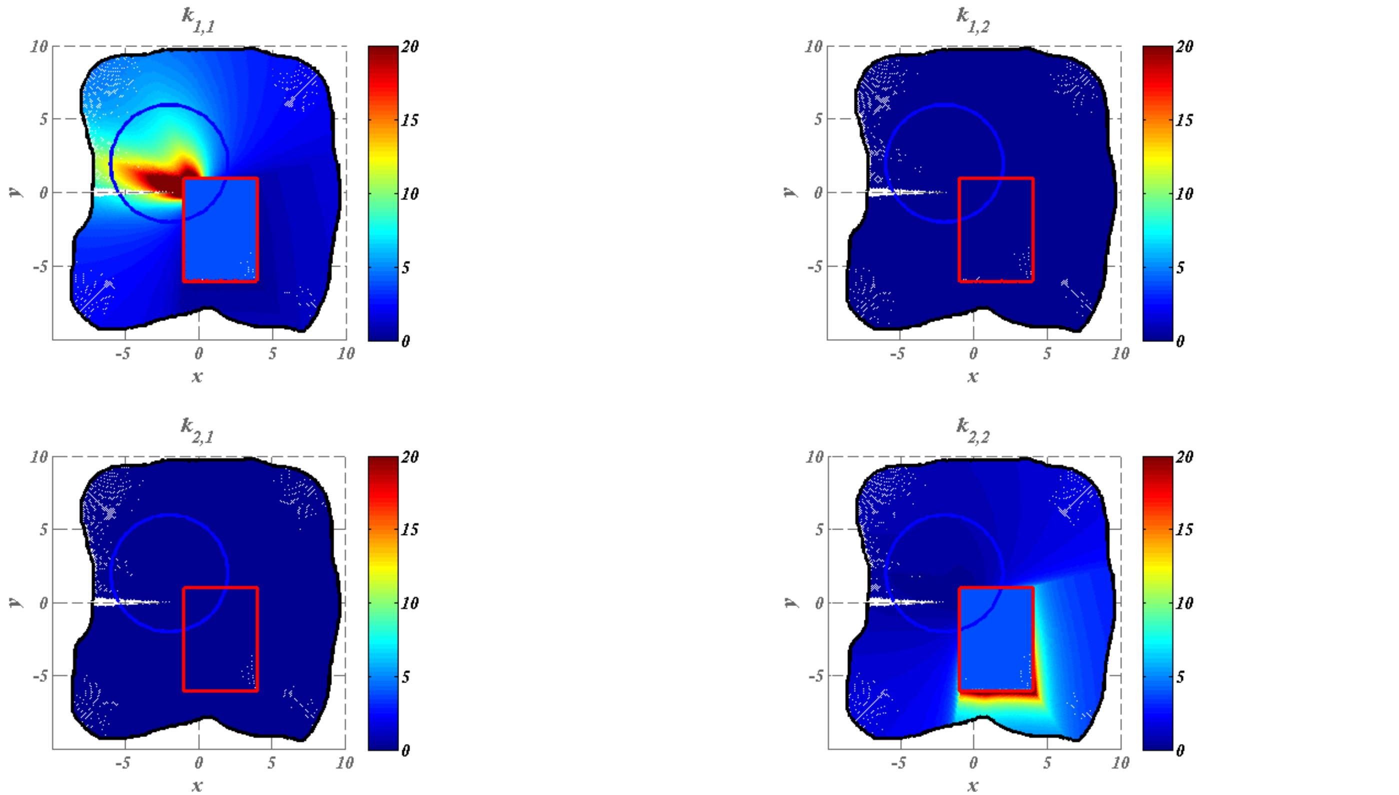
Conductivities are shape dependent

In case where the physical parameters