Relative Trace Formula, Periods, L-Functions, Harmonic Analysis and Langlands Functoriality

23-27 May 2016

James Arthur: Beyond Endoscopy and elliptic terms in the trace formula.

Beyond endoscopy it the strategy put forward by Langlands for applying the trace formula to the general principle of functoriality. Subsequent papers by Langlands (one in collaboration with Frenkel and Ngo), together with more recent papers by Altug, have refined the strategy. They all emphasize the importance of understanding the elliptic terms on the geometric side of the trace formula.

We shall discuss the general strategy, and how it pertains to these terms. We shall then describe some of the difficuties, and some possible ways of dealing with them.

Raphael Beuzart-Plessis: The local Gan-Gross-Prasad conjecture for unitary groups.

The local Gan-Gross-Prasad conjectures concern certain branching or restriction problems between representations of real or p-adic Lie groups. In its simplest form it predicts certain multiplicity-one results for "extended" L-packets. In a recent series of papers, Waldspurger has settled the conjecture for special orthogonal groups over p-adic field. In this talk, I will present a proof of the conjecture for unitary groups which has the advantage of working equally well over archimedean and non-archimedean fields.

Masaaki Furusawa: On special Bessel periods and the Gross-Prasad conjecture for $\mathrm{SO}(2n+1) \times \mathrm{SO}(2)$

We consider one instance of the Gross-Prasad conjecture. Our main result is that if an irreducible cuspidal automorphic representation π of an odd dimensional special orthogonal group, whose local component at some finite place is generic, admits the special Bessel model corresponding to a quadratic extension E, then the central L-value $L(1/2, \pi)L(1/2, \pi \times \chi)$ does not vanish. Here χ denotes the quadratic character corresponding to E. As an application, we obtain the equivalence between the non-vanishing of the special Bessel periods and that of the corresponding central L-values when π corresponds to a full modular holomorphic Siegel cusp form of degree two and E is an imaginary quadratic extension of $\mathbb{Q}$.
This is joint work with Kazuki Morimoto.

Wee-Teck Gan: Theta lifts of tempered representations and Langlands parameters.

In joint work with Hiraku Atobe, we determine the theta lifting of irreducible tempered representations for symplectic-metaplectic–orthogonal and unitary dual pairs in terms of the local Langlands correspondence. The main new tool for proving our result is the recently established local Gross-Prasad conjecture.

Nadia Gurevic: Poles of the standard L-function for G2 and the image of functorial lifts.

the exceptional group of type G_2 is a member of various dual pairs. The associated global theta lifts are functorial and their images can often be described in terms of poles of L-functions. We shall use a recent construction of the Rankin-Selberg integral for the standard L-function for G2 to describe some of these images.
This is joint work with Avner Segal.

Jeffrey Hakim: Constructing Tame Supercuspidal Representations.

I will give a new approach to Jiu-Kang Yu's construction of tame supercuspidal representations of p-adic reductive groups. Connections with distinguished representations and the theory of cuspidal Deligne-Lusztig representations of finite groups of Lie type are also discussed.

Michael Harris: Special values of Rankin-Selberg L-functions and automorphic periods.

This is a report on work of the author, Grobner, and Lin on an automorphic interpretation of Deligne's conjecture on critical values of Rankin-Selberg L-functions of cohomological automorphic representations of GL(n) x GL(n'), n > n', over a CM field. The critical values are expressed as algebraic multiples of automorphic periods on unitary groups, and the results are compared with Deligne's conjecture on the critical values of the corresponding motives.

Atsushi Ichino: The automorphic discrete spectrum of Mp(2n)

In his 1973 paper, Shimura established a lifting from half-integral weight modular forms to integral weight modular forms. After that, Waldspurger studied this in the framework of automorphic representations and classified the automorphic discrete spectrum of the metaplectic group Mp(2), which is a nonlinear double cover of SL(2), in terms of that of PGL(2). We discuss a generalization of his classification to the metaplectic group Mp(2n) of higher rank.
This is joint work with Wee Teck Gan.

Dihua Jiang: On the Central Value of Tensor Product L-functions and the Langlands Functoriality.

It is better known that the poles of automorphic L-functions are closely related to the Langlands functoriality for automorphic forms. In this talk, we are going to explain the relation between the central value of tensor product L-functions and the Langlands functoriality, which is less known in general.
This is a report on my work joint with Baiying Liu and Lei Zhang.

Wen-Wei Li: Prehomogeneous zeta integrals with generalized coefficients.

I will try to justify a formalism of local zeta integrals associated to affine spherical embeddings by using the prehomogeneous vector spaces as a testing ground. These can be viewed as natural variants of the usual prehomogeneous local zeta integrals, and their basic analytic properties will be discussed in the archimedean case.

Nadir Matringe: Distinction of the Steinberg representation for GL(n) and its inner forms.

Let F be a non archimedean local field, E a quadratic extension of F, and D a division algebra over F. If χ is a character of E^*, we will give a necessary and sufficient condition on χ for the Steinberg representation St(χ) of GL(m,D$\otimes$ E) to be distinguished by GL(m,D), checking a conjecture of D. Prasad in this case.

Fiona Murnaghan: Tame relatively supercuspidal representations.

Let G be a connected reductive p-adic group that splits over a tamely ramified extension. Let H be the fixed points of an involution of G. An irreducible smooth H-distinguished representation of G is H-relatively supercuspidal if its relative matrix coefficients are compactly supported modulo H Z(G). (Here, Z(G) is the centre of G.) We will describe some relatively supercuspidal representations whose cuspidal supports belong to the supercuspidals constructed by J.K. Yu.

