RECENT TRENDS IN NONLINEAR EVOLUTION EQUATIONS

April 4 - 8, 2016

Hajer Bahouri: Dispersive estimates for the Schrödinger equation on 2-step stratified Lie groups.

The present work is dedicated to the proof of dispersive estimates on 2-step stratified Lie groups, for the linear Schrödinger equation involving a sublaplacian. It turns out that the Schrödinger propagator on 2-step stratified Lie groups behaves like a wave operator on a space of the same dimension as the center of the group and like a Schrödinger operator on a space of the same dimension as the radical of the canonical skew-symmetric form. This unusual behavior (in the case when the radical is not trivial) of the Schrödinger propagator makes the analysis of the explicit representations of the solutions tricky and gives rise to uncommon dispersive estimates. It will also appear from our analysis that the optimal rate of decay is not always in accordance with the dimension of the center as it is the case for H-type groups: we will exhibit examples of 2-step stratified Lie groups with center of any dimension and for which no dispersion phenomenon occurs for the Schrödinger equation. We will identify a generic condition under which the optimal rate of decay is achieved.

Fabrice Béthuel: On the formation of one-dimensional interfaces in two dimensional multiple-well gradient problems.

The formation of codimension-one interfaces for multiwell gradient-driven problems is well-known and established in the scalar case, where the equation is is often referred to as the Allen-Cahn equation. The vectorial case in contrast is quite open. due to the lack of known appropriate monotinicity formula. In this talk, I will focus on elliptic case in two-dimensions, and discuss the concentration on one-dimensional sets.

Isabeau Birindelli: Symmetry of solutions for fully non linear PDE.

I will talk about the property inherited by a viscosity solution of a rotationally invariant equation of the form F(x,D2u)+f(x,u) = 0, in relation with the spectral property of the ?improperly called? linearized operator at u. We will show applications of our symmetry results to obtain bounds on the spectrum and to deduce properties of possible nodal eigenfunctions for the operator . In the same contest other symmetry questions will be treated such as Faber Krahn inequalities and overdetermined problems.

Walter Craig: Vortex filament dynamics.

An important problem in mathematical hydrodynamics addresses the evolution of vortex filaments as solutions of the Euler equations. This is a setting with a strong analogy to Hamiltonian dynamical systems. I will give an analysis of a system of model equations for the dynamics of near-parallel vortex filaments in a three dimensional fluid. These equations can be formulated as a Hamiltonian system of partial differential equations, and the talk will describe some aspects of a phase space analysis of solutions, including the construction of periodic and quasi-periodic orbits via a version of KAM theory for PDEs, and a topological principle to count multiplicity of solutions. This is ongoing joint work with C. Garcia (UNAM) and C.-R. Yang (McMaster and the Fields Institute).

Panagiota Daskalopoulos: Ancient solutions to geometric flows.

We will give a survey of recent research progress on ancient or eternal solutions to geometric flows such as the Ricci flow, the Mean Curvature flow and the Yamabe flow. We will address the classification of ancient solutions to parabolic equations as well as the construction of new ancient solutions from the gluing of two or more solitons.

Manuel Del Pino: Blow-up by bubbling in some critical parabolic equations.

We construct solutions that blow up in the form of energy invariant, time dependent space scalings of steady states for some problems where such steady states exist. We do that for the heat flow of the Yamabe equation, for 2d harmonic map flow into S2 and for the Keller Segel equation in the plane, with no symmetry assumptions.

Erwan Faou: Numerical solitons.

I will address the questions of existence and stability of standing and travelling waves in discrete NLS models their time discretizations.
This is a joint work with D. Bambusi, B. Grébert and A. Maspero.

Patrick Gérard: Resonant two-soliton interaction for the one dimensional half wave equation.

The one dimensional half wave equation is an interesting example of a nonlinear wave equation with vanishing dispersion, displaying arbitrarily small mass solitons. I will discuss how, in some resonant regime, the interaction of two such solitons leads to long time transition to high frequencies.
This talk is issued from a jointwork with Enno Lenzmann, Oana Pocovnicu and Pierre Raphael.

François Hamel: Transition fronts for monostable reaction-diffusion equations.

The standard notions of reaction-diffusion traveling fronts can be viewed as examples of generalized transition fronts. These notions involve uniform limits, with respect to the geodesic distance, to a family of hypersurfaces which are parametrized by time. The existence of transition fronts has been proved in various contexts where the standard notions of fronts make no longer sense. Even for homogeneous equations, fronts with varying speeds are known to exist. In this talk, I will first review the various notions of standard and transition fronts and I will then report on some recent existence results and qualitative properties of transition fronts for monostable homogeneous and heterogeneous one-dimensional equations. I will also discuss their asymptotic past and future speeds. The talk is based on some joint works with Luca Rossi.

Michael Jenskinson: On-Site and Off-Site Bound States of the Discrete Nonlinear Schroedinger Equation and the Peierls-Nabarro Barrier.

