SPECTRAL THEORY OF NOVEL MATERIALS

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Claude Amra and Michel Bellieud: Heat flow with thermal metamaterials.

Analogies between thermal diffusion and optical propagation in metal optics [1] allowed to generalize transformation optics [2,3] to the field of thermal [4-7], hence expanding cloaking and mimicking principles to thermodynamics. As a result, thermal metamaterials supported on homogenization techniques have been emphasized for the management of heat flow.

In this talk we first present our recent work on thermal metamaterials, including theoretical, numerical and technological aspects. Realistic applications are discussed within the framework of given objects to cloak or mimic. Heat conduction and thermal radiation are considered.

The second part of the talk concerns the design of metamaterials. Homogenization and singular perturbations problems in stratified media have been studied in [8,9]. These results are revisited and extended in the setting of both the heat equation and the equations of linear elasticity.

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Nicolas Bonod: Effective medium theory to mimic the fields scattered by metallic particles with insulators.

Metallic particles hosting localized surface plasmons open novel routes to tailor light propagation due to their ability to strongly enhance the electromagnetic fields at the nanoscale [1]. The dielectric permittivity of noble metals is dominated by the free electrons in the conduction band leading to a negative real part and positive imaginary part, while insulators (without free electrons) are characterized by dielectric permittivity with positive real and imaginary parts.

By using an effective medium theory applied to spherical scatterers, we show that the strong electric field scattered by a silver nanoparticle excited at its surface plasmon resonance (strong oscillation of free electrons) can be identically reproduced by a particle of same size (compared with the wavelength) but made of positive dielectric (without free electrons) [2].

Light scattering of metallic nanoparticles can be described by the first order electric coefficient: $a_1 = \frac{j_1(z_b)}{h_1^{(+)}(z_b)} \frac{\varepsilon_s \varphi_1^{(1)}(z_b) - \varepsilon_b \varphi_1^{(1)}(z_s)}{\varepsilon_s \varphi_1^{(+)}(z_b) - \varepsilon_b \varphi_1^{(1)}(z_s)}$, where $z_i = k_i R$ with $k_i^2 = \varepsilon_i\ k_0^2$, k_0 being the wavenumber in vacuum, R the radius of the particle, $\varepsilon_b = 1$ and ε_s are the permittivities of the background and particle respectively. The spherical Bessel and Hankel functions are respectively denoted by j_1 and $h_1^{\pm}$, with $\pm$ representing the outgoing $(+)$ and incoming (-) Hankel functions. The φ_1 functions are defined as $\varphi_1^{(\pm)}(z) \equiv \frac{\left[z h_1^{(\pm)}(z)\right]'}{h_1^{(\pm)}(z)}$, $\varphi_1^{(1)}(z) \equiv \frac{[z j_1(z)]'}{j_1(z)}$.

Positive dielectrics can reproduce the fields scattered by a silver particle if the scattering coefficients are equal : $a_1(z_0, \varepsilon_{metal}) = a_1(z_0, \varepsilon_{insulator})$, which can be simplified after some algebraic manipulations to the following complex transcendental equation :

$$\frac{\varphi_1^{(1)}(z_{metal})}{\varepsilon_{z_{metal}}} = \frac{\varphi_1^{(1)}(z_{insulator})}{\varepsilon_{z_{insulator}}}. \tag{1}$$

We solved this expression by using the factorized expression of the Bessel function, which allowed to obtain a new formulation of the special function[2-4]: $\varphi_n^{(1)}(z) = n + 1 + \sum_{\alpha=1}^{\infty} \frac{2z^2}{z^2 - a_{n,\alpha}^2}$, where $a_{n,\alpha}$ are the (tabulated) zeros of the Bessel function $j_n(x)$. By taking only the lowest zero, $a_{1,1}$, Eq. 1 can be cast:

$$\varepsilon_{\text{eq}} \quad = \quad \left(\frac{a}{z_0}\right)^2 \frac{1 - (z_0/a)^2 \varepsilon_{\text{in}}}{1 - (z_0/b)^2 \varepsilon_{\text{in}}}. \tag{2}$$

A study of the sign of the dielectric permittivity of the insulator, $\varepsilon_{z_{insulator}}$, shows that when $\varepsilon_{z_{metal}}$ depicts the dielectric permittivity of silver, $Re[\varepsilon_{z_{insulator}}]$ is positive (see Fig.1). In other words, the electromagnetic response of silver nanoparticles when excited at the surface plasmon resonance frequency can be reproduced with particles made of insulators [2].

