DYNAMICS OF EVOLUTION EQUATIONS

March 21–25, 2016

HALE LECTURES

Peter Polacik: *Dynamics of bounded solutions of parabolic equations on the real line.*

We consider parabolic equations of the form $u_t = u_{xx} + f(u)$ on the real line. Unlike their counterparts on bounded intervals, these equations admit bounded solutions whose large-time dynamics is not governed by steady states. Even with respect to the locally uniform convergence, the solutions may not be quasiconvergent, that is, their omega-limit sets may contain nonstationary solutions.

We will start this lecture series by exhibiting several examples of non-quasiconvergent solutions, discussing also some entire solutions appearing in their omega-limit sets. Minimal assumptions on the nonlinearity are needed in the examples, which shows that non-quasiconvergent solutions occur very frequently in this type of equations. Our next goal will be to identify specific classes of initial data that lead to quasiconvergent solutions. These include localized initial data (joint work with Hiroshi Matano) and front-like initial data. Finally, in the last part of these lectures, we take a more global look at the solutions with such initial data. Employing propagating terraces, or stacked families of traveling fronts, we describe their entire spatial profile at large times.

SPECIAL TALK

Constantine Dafermos: *Jack Hale*

THEMATIC AFTERNOONS

Wilhelm Schlag: *Invariant manifolds and dispersive nonlinear evolution PDE's.*

Sylvain Crovisier and Nicolas Gourmelon: *Perturbative techniques of the dynamics in the C^1-topology.*

These lectures will address the dynamics of vector fields or diffeomorphisms of compact manifolds. For the study of generic properties or for the construction of examples, it is often useful to be able to perturb a system. This generally leads to delicate problems: a local modification of the dynamics may cause a radical change in the behavior of the orbits. For the C^1-topology, various techniques have been developed which allow to perturb while controlling the dynamics: closing and connection of orbits, perturbation of the tangent dynamics... We derive various applications to the description of C^1-generic diffeomorphisms.

Ludovic Rifford: *Geometric control and dynamics.*

The geometric control theory is concerned with the study of control systems in finite dimension, that is dynamical systems on which one can act by a control. After a brief introduction to controllability properties of control systems, we will see how basic techniques from control theory can be used to obtain for example generic properties in Hamiltonians dynamics.

Afternoon session: biological dynamics

Zhilan Feng: *Emerging disease dynamics in a model coupling within-host and between-host systems*

Epidemiological models and immunological models have been studied largely independently. However, the two biological processes (within- and between-host interactions) occur jointly and models that couple the two processes may generate new biological insights. Particularly, the threshold conditions for disease control may be dramatically different when compared with those generated from the epidemiological or immunological models separately. We developed and analyzed an ODE model, which links an SI epidemiological model and an immunological model for pathogen-cell dynamics. When the two sub-systems are considered in isolation, the dynamics are standard and simple. That is, either the infection-free equilibrium is stable or a unique positive equilibrium is stable depending on the relevant reproduction number being less or greater than 1. However, when the two sub-systems are explicitly coupled, the full system exhibits more complex dynamics including backward bifurcations; that is, multiple positive equilibria exist with one of which being stable even if the reproduction number is less than 1. The biological implications of such bifurcations are illustrated using an example concerning the spread and control of toxoplasmosis.

Tibor Krisztin: *Differential equations with queuing delays*

We consider a system of delay differential equations modeling queuing processes. These types of processes appear in computer networks, road networks, or when customers are waiting for service, or patients for treatment. The state dependent time delays are determined by algebraic equations involving the length of the queues. For the length of the queues discontinuous differential equations hold. We formulate an appropriate framework to study the problem, and show that the solutions define a continuous semiflow in the phase space. We study the stability of equilibria, and in some cases prove the existence of slowly oscillating periodic solutions.