can be adjusted,
mimetism is immediate



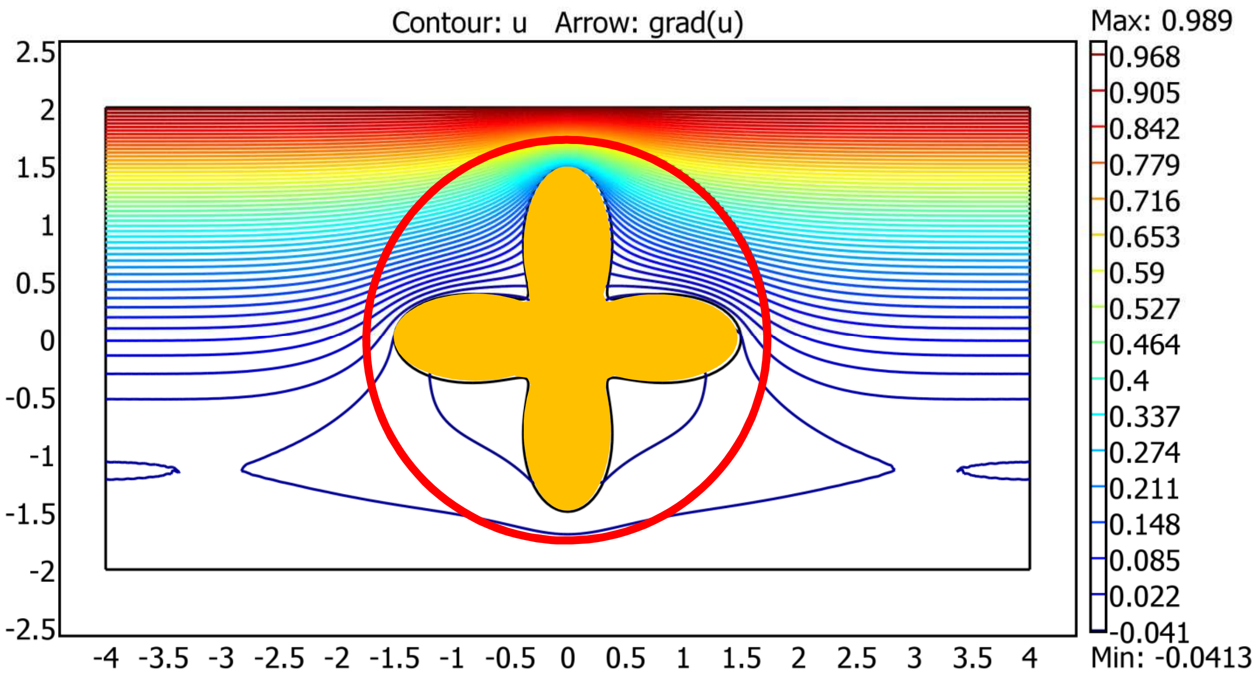
Mimetism conductivity matrix:

From rectangle to circle



Another heat mimetism: flower to ellipse

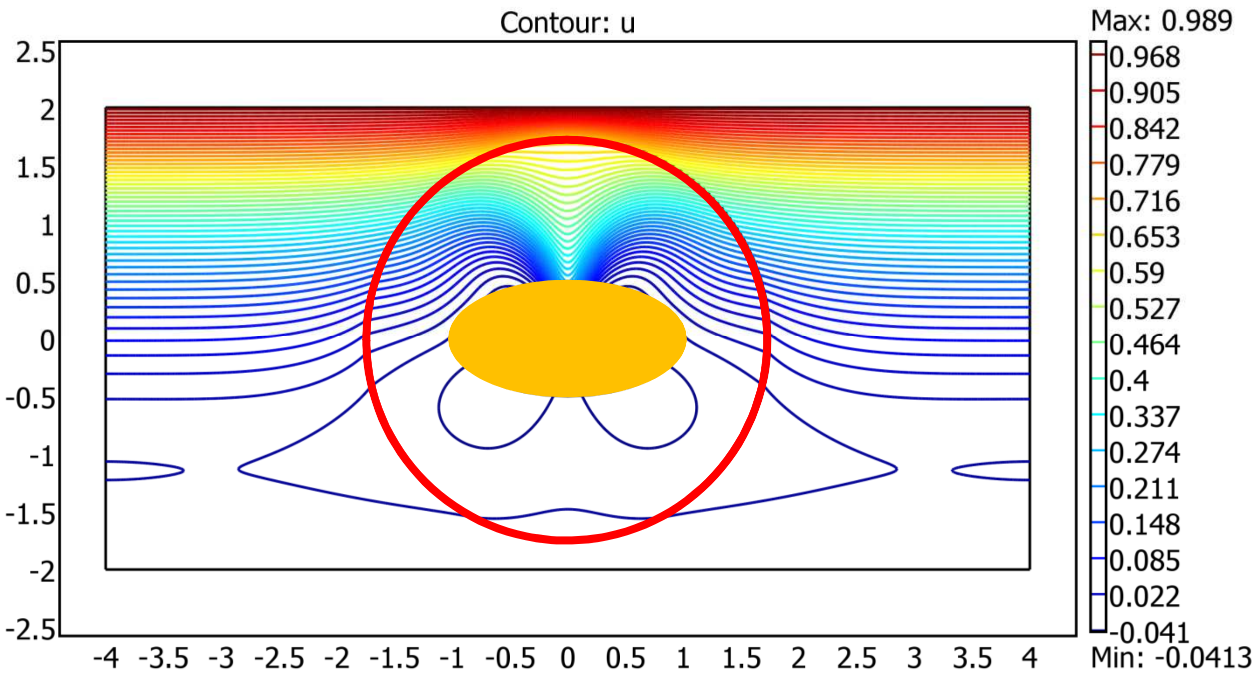
Harmonic regime



Same field outside the (red) cloak frontier
(homogeneous cloak)

Heat mimetism: flower to ellipse

Harmonic regime



Same field outside the (red) cloak frontier
(heterogenous anisotropic cloak)

… but it would be more practical

to work with given objects

- *we do not choose what we have to hide*
- *we do not choose what we have to mimik*

The case of given objects

Conductivity is given for the objects to hide and mimik

$$k_\Omega = (1/\det M_H)\, M_H\, k_\omega\, M_H^T$$

⇩

The tensor is forced: $M_H = M_H(k_{\Omega,} k_\omega)$

⇩

The set of transformations is reduced: $H \in \{H_\alpha(k_{\Omega,} k_\omega)\}$

⇩

Class of shapes that can be mimicked: $\partial\omega \in \mathbf{H}_\alpha^{-1}\, (\partial\Omega)$

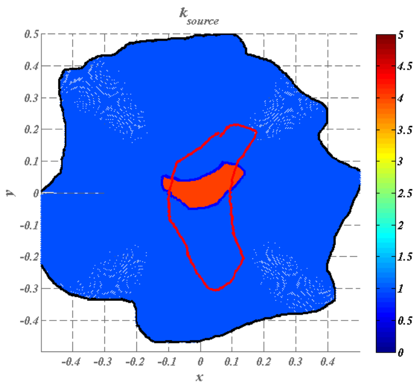
… and this should be compatible with the ρC conditions