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B. Malcolm Brown: Uniqueness for an inverse problem in electromagnetism with partial data.

A uniqueness result for the recovery of the electric and magnetic coefficients in the time-harmonic Maxwell equations from local boundary measurements is proven. No special geometrical condition is imposed on the inaccessible part of the boundary of the domain, apart from imposing that the boundary of the domain is C 1,1. The coefficients are assumed to coincide on a neighbourhood of the boundary, a natural property in applications
Joint work with: Marco Marletta and Juan Reves.

Jochen Brüning: Heat kernel estimates on arbitrary Riemannian manifolds.

We consider a self-adjoint non-negative operator H in a Hilbert space $\mathsf{L}^2(X, \mathrm{d}\mu)$. We assume that the semigroup $(\mathrm{e}^{-tH})_{t>0}$ is defined by an integral kernel, p, which allows an estimate of the form $p(t,x,x) \leq F_1(x)F_2(t)$ for all $(x,t) \in X \times \mathbb{R}_+$; we refer to F_1 as the *control function*. We show that such an estimate leads to rather satisfying abstract results on relative compactness of perturbations of H by potentials. It came as a surprise to us, however, that such an estimate holds for the Laplace-Beltrami operator on *any* Riemannian manifold. In particular, using a domination principle, one can deduce from the latter fact a very general result on the relative compactness of perturbations by potentials of the Bochner Laplacian associated with a Hermitian bundle (E, h^E, ∇^E) over an arbitrary Riemannian manifold (M, g); in fact, only quantities of order zero in g enter in the estimates. These results can be extended to weighted Riemannian manifolds, where under lower curvature bounds on the α-Bakry-Émery tensor one can construct quite explicit control functions, and to any weighted graph, where the control function is expressed in terms of the vertex weight function. It should be possible to extend this heat kernel estimate also to Bochner Laplacians of Hermitian vector bundles over any Riemannian manifold.

Jean-Bernard Bru: From the 2nd Law of Thermodynamics to AC-Conductivity Measures of Interacting Fermions in Disordered Media.

We study the dynamics of interacting lattice fermions with random hopping amplitudes and random static potentials, in presence of time–dependent electromagnetic fields. The interparticle interaction is short-range and translation invariant. Electromagnetic fields are compactly supported in time and space. I will explain how the limit of infinite space supports (macroscopic limit) of electromagnetic fields yields Ohm and Joule's laws in the AC-regime. An important outcome is the extension to interacting fermions of the notion of macroscopic AC–conductivity measures, known so far only for free fermions with disorder. Such excitation measures result from the 2nd law of thermodynamics and turn out to be Lévy measures. As compared to the Drude (Lorentz–Sommerfeld) model, widely used in Physics, the quantum many-body problem studied here predicts a much smaller AC-conductivity at large frequencies. This indicates (in accordance with experimental results) that the relaxation time of the Drude model, seen as an effective parameter for the conductivity, should be highly frequency-dependent. Meanwhile, this study also shows the validity of Ohm's law at the atomic scale for a purely quantum system, which has experimentally been verified in 2012. Such a behavior was unexpected.
Joint work with: W. de Siqueira Pedra.

Kirill Cherednichenko: Asymptotic behaviour of the spectra of systems of Maxwell equations in periodic composite media with high contrast.

I shall discuss the behaviour of the spectrum of the system of Maxwell equations of electromagnetism, with rapidly oscillating periodic coefficients, subject to periodic boundary conditions on a "macroscopic" domain $(0,T)^d, T > 0$. We consider the case when the contrast between the values of the coefficients in different parts of their periodicity cell increases as the period of oscillations η goes to zero. We show that the limit of the spectrum as $\eta \to 0$ contains the spectrum of a "homogenised" system of equations that is solved by the limits of sequences of eigenfunctions of the original problem. We investigate the behaviour of this system and demonstrate phenomena not present in the scalar theory for polarised waves.
Joint work with Shane Cooper (University of Bath).

Giuseppe De Nittis: Topological nature of the Fu-Kane-Mele invariants.