Xingfu Zou: *A DDE model modelling the fear effect in predator-prey interactions with adaptive avoidance of predators.*

Recent field experiment of vertebrates showed that mere presence of a predator would cause a dramatic change of a prey's demography. Fear of predators increases prey's survival probability but leads to a cost in prey's reproduction. Based on the experimental findings, we propose an age structured predator-prey model with the cost of fear and adaptive avoidance of predators. Mathematical analyses show that maturation delay between juvenile prey and adult prey can induce stability change of an equilibrium.

Particularly, the positive equilibrium may lose its stability with a relatively large value of delay and regain stability if the delay is further larger. Numerical simulations show that either strong adaptation of adult prey or large cost of fear have destabilizing effect while large population of predators has a stabilizing effect on the predator-prey interactions under consideration. Numerical simulations also indicate that adult prey needs stronger anti-predator response if population of predators is larger and needs weaker anti-predator response if the cost of fear is larger.

Huaiping Zhu: *Modeling and dynamics of mosquito-borne diseases*

Mosquitoes are the cause of a lot trouble these days, including malaria, dengue, West Nile virus, La Crosse, and of course, recently the Zika. In this talk I will first briefly introduce our predictive modeling and forecasting studies of culex mosquitoes and West Nile virus for public health agencies in Canada. Then, I will continue to discuss backward bifurcation and bifurcations leading to oscillations for the compartmental models of mosquito-borne diseases. I will end the talk by connecting the fast-slow dynamics of some model with the bifurcation and cyclicity of degenerate graphics which are closely related to the Hilbert's 16th problem for quadratic vector fields.

Jianhong Wu: *Hale-Waltman's persistence theory applied to avian influenza spread: what remains?*

Hale-Waltman's theory on persistence of infinite dimensional dynamical systems provides a general geometric framework to use key concepts and approaches of dynamical systems to address an important issue of species extinction or persistence. This theory, when applied to any specific biological system, often provides the sharpest condition of extinction, which coincides with the threshold phenomena observed in many epidemic models. We use a delay differential system with periodic coefficients arising from bird immigration and avian influenza spread to demonstrate the power of Hale-Waltman's theory. We also discuss remaining challenges relevant to the predictability of inter-epidemic duration, and the distinction between the mathematical concept of persistence and the epidemiological stochastic reality infection dynamics.

PLENARY SPEAKERS

Marie-Claude Arnaud: *A multi-dimensional Birkhoff theorem for non autonomous Hamiltonian flows.*

For Hamiltonian and symplectic Dynamics, the invariant Lagrangian submanifolds are a central object of study. Here we will focus on the so-called Tonelli Hamiltonian flows that are defined on the cotangent bundle of a manifold, and explain that under some topological appropriate assumptions, an invariant Lagragian submanifold has to be a graph.

This is a joint work with Andrea Venturelli.

José M. Arrieta: *Asymptotic behavior of degenerate logistic parabolic equations.*

We analyze the asymptotic behavior of positive solutions of parabolic equations with a class of degenerate logistic nonlinearities of the type $\lambda u - n(x)u^\rho$, with $n(\cdot) \geq 0$ and $\rho > 1$. An important characteristic of this work is that the region where the logistic term degenerates, that is $K_0 = \{x : n(x) = 0\}$, may be non smooth.

We analyze conditions on λ, ρ, $n(\cdot)$ and K_0 guaranteeing that the solution starting at a positive initial condition remains bounded or blows up as time goes to infinity. Specially we try to understand how topological or geometric properties of the set K_0 affect the asymptotic dynamics of the solutions. As a matter of fact, this behavior may not be the same in different parts of K_0, even when K_0 is connected.

This is a joint work with Rosa Pardo and Anibal Rodríguez-Bernal from Madrid.

Patrick Bernard: *Variational and viscosity solutions of the Hamilton-Jacobi equation*

I will describe these notions of solutions and give some elements of comparison between them.