We construct several families of symmetric localized standing waves (breathers) to the one-, two-, and three-dimensional discrete nonlinear Schrödinger equation (DNLS) with cubic nonlinearity using bifurcation methods about the continuum limit. Such waves and their energy differences play a role in the propagation of localized states of DNLS across the lattice. The energy differences, which we prove to exponentially small in a natural parameter, are related to the "Peierls-Nabarro Barrier" in discrete systems, first investigated by M. Peyrard and M.D. Kruskal (1984). These results may be generalized to different lattice geometries and inter-site coupling parameters. Finally, we discuss the local stability properties of such bound states. This is joint work with Michael I. Weinstein.

Felipe Linares: On special regularity properties of solutions to the k-generalized Korteweg-de Vries equation.

We will discuss special regularity properties of solutions to the IVP associated to the k-generalized KdV equations. In [1] we show that for data $u_0 \in H^{3/4+}(\mathbb{R})$ whose restriction belongs to $H^k((b,\infty))$ for some $k \in \mathbb{Z}^+$ and $b \in \mathbb{R}$, the restriction of the corresponding solution $u(\cdot,t)$ belongs to $H^k((\beta,\infty))$ for any $\beta \in \mathbb{R}$ and any $t \in (0,T)$. Thus, this type of regularity propagates with infinite speed to its left as time evolves. This kind of regularity can be extended to a general class of nonlinear dispersive equations. Recently, in [2] we proved that the solution flow of the k-generalized KdV equation does not preserve other kind of regularities exhibited by the initial data u_0. If time allows we will discuss propagation of regularity in solutions of related dispersive equations.

References

[1] P. Isaza, F. Linares, and G. Ponce, *On the propagation of regularity and decay of solutions to the k-generalized Kortweg-de Vries equation*, Comm. Partial Diff. Eqs. **40** (2015), 1336–1364.

[2] F. Linares, G. Ponce, and D. Smith, *On the regularity of solutions to a class of nonlinear dispersive equations*, preprint.

Nikolai Nadirashvili: On level sets of solutions of elliptic and parabolic equations.

We discuss regularity, geometry and metric properties of nodal and level sets of solutions to second order elliptic and parabolic equations.

Tohru Ozawa: Quadratic Interactions in Dispersive Systems.

This talk is based on my recent jointwork with K. Fujiwara and S. Machihara. We consider the Cauchy problem for semirelativistic equations with quadratic interaction. It is shown that the standard iteration scheme of Picard fails for this system at the level of energy space $H^{1/2}$. We prove the global well-posedness in $H^{1/2}$ from the viewpoint of Hamilton structure, without compactness argument.

Jean-Claude Saut: Full dispersion water waves models.

When deriving asymptotic models for water waves (or other dispersive physical systems) one usually approximates the dispersion relation via Taylor expansion at a given wave number, yielding models with "good" dispersive properties. The so-called Full dispersion models are obtained when keeping the original dispersion relation, hoping to get a good approximation for a wider range of frequencies. One shortcoming is that one obtains "weakly dispersive" equations or systems (with nonlocal dispersion), lacking the nice dispersive properties of the classical models (KdV, KP, NLS, Davey-Stewartson,...). The aim of the talk is to survey recent results and open questions on Full dispersion models.

Juan Luis Vazquez: The theory of nonlinear diffusions with fractional operators.

In this talk I will report on some of the progress made by the author and collaborators on the topic of nonlinear diffusion equations involving long distance interactions in the form of fractional Laplacian operators. The nonlinearities are of the following types: porous medium, fast diffusion or p-Laplacian. Results cover well-posedness, regularity, free bouncadaries, asymptotics, extinction, and others. Differences with standard diffusion have been specially examined.

Luis Vega: Hardy Uncertainty Principle and Carleman Inequalities.

In the first half of the talk I shall present some recent work done in collaboration with L. Escauriaza, C. Kenig, and G. Ponce about the application of Hardy's uncertainty principle to obtain lower bounds for a diffusion equation with bounded perturbations of zero order that can depend on time. In the second half I will sketch a related work done with A. Fernandez-Bertolin on the Schrödinger equation associated to the discrete laplacian.The main technique is to use Carleman inequalities to prove convexity properties of appropriately chosen quantities.

Hatem Zaag: Existence and stability of a blow-up solution for a heat equation with a critical nonlinear gradient term.

We consider a nonlinear heat equation with a double source: a power of the solution and a power of its gradient, in the critical case. We construct a solution blowing up in finite time only at the origin, and give its profile. To do so, we linearize the similarity variables' version around the profile, and show that the negative part of the spectrum can be controlled thanks to the bounding effect of the linear operator. We are left only with the positive directions of the solution, in finite number. They are controlled near zero thanks to the use of index theory. The interpretation of the parameters of the finite-dimensional problems in terms of the blow-up time and the blow-up point give us the stability of the constructed solution with respect to initial data. Remarkably, our solution shows a new type of behavior, unknown for the semilinear heat equation (with power nonlinearity).