Condensed matter electronic systems endowed with odd time-reversal symmetry (TRS) (a.k.a. class AII topological insulators) show topologically protected phases which are described by an invariant known as Fu-Kane-Mele index. The construction of this invariant, in its original form, is specific for electrons in a periodic background and is not immediately generalizable to other interesting physical models where different forms of TRS also play a role. By exploiting the fact that system with an odd TRS (in absence of disorder) can be classified by Quaternionic vector bundles, we introduce a Quaternionic topological invariant, called FKMM-invariant, which generalizes and explains the

topological nature of the Fu-Kane-Mele index. We show that the FKMM-invariant is a universal characteristic class which can be defined for Quaternionic vector bundles in full generality, independently of the particular nature of the base space. Moreover, it suffices to discriminate among different topological phases of system with an odd TRS in low dimension. As a particular application we describe the complete classification over a big class of low dimensional involutive spheres and tori. We also compare our classification with recent results concerning the description of topological phases for two-dimensional adiabatically perturbed systems.
Joint work with: K. Gomi.

Stefan Enoch: Metamaterials, transformation optics for waves engineering.

We will introduce the concepts of metamaterials and transformation optics. Then, we will show examples of the new engineering possibilities offered by these tools. We will also present how surface plasmons could be manipulated in the same way [1]. Using simple shallow water model, we will show that transposition of optical invisibility concepts to water waves is straightforward. Then, experiments on water waves cloaks and carpets will be presented [2]. A second type of waves we will consider is bending waves. We will show thanks to a thin-plate model that cloaking of waves on such plates is possible with potential applications to anti-vibrating systems [3]. It also opens the way to earthquake protection [4].

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Alexander Figotin and Aaron Welters: Overdamping in gyroscopic systems composed of high-loss and lossless components.

Using a Lagrangian framework, we study overdamping phenomena in gyroscopic systems composed of two components, one of which is highly lossy and the other is lossless. The losses are accounted for by a Rayleigh dissipative function. We prove that selective overdamping is a generic phenomenon in Lagrangian systems with gyroscopic forces and give an analysis of the overdamping phenomena in such systems. Central to the analysis is the introduction of the notion of a dual Lagrangian system and this yields significant improvements on some results on modal dichotomy and overdamping.

Dirk Hundertmark: Discrete diffraction managed solitons: Threshold phenomena and rapid decay for general nonlinearities.

We study solitary breather type solutions for a discrete nonlinear Schrödinger type equation. This equation is known to model a wide range of effects ranging from molecular crystals to biophysical systems. Our interest for this equation comes from the fact that it models an array of nonlinear waveguides, the hopping terms corresponding to the interaction of neighbouring waveguides. We consider a variant of this model, the diffraction management equation, where the diffraction is periodically modulated along the waveguides. In experiments stable low power pulses in these waveguide arrays have been observed.

We prove a threshold phenomenon for the existence/non-existence of energy minimizing solitary solutions of the diffraction management equation for strictly positive and zero average diffraction. Our methods allow for a large class of nonlinearities, they are, for example, allowed to change sign, and we impose the weakest possible conditions on the local diffraction profile, it only has to be locally integrable. The solutions are found as minimizers of a nonlinear and nonlocal variational problem which is translation invariant. There exists a critical threshold such that minimizers for this variational problem exist if their power is bigger than the critical threshold and no non-trivial minimizers exist with powers below the critical threshold. We also give simple criteria for the finiteness and strict positivity of the critical threshold. Our proof of existence of minimizers is rather direct and avoids the use of Lions' concentration compactness argument.

Furthermore, we give precise quantitative lower bounds on the exponential decay rate of the diffraction management solitons, which confirm the physical heuristic prediction for the asymptotic decay rate. Moreover, for ground state solutions, these bounds give a quantitative lower bound for the divergence of the exponential decay rate in the limit of vanishing average diffraction. For zero average diffraction, we prove quantitative bounds which show that the solitons decay much faster than exponential. Our results considerably extend and strengthen previous results.
Joint work with: Mi-Ran Choi and Young-Ran Lee, Sogang University, Seoul.

Muamer Kadic: Mechanical Metamaterials.