Nicolas Burq: *Stabilization of wave equations with rough damping*

For the damped wave equation on a compact manifold with continuous dampings, the geometric control condition is necessary and sufficient for uniform stabilization. In this talk, on the two dimensional torus, in the special case where $a(x) = \sum_{j=1}^N a_j 1_{x \in R_j}$ (R_j are rectangles), we give a very simple necessary and sufficient geometric condition for uniform stabilization. We also propose a natural generalization of the geometric control condition which makes sense for L^∞ dampings. We show that this condition is always necessary for uniform stabilization (for any compact (smooth) manifold and any L^∞ damping), and we prove that it is sufficient in our particular case on $\mathbb{T}^2$ (and for our particular damping).

This is a joint work with P. Gérard (Université Paris-Sud).

Rafael de la Llave: *Quasi-periodic solutions for state dependent delay equations.*

We consider delay differential equations in which the delay depends on the state of the system. These equations appear naturally in electrodynamics for particles interacting with retarded potentials (the delay is proportional to the distance) as well as in several biological models. We note that for these equations the phase space is infinite dimensional and not easy to describe. Questions such existence, uniqueness, dependence on parameters are still puzzling. We develop a theory of quasi-periodic solutions that bypasses the questions of existence for general initial data. We develop a functional equation for the quasi-periodic equations and study them by functional analysis methods. The main results are stated in an a posteriori format that states that given approximate solutions that satisfy some explicit non-degeneracy solutions, there are true solutions nearby. This can be used to justify some numerical solutions that have been produced. We show that in a one-parameter family, there are smooth solutions. Furthermore, we can find a large measure set where the quasi-periodic solution are analytic. We conjecture that this regularity is optimal. We also develop a theory of stable/unstable manifolds.

This is joint work with Xiaolong He.

Luca Dieci: *Considerations on sliding motion for piecewise smooth systems of Filippov type.*

In this talk, we give an overview of recent results for piecewise smooth systems of Filippov type. Emphasis will be on (partial) sliding motion on the intersection of 2 manifolds under general attractivity properties, and on conditions leading to abandon sliding motion. Discussion of appropriate behavior of a solution trajectory will be given, and the consequences for the selection of a Filippov vector field will be discussed.

Based on joint works with C. Elia and F. Difonzo.

Bernold Fiedler: *Sturm global attractors which are 3-balls - Part 2.*

In our joint talks with Carlos Rocha, we consider the Sturm global attractors $\mathcal{A}$ of dissipative scalar reaction-advection-diffusion equations

$$u_t = u_{xx} + f(x, u, u_x)$$

on the unit interval with Neumann boundary conditions. This follows the grand tradition of Brunovský, Chafee, Conley, Henry, Hale, Infante, and many others. We assume all equilibria to be hyperbolic. Their unstable manifolds decompose $\mathcal{A}$ into a regular cell complex. (This is false, in general, but a theorem for Sturm global attractors $\mathcal{A}$.)

We call $\mathcal{A}$ a 3-ball, if it is the closure of the unstable manifold of a single equilibrium $v{=}\mathcal{O}$ of Morse index three. On the 2-sphere surface of the 3-ball complex, *any* regular cell complex can be realized.

In our talks we will attempt to describe *all* ways to realize any given 3-ball regular cell complex as a Sturm global attractor $\mathcal{A}$. We start from a Schoenflies decomposition of the 2-sphere surface into planar *hemispheres.* Their separating *meridian circle* is the boundary of the two-dimensional fast unstable manifold of $\mathcal{O}$. The *poles* on the meridian, alias the boundary of the fastest unstable manifold of $\mathcal{O}$, are the maximal and minimal equilibria in the monotone order.

Each hemisphere, separately, turns out to be a planar Sturm global attractor $\mathcal{A}_{\pm}$. We characterize their matching conditions along the shared meridian circle, both, geometrically and in terms of the meanders of the associated shooting curve for the Neumann equilibrium problem. In particular we construct the meander permutations of all the infinitely many Sturm 3-balls $\mathcal{A}$, in finite time.