Omer Offen: On gamma factors, root numbers and distinction.

Let E/F be a quadratic extension of p-adic fields. A representation of $GL_n(E)$ is distinguished if it admits a non-zero $GL_n(F)$ invariant linear form. For a supercuspidal representation this property can be characterized by local invariants in two different ways. By a result of Ok: Distinction is equivalent to, Rankin-Selberg gamma factor at one half equals one for 'enough' distinguished twists. By a result of Kable: Distinction is equivalent to the Asai L-function having a pole at zero. We will consider possible generalizations of the first characterization to non-supercuspidals.
This is joint work in progress with Nadir Matringe.

Eric Opdam: On the spherical automorphic spectrum supported in the Borel subgroup.

Let G be a split connected reductive group defined over a global field F, with maximal split torus T, and let $\mathbf{K} = \prod_v K_v \subset G(\mathbb{A})$ be a maximal compact subgroup such that $K_v = G(O_v)$ at every nonarchimedian place v. Let χ be an everywhere unramified automorphic character of T. In this talk I will explain how the spectral decomposition of $L^2(G(F)\backslash G(\mathbb{A}))^{\mathbf{K}}_{[T,\chi]}$ reduces to the known spectral decomposition of the (anti)spherical subalgebra of a certain (graded) affine Hecke algebra. The proof uses besides standard analytic properties of the Dedekind L-function, known properties of so-called residue distributions, which were introduced to study the Plancherel decomposition of (graded) affine Hecke algebras, and a result by M. Reeder on the support of the weight spaces of the anti-spherical discrete series representations of affine Hecke algebras. Both these ingredients are of a purely local nature.
This talk is based on joint work with M. De Martino and V. Heiermann.

Jean-Loup Waldspurger: Caractères des représentations de niveau 0.

On considère une représentation admissible de longueur finie d'un groupe réductif connexe défini sur un corps local non-archimédien de caractéristique nulle. On suppose qu'elle est de niveau 0. On exprime alors son caractère comme une somme d'intégrales orbitales pondérées de fonctions très cuspidales explicites. La preuve reprend un article de Courtès en y insérant les résolutions de Schneider et Stuhler, dans une forme précisée par Meyer et Solleveld.

Chen Wan: Multiplicity one theorem for the Ginzburg-Rallis model.

Following the method developed by Waldspurger and Beuzart-Plessis in their proof of the local Gan-Gross-Prasad conjecture, we were able to prove the multiplicity one theorem on Vogan L-packet for the Ginzburg-Rallis model. In some cases, we can also relate the multiplicity to the central value of epsilon factor.

Hang Xue: Approximating smooth transfer in Jacquet–Rallis relative trace formulas.

I will explain the Gan-Gross-Prasad conjecture for $U(n) \diamond U(n+1)$ and the Jacquet–Rallis relative trace formula approach. I will explain a simple observation which enables us to remove the hypothesis at the archimedean place in a previous result of Wei Zhang.

Shunsuke Yamana: On the lifting of Hilbert cusp forms to Hilbert-Siegel cusp forms.

Starting from a Hilbert cusp form, I will construct a Hilbert-Siegel cusp form by giving its Fourier expansion explicitly. This is a joint work with Tamotsu Ikeda and a generalization of the lifting of elliptic cusp forms constructed by him in 2001 in classical language.

Shou-Wu Zhang: Congruent number problem and BSD conjecture.

A thousand years old problem is to determine when a square free integer n is a congruent number ,i,e, the areas of right angled triangles with sides of rational lengths. This problem has a some beautiful connection with the BSD conjecture for elliptic curves $E_n : ny^2 = x^3 - x$. In fact by BSD, all n= 5, 6, 7 mod 8 should be congruent numbers, and most of n=1, 2, 3 mod 8 should not be congruent numbers. Recently, Alex Smith has proved that at least 41.9% of n=1,2,3 satisfy (refined) BSD in rank 0, and at least 55.9% of n=5,6,7 mod 8 satisfy (weak) BSD in rank 1. This implies in particular that at last 41.9% of n=1,2,3 mod 8 are not congruent numbers, and 55.9% of n=5, 6, 7 mod 8 are congruent numbers. I will explain the ingredients used in Smith?s proof: including the classical work of Heath-Brown and Monsky on the distribution F_2 rank of Selmer group of E_n, the complex formula for central value and derivative of L-fucntions of Waldspurger and Gross-Zagier and their extension by Yuan-Zhang-Zhang, and their mod 2 version by Tian-Yuan-Zhang.

Wei Zhang: Cycles on the moduli of Shtukas and Taylor coefficients of L-functions.

This is joint work with Zhiwei Yun. We prove a generalization of Gross-Zagier formula in the function field setting. Our formula relates self-intersection of certain cycles on the moduli of rank two Shtukas to higher derivatives of L-functions for GL(2).

Michal Zydor: The Jacquet-Rallis trace formula.

The Jacquet-Rallis relative trace formula was introduced as a tool towards solving the global conjectures of Gan-Gross-Prasad for unitary groups. I will present some recent progress in developing the full formula. I will show how to extend the transfer of regular orbital integrals to singular geometric terms and I will discuss some applications to the Gan-Gross-Prasad conjecture. (Joint with Pierre-Henri Chaudouard)