Metamaterials are man-made structures that obtain their unusual (meta=beyond) effective properties from geometrical structure rather than chemistry. Using 3D printing, metamaterials offer an opportunity to realize different properties and functionalities by micro-structuring a single material. Here, we will focus on two recent examples: controlling deformations [1,2] using transformational physics and absorbing energy [3] using mechanical instabilities. We will first review the different steps of protecting objects using transformational physics [4] in linear mechanics from the quasi-static case (using the concept of neutral inclusions for pentamode materials) to dynamic elastic cloaks (design by direct lattice transformations). We will show how to characterize their performances using Finite Element Method (FEM) from static to dynamic case via a scattering formalism [5,6] dealing with virtual sources and adaptive PMLs (Perfectly Matched Layers). Aiming to go beyond the linear regime, we will show how reusable mechanical metamaterials can be designed to absorb mechanical (shock) energy and hence protect from external loads. Usually shock absorbers are based on viscoelasticity or plastic (irreversible) deformations. Here, we present a new class of metamaterials which is based on buckling (instabilities) and that behaves as fully reusable mechanical metamaterials. These metamaterials have successfully been fabricated and characterized, showing promising performance.

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Yosuke Kubota: Twisted equivariant K-theory and topological phases.

The classification of topological phases in each Altland-Zirnbauer symmetry class is related to one of 2 complex or 8 real K-theory by Kitaev. A more general framework, in which we deal with systems with an arbitrary symmetry of quantum mechanics specified by Wigner's theorem, is introduced by Freed and Moore by using a generalization of twisted K-theory. In this talk, we introduce the definition of twisted K-theory in the sense of Freed-Moore for C^*-algebras, which gives a framework for the study of topological phases of non-periodic systems with a symmetry of quantum mechanics. Moreover, we introduce uses of basic tools in K-theory of operator algebras such as inductions and the Green-Julg isomorphism for the study of topological phases.

Max Lein: Rigorous Analogies Between Quantum Systems and Certain Wave Equations.

The heuristic similarities of certain wave equations with analogous quantum systems has allowed physicists to successfully predict and later observe novel physical effects. Examples are one-way edge modes in periodic light conductors and magnonic crystals which resemble those found in topological insulators from condensed matter physics. However, few of these *analogies to quantum systems* have been derived from first principles, and mathematical results are scarcer still. In this talk I will explore how to go about making these heuristic arguments rigorous, with an emphasis on shared mathematical structures. The main idea is to recast the dynamical equations in the form of a Schrödinger equation, $\mathrm{i}\partial_t \psi = M\psi$. But there are a number of differences compared to quantum systems: (1) The generator $M = W\,D$ is the *product* of "weights" W and a differential operator D; also perturbations are multiplicative. Operators of this type are much less understood and have very interesting mathematical features. And (2) the physically relevant fields (e. g. electromagnetic fields) are *real* whereas quantum wave functions are complex. Hence, applying certain tools such as the classification theory of *complex* vector bundles requires some care. I will illustrate these points with examples from electromagnetism. Joint work with Giuseppe De Nittis and Carlos Villegas Blas.

Mikyoung Lim: Spectrum of the Neumann-Poincaré operator and plasmon resonance.

In this talk we consider spectral properties of the Neumann-Poincaré (NP) operator on planar domains with corners. Recently there is rapidly growing interest in the spectral properties of the NP operator due to its relation to plasmonics and cloaking by anomalous localized resonance: Plasmon resonance occurs at eigenvalues of the NP operator and anomalous localized resonance occurs at the accumulation point of eigenvalues, respectively. We show that the rate of resonance at continuous spectrum is different from that at eigenvalues, and then derive a method to distinguish continuous spectrum from eigenvalues. We analyse the spectrum of intersecting disks which has two corners and show computational experiments which provide the spectral properties of domains with corners. For the computations we use a modification of the Nyström method which makes it possible to construct high-order convergent discretisations of the NP operator on domains with corners.

Ling Lu: Topological and invisible photonic crystals.

I will show experimental realizations and theoretical predictions of 2D and 3D photonic crystals with topologically protected edge and surface states.

I will also discuss the realization of an invisible material having identical electromagnetic properties as air at a desired frequency.

Thierry Martin: Electronic quantum optics with quantum Hall edge channels and topological insulators.