Thierry Gallay: *Remarks on Nonlinear Stabilization.*

If a map or a flow has an equilibrium point that is linearly exponentially unstable, under which conditions may we deduce that the equilibrium is unstable in the nonlinear sense too? Quite surprisingly, this natural question remains essentially open, despite partial results dating back to the pioneering work of Lyapunov. In this talk I discuss various aspects of this interesting problem, and I present some modest recent contributions which, I hope, give an idea of what the general solution may be.

Patrick Gérard: *A review of the cubic Szegő equation.*

In this talk, I will review properties of the cubic Szegő equation, a Hamiltonian evolution equation on the Hardy space of holomorphic functions on the disc, which displays both integrability and instability features, thanks to a special Lax pair structure involving Hankel operators from classical analysis. This talk is based on a series of results in collaboration with Sandrine Grellier and Herbert Koch.

Mariana Haragus: *Stability of periodic waves in water-wave models.*

The classical water-wave problem is concerned with the irrotational flow of a perfect fluid with constant density, subject to the forces of gravity and surface tension. The governing equations are the Euler equations in a domain bounded below by a flat bottom and above by a free surface. In this talk we discuss the stability of several classes of two-dimensional periodic traveling waves, for the Euler equations or for some simpler model equations arising in the regime of long waves. We focus on the questions of transverse and modulational linear stability/instability. The results rely upon spectral criteria which apply to rather general classes of reversible or Hamiltonian systems.

Guillaume James: *Discrete breathers in granular chains.*

Granular chains made of aligned beads interacting by contact (e.g. Newton's cradle) are widely studied in the context of impact dynamics and acoustic metamaterials. While much effort has been devoted to the theoretical and experimental analysis of solitary waves in granular chains, there is now an increasing interest in the study of breathers (spatially localized oscillations) in granular systems. Due to their oscillatory nature and associated resonance phenomena, static or traveling breathers exhibit much more complex dynamical properties compared to solitary waves. Such properties have strong potential applications for the design of acoustic metamaterials allowing to efficiently damp or deviate shocks and vibrations. In this talk, we review recent results and open problems concerning the dynamics of breathers in granular systems and their approximation through modulation equations.

[1] G. James, Nonlinear waves in Newton's cradle and the discrete p-Schrödinger equation, Math. Models Meth. Appl. Sci. 21 (2011), 2335-2377.
[2] G. James, P.G. Kevrekidis and J. Cuevas, Breathers in oscillator chains with Hertzian interactions, Physica D 251 (2013), 39-59.
[3] B. Bidégaray-Fesquet, E. Dumas and G. James, From Newton's cradle to the discrete p-Schrödinger equation, SIAM J. Math. Anal. 45 (2013), 3404-3430.
[4] L. Liu, G. James, P. Kevrekidis and A. Vainchtein, Nonlinear waves in a strongly resonant granular chain (2015), arXiv:1506.02827 [nlin.PS].

Min Ji: *An integral identity, measure estimates for stationary Fokker-Planck equations.*

We consider the stationary Fokker-Planck equations in a general domain of $\mathbb{R}^n$ with L^p_{loc} drift term and $W^{1,p}_{loc}$ diffusion term for any $p > n$. Fokker-Planck equation is a PDE of parabolic type, which comes from the stochastic differential equation. By deriving an integral identity, we give several measure estimates for stationary solutions in an exterior domain with respect to the diffusion and Lyapunov-like or anti-Lyapunov-like functions. These estimates will be useful to problems such as the existence and non-existence of stationary solutions in a general domain as well as the concentration and limit behaviors of stationary solutions as diffusion tends to zero.