Numerical calculation in progress…

However simple analytical calculation can be done

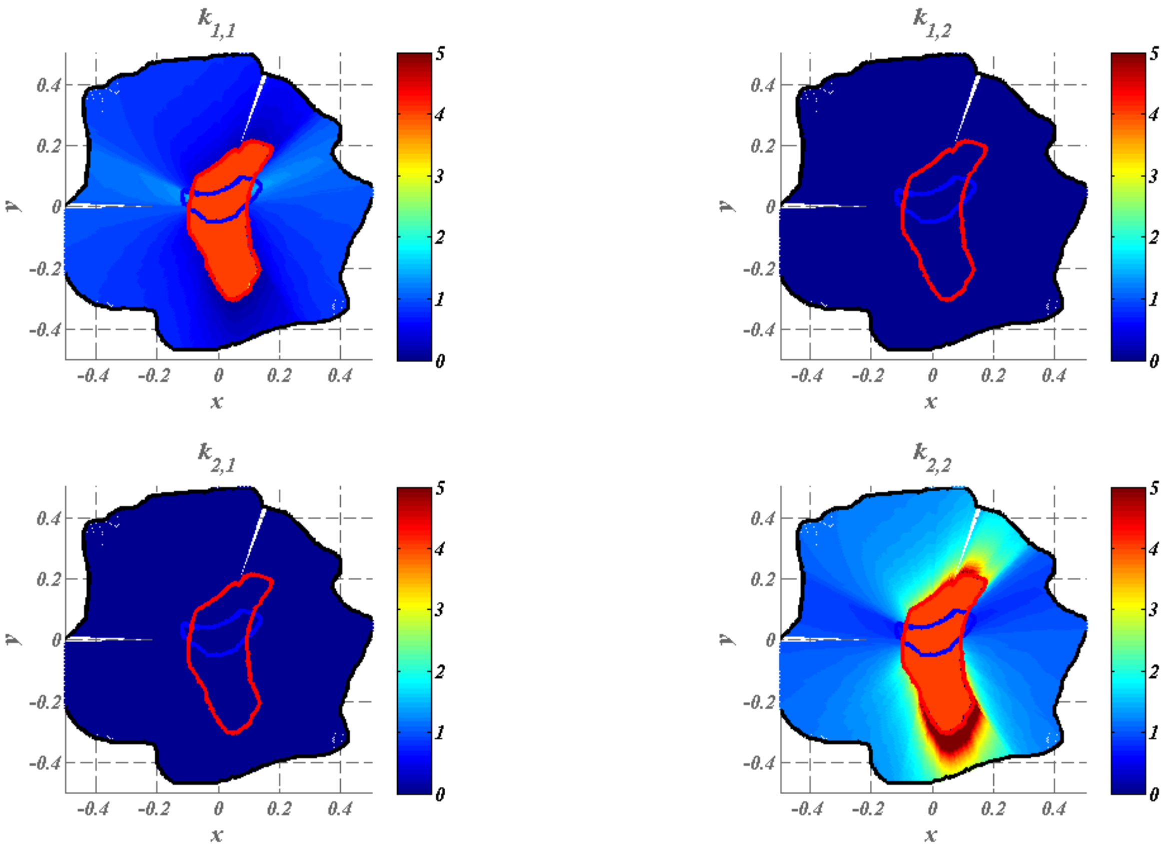
to solve the inverse problem in case of:

- A diagonal conductivity of the object to hide
- An isotropic object to mimik



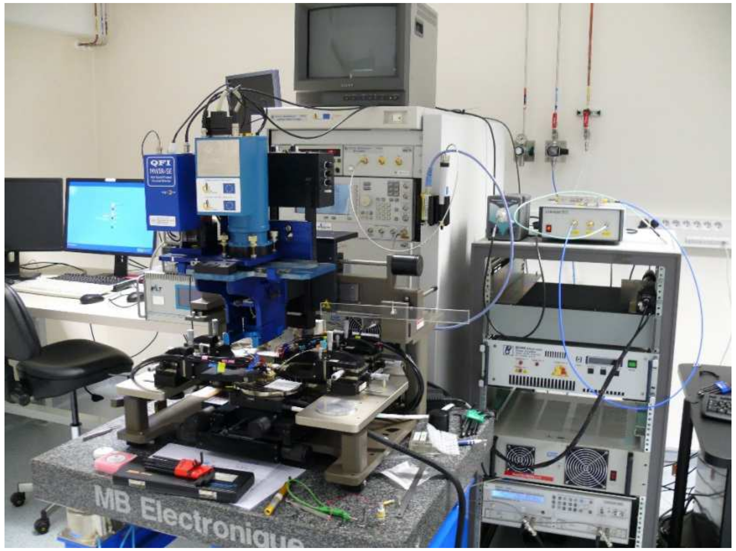
Example with star domains

Mimetism conductivity matrix



IV- Conclusion

- Now working on thermal radiation
- Devices are under progress



Homogeneization appears to be the major point:

- ✓ choice of solutions
- ✓ resulting N-efficiency
- ✓ sensitivity to design…

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And many others…