Electronic quantum optics attempts to mimic photons collisions scenarios of quantum optics in a condensed matter setting, with single electron source for instance propagating along the edge states of a semiconductor device. Electrons differ from photon because of their statistics, because they repel each other and because they are accompanied by a Fermi sea. We will start by summarizing the results of the nanophysics team in the integer quantum Hall regime. Next, we will concentrate on edge states of a two dimensional topological insulator. The edge states of a two-dimensional topological insulator are characterized by their helicity, a very remarkable property which is related to the time-reversal symmetry and the topology of the underlying system. We theoretically investigate a Hong-Ou-Mandeltype setup as a tool to probe it. Collisions of two electrons with the same spin show a Pauli dip, analogous to the one obtained in the integer quantum Hall case. Moreover, the collisions between electrons of opposite spin also lead to a dip, known as Z2 dip, which is a direct consequence of the constraints imposed by time-reversal symmetry. In contrast to the integer quantum Hall case, the visibility of these dips is reduced by the presence of the additional edge channels, and crucially depends on the properties of the quantum point contact. As a unique feature of this system, we show the possibility of three-electron interference, which leads to a total suppression of the noise independently of the point contact configuration. This is assured by the peculiar

interplay between Fermi statistics and topology. This work intends to extend the domain of applicability of electron quantum optics.
Joint work with: K. Gomi.

Shinichiroh Matsuo: Gysin maps and bulk-edge correspondence.

We propose a yet another definition of KR-groups, which combines those of Atiyah and Karoubi and gives a simple proof of the Bott periodicity. Using the new definition, we can formulate the bulk-edge correspondence for free fermion systems as the functoriality of the Gysin map.
This is joint work with M. Furuta, S. Hayashi, M. Kotani, Y. Kubota, and K. Sato.

Owen D. Miller: Material-dictated upper bounds for scattering response in linear systems.

First I describe a simple energy-conservation approach that establishes geometry-independent upper bounds to optical response in absorptive systems. For a material with a (local, linear) susceptibility χ at a fixed frequency, the "optical theorem" leads to bounds that depend on $|\chi|^2/\operatorname{Im}\chi$ and the source/scatterer separation distance, but which are independent of shape or periodicity and apply equally well to complex sub-wavelength structures such as metamaterials (with the bounds depending only on the constituent materials). I will show that generalizations of this approach—finding convex constraints for induced currents—can further yield limits to the power–bandwidth product per resonance of any scatterer (not necessarily absorptive), and can be generalized to linear elastodynamics as well as coupled multiphysics systems. I will discuss efforts to achieve optimal performance for a variety of scattering processes, including absorption, scattering, local density of states, and radiative heat transfer, demonstrating the utility of the bounds for providing a global view of what is possible in highly nonconvex and exponentially large design spaces.
Joint work with Steven G. Johnson.

Bruno Nachtergaele: Stability of Frustration-Free Ground States of Quantum Spin Systems.

We study frustration-free quantum lattice systems with a non-vanishing spectral gap above one or more (infinite-volume) ground states. The ground states are called stable if arbitrary perturbations of the Hamiltonian that are uniformly small throughout the lattice have only a perturbative effect. In the past several years such stability results have been obtained in increasing generality. We review results by Bravyi-Hastings, Bravyi-Hastings-Michalakis, and Michalakis-Zwolak, as well as some recent refinements.
This is joint work with Robert Sims and Amanda Young.

Hagen Neidhardt: Scattering matrix and Dirichlet-to-Neumann maps.

A general representation formula for the scattering matrix of a scattering system consisting of two self-adjoint operators in terms of an abstract operator valued Titchmarsh-Weyl M-function is proved. This result is applicable to scattering problems for different selfadjoint realizations of Schrödinger operators on unbounded domains, and Schrödinger operators with singular potentials supported on hypersurfaces. In both applications the scattering matrix is expressed in an explicit form with the help of Dirichlet-to-Neumann maps.

Hoai-Minh Nguyen: Applications and stabilities of negative index materials.

Negative index materials are artificial structures whose refractive index are negative over some frequency range. These materials were first investigated theoretically by Veselago in 1964 and their existence was confirmed experimentally by Shelby et al. in 2001. In the first part of this talk, I explain how and why one can use negative index materials to do cloaking and superlensing using complementary media, and cloaking a source and an object via anomalous localized resonance. In the second part of the talk, I discus various conditions on the stabilities of these materials. From mathematics point of view, the study of negative index materials involves the study of Maxwell or Helmholtz equations with sign changing coefficients. Hence the ellipticity and the compactness are lost in general. Moreover, the localized resonance, i.e., the fields blow up in some regions and remain bounded in some others as the loss (the viscosity) goes to 0, might occur.

Toshiaki Omori: A discrete surface theory on 3-valent graphs embedded in 3-dimensional Euclidean space.