Michael Li: *Threshold results for deterministic and stochastic epidemic models.*

A hallmark of deterministic epidemic models is the threshold result: the disease dies out if the basic reproduction number R_0 is less than 1, and the disease persists if $R_0 > 1$. Standard stochastic epidemic models, on the other hand, predicts that the disease always dies out with probability 1 as $t \to \infty$. Such a difference in the predicted long-time behaviours by deterministic and stochastic models was also present in chemical kinetic dynamics and known as Keizer's Paradox.

We incorporated stochastic disease incidences into a standard continuous time Markov chain model of SIS type for the transmission of an infectious disease, and considered the associated master equation. The disease-free state is no longer absorbing and a positive stationary distribution (PSD) exists and is globally asymptotically stable. By examining the profile of the PSD as the population size N increases, we prove that, in a Limit Threshold Theorem, the sharp threshold result of deterministic epidemic models can be obtained from a class of stochastic models as $N \to \infty$, in the sense of a "thermodynamic limit".

Kening Lu: *SRB measures, entropy, and horseshoes for infinite dimensional dynamical systems.*

This talk contains three parts: (1) the existence of SRB measures and their properties for infinite dimensional dynamical systems; (2) The existence of strange attractors with SRB measures for parabolic PDEs undergoing Hopf bifurcations driven by a periodic forcing with applications to the Brusselator; (3) Positive entropy implying the existence of horseshoes for infinite dimensional dynamical systems. The first part is based on joint works with Zeng Lian and Peidong Liu, the second part is a joint work with Qiudong Wang and Lai-Sang Young, and the third part is a joint work with Wen Huang.

John Mallet-Paret: *Regularity of Solutions of Delay Differential Equations: C^∞ versus Analytic.*

While delay differential equations with variable delays may have a superficial appearance of analyticity, it is far from clear in general that a global bounded solution $x(t)$ (namely, a bounded solution defined for all time t, such as a solution lying on an attractor) is an analytic function of t. Indeed, very often such solutions are not analytic, although they are often C^∞. In this talk we provide sufficient conditions both for analyticity and for non-analyticity (but C^∞ smoothness) of such solutions. In fact these conditions may occur simultaneously for the same solution, but in different regions of its domain, and so the solution exhibits co-existence of analyticity and non-analyticity. In fact, we show it can happen that the set of non-analytic points t of a solution $x(t)$ can be a generalized Cantor set.

Konstantin Mischaikow: *Dynamic signatures generated by regulatory networks.*

Consider a regulatory network presented as a directed graph with annotated edges that indicate if the first node is up-regulating or down-regulating the second node. What kind of dynamics can this network generate? While this may seem to be an inadequately posed question it arises fairly often in biological contexts. Our motivation for addressing it arises from gene regulatory networks where we assume that the nodes represent genes and act as switches. However, we do not assume that we know the appropriate parameter values let alone the nonlinear reactions that govern the switches. Nevertheless, as I will describe in this talk, for moderate sized networks we can give a mathematically justifiable, computationally tractable, description of the global dynamics for a large class of ode models and a wide range of parameter values.

Michela Procesi: *Quasi-periodic solutions with beating effects for the quintic NLS on the circle.*

I will discuss the existence and linear stability of a class of quasi-periodic solutions for the quintic NLS on the circle. Such solutions arise from the resonances of the NLS normal form and exhibit a periodic transfer of the Sobolev norm between Fourier modes.

This is a joint work with E. Haus.

Carlos Rocha: *Sturm global attractors which are 3-balls - Part 1.*

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on the unit interval with Neumann boundary conditions. This follows the grand tradition of Brunovský, Chafee, Conley, Henry, Hale, Infante, and many others. We assume all equilibria to be hyperbolic. Their unstable manifolds decompose $\mathcal{A}$ into a regular cell complex. (This is false, in general, but a theorem for Sturm global attractors $\mathcal{A}$.)

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Armen Shirikyan: *Global controllability to trajectories for the viscous Burgers equation*

We study the problem of global controllability by an external force for the viscous Burgers equation on a bounded interval. Assuming that the force is localised in space, we prove that any non-stationary trajectory can be exponentially stabilised. We next discuss various consequences of this result, including global exact controllability to trajectories and approximate controllability by a localised low-dimensional control.

Sjoerd Verduyn Lunel: *Perturbation theory for semigroups of operators that are not strongly continuous (with an application to show that Jack Hale was right after all)*

A neutral delay differential equation can be written as a system of a retarded delay differential equation coupled with a difference equation. In contrast to the case for retarded delay equations, there is not yet an effective perturbation theory for difference equations and more general for delay differential-algebraic equations available.

For a given neutral delay differential equation, one defines a semigroup of operators by shifting along the solution. The need to develop a perturbation theory for such solution semigroups arises in the context of stability and control problems. The answer employs the variation-of-constants formula, which involves integration in a Banach space. The reason that there does not yet exist a perturbation theory for delay differential-algebraic equations is related to the fact that in general solutions of such equations lack smoothness properties, see [3]. A similar problem arises if one would like to study retarded differential equations on spaces of discontinuous functions.

The aim of this talk is to develop a perturbation theory for semigroups of operators that are not necessarily strongly continuous. A first key idea is to describe a perturbation at the generator level by a cumulative output map [1] with finite dimensional range and to construct the semigroup by solving a linear finite dimensional convolution equation. Once this is done, all that remains is to explicate the integral in the variation-of-constants formula in the nonsmooth case. Already in 1953 Feller [2] emphasized that one might use the Lebesgue integral for scalar valued functions of time obtained by pairing a linear semigroup acting on an element with an element of the dual space and that there is no need to require strong continuity. More recently this point of view was elaborated by Kunze [4] who defined a Pettis type integral in the framework of a norming dual pair of spaces. We combine and develop these ideas further to arrive at a perturbation theory for semigroups that are not necessarily strongly continuous.

We illustrate our perturbation with some results about stability and control of difference equations with applications to boundary control of partial differential equations, and finally we will illustrate why Jack Hale was right in his treatment of delay differential equations involving the discontinuous fundamental solution in the variation-of-constants formula.

This is joint work with Odo Diekmann.

[1] O. Diekmann, M. Gyllenberg and H.R. Thieme, Perturbing semigroups by solving Stieltjes Renewal Equations, Diff. Int. Equa. 6 (1993) 155-181.
[2] W. Feller, Semi-groups of transformations in general weak topologies, Ann. of Math. 57 (1953) 287-308.
[3] J.K. Hale and S.M. Verduyn Lunel, Effects of small delays on stability and control, In: Operator Theory and Analysis, The M.A. Kaashoek Anniversary Volume (eds. H. Bart, I. Gohberg and A.C.M. Ran), Operator Theory: Advances and Applications 122, Birkhäuser, 2001.
[4] M. Kunze, A Pettis-type integral and applications to transition semigroups, Czech. Math. J. 61 (2011) 437-459.

Sergey Zelik: *Smoothness of inertial manifolds revisited.*

It is well-known that the dissipative systems generated by semi-linear parabolic equations possess inertial manifolds if the so-called spectral gap condition is satisfied. However, the obtained manifolds are usually only C^1-smooth and for more regular (say, C^2) manifolds, we need essentially stronger spectral gap conditions which are not satisfied in general for parabolic equations generated by elliptic differential operators. In the present talk we show that the problem may be overcome if we relax the invariance property by preserving the dynamics on the attractor only and allow to correct the vector field in the directions transversal to the attractor. Then, for every given k, we show how to construct a C^k-smooth inertial manifold with exponential tracking property if the usual spectral gap condition is satisfied.

SHORT TALKS

Alex Blumenthal: *Entropy, volume growth and SRB measures for Banach space mappings.*

In this talk, we discuss the extension of the characterization of SRB measures, as those for which volume growth on the unstable foliation coincides with metric entropy, to the setting of "dissipative" Banach space mappings admitting a compact attractor. This work generalizes previously known results, due to Ledrappier and others, for diffeomorphisms of finite dimensional Riemannian manifolds, and is applicable to dynamical systems defined by large classes of dissipative parabolic PDEs.