Toward a new insight into the physical properties of carbon materials of negatively-curved type, especially of Mackay-like carbon materials, we have recently developed a new discrete surface theory on 3-valent embedded graphs in 3-dimensional Euclidean space. In this talk, we shall start with the definitions of the mean curvature and the Gauss curvature of 3-valent discrete surfaces, continue with their several fundamental properties, including variational principles for the area functional, then introduce several basic examples. We shall also introduce an example of a discrete minimal surface which is constructed from a standardly realized (due to Kotani-Sunada) discrete surface. Additionally, if time permitted, we discuss convergence of a sequence of 3-valent discrete surfaces which are coming from the so-called Goldberg-Coxeter construction.
This talk is based on joint work with Motoko Kotani and Hisashi Naito.

Nicolas Rougerie: Emergent anyons in quantum Hall physics.

Anyons are by definition particles with quantum statistics different from those of bosons and fermions. They can occur only in low dimensions, 2D being the most interesting case. They have hitherto remained hypothetical, but there is good theoretical evidence that certain quasi-particles occuring in quantum Hall physics should behave as anyons.

In this talk I shall consider the case of tracer particles immersed in a so-called Laughlin liquid. I will argue that, under certain circumstances, these become anyons. This is made manifest by the emergence of a particular effective Hamiltonian for their motion. The latter is notoriously hard to solve even in simple cases, and well-controled simplifications are highly desirable. I will discuss a possible mean-field approximation, leading to a one-particle energy functional with self-consistent magnetic field.
Joint work with Douglas Lundholm.

Stephen P. Shipman: Spectrally embedded bound states for quantum graphs.

As a rule, a second-order self-adjoint periodic differential operator with a local defect cannot admit an eigenvalue embedded in its continuous spectrum. This rule essentially extends to periodic quantum graphs, where the result relies on the irreducibility of the Fermi surface (dispersion relation). But there are certain periodic graphs, such as bilayer graphene, that do allow spectrally embedded eigenfunctions under local perturbations. Construction of the eigenfunctions utilizes the separability of the Fermi surface for all energies, which in turn comes from the symmetry group of the graph, which cannot be realized as a group of symmetries of Euclidean space. This presentation will show the construction of embedded eigenvalues for quantum graphs and discuss the connections to reducibility of the Fermi surface and the symmetries of the graph.

Aaron Welters: Analyticity of the Dirichlet-to-Neumann Map for Maxwells Equations in Passive Composite Media.

In this talk I will discuss the analyticity properties of the electromagnetic Dirichlet-to-Neumann (DtN) map for the time-harmonic Maxwells equations for passive linear multi-component media. I will also discuss the connection of this map to Herglotz functions for isotropic and anisotropic multicomponent composites. The focus of the discussion will be on two different types of geom- etry, namely, layered media and bounded media (with Lipschitz domains). For these geometries I will derive the analyticity properties of the associated DtN map in terms of the transfer matrix for layered media and, for bounded media, using a variational formulation of the time-harmonic Maxwells equations.
This is joint work with Graeme Milton (Univ.of Utah) and Maxence Cassier (Univ.of Utah).

John R. Willis: Negative refraction in a laminate.

This talk relates to the reflection and transmission of waves at a plane interface between a homogeneous elastic half-space and a half-space of elastic material that is periodically laminated. The lamination is always in the direction of the x_1-coordinate axis and the displacement is always longitudinal shear, so that the only non-zero displacement component is $u_3(x_1, x_2, t)$. After an initial discussion of Floquet–Bloch waves in the laminated material, brief consideration is given to the reflection-transmission problem, when the interface between the two media is the plane $x_1 = 0$. Nothing unusual emerges: there are just a single reflected wave and a single transmitted wave, undergoing positive group-velocity refraction. Then, the problem is considered when the interface between the two media is the plane $x_2 = 0$. The periodic structure of the interface induces an infinite set of reflected waves and an infinite set of transmitted waves. All need to be taken into account, but most decay exponentially away from the interface. It had previously been recognised that, if the incident wave had appropriate frequency and angle of incidence, a propagating transmitted wave would be generated that would undergo negative group-velocity refraction – behaviour usually associated with a metamaterial. It is demonstrated by example that there can be, in addition, a propagating transmitted wave with smaller wavelength but larger group velocity, that undergoes positive group-velocity refraction. The presentation will conclude with a brief discussion of this finding, including its implications for the utility (or not) of "effective medium" theory.