Emphasis will be placed on potential complications in the infinite-dimensional setting, including: (1) the lack of a natural, differentiably-varying d-dimensional volume element; and (2) the derivative of the mapping at a point may exhibit arbitrarily strong contraction, e.g., if derivatives are compact linear operators.

This is a joint work with Lai-Sang Young.

Pavol Brunovský: *Dichotomy, the closed range theorem and optimal control.*

Necessary conditions for infinite horizon optimal controls problem can be obtained by the alternative theorem. This theorem requires that the range of a shift operator on a functional space is closed. It will be shown that this is the case if the dynamics of the problem is hyperbolic but may fail to be so if it is not.

Alexandre Carvalho: *Structural stability of uniform attractors: topological and geometrical.*

We present a careful description of the relationship between pullback and uniform attractors, leading to a detailed description of the uniform attractor and providing the understanding of its dynamical structures. That description is used to show continuity (upper and lower semicontinuity) and structural stability (topological and geometrical) of uniform attractors, at least for a non-autonomous perturbation of a semigroup.

Alain Haraux: *Some simple problems for the next generations.*

A list of open problems on global behaviour in time of some evolution systems, mainly governed by P.D.E, is given together with some background information explaining the context in which these problems appeared.

Russell Johnson: *Generalized reflectionless potentials and the K-dV equation.*

The set GR of generalized reflectionless Schroedinger potentials was introduced and studied by Lundina in 1985. It contains the classical reflectionless potentials, and (certain translates of) the algebro-geometric potentials of Dubrovin-Novikov-Matveev. Basic facts concerning GR were stated by Marchenko in 1991. About ten year later, Kotani used the infinite-dimensional Grassmannian of Sato-Segal-Wilson to show that each potential q(x) in GR gives rise to a solution u(t,x) of the Korteweg - de Vries equation, with initial value q(x), which is meromorphic in the complex (t,x) plane.

We will consider stationary ergodic subsets of GR which consist of potentials which are reflectionless in the sense of Craig. It turns out that one can determine subsets of this type which are counterexamples to the so-called Kotani-Last conjecture; they are almost automorphic in the sense of Bochner-Veech, but not almost periodic. Other subsets of this type have a Lyapunov exponent with singular behavior. We will discuss as many examples as time permits.

Hüseyin Koçak: *Existence of connecting orbits with numerics.*

Several general results for establishing the existence of connecting orbits from numerically computed approximate such orbits will be presented. Connecting orbits will include, transversal homoclinic points, connecting orbits to periodic orbits, orbits connecting equilibria, and Silnikov orbits. Examples of various connecting orbits will be demonstrated on specific systems of maps and ordinary differential equations.

Xiao-Biao Lin: *Codiagonalization of matrices and existence of multiple homoclinic solutions.*

Waves in periodic media can be modeled by periodically perturbed homoclinic solutions. Consider a degenerate homoclinic orbit γ asymptotic to a hyperbolic equilibrium, and the linear variational equation around γ with $d \geq 1$ bounded linearly independent solutions. The case $d = 1$ has been extensively studied. In 1984, Hale proposed to study the degenerate cases where $d \geq 2$. In this talk, we will present two general methods to study systems of two quadratic equations. We show that the quadratic system can have up to 4 real valued solutions. We then use our result to study periodically perturbed homoclinic orbits with $d = 3$. Among the three bifurcation equations, one is due to the homoclinic tangency along the orbital direction. Under generic conditions on perturbations of the normal and tangential directions of the homoclinic orbit, we show up to 4, or 8 homoclinic orbits can be created.

Carmen Núñez: *Dissipativity in nonautonomous linear-quadratic control processes.*

This talk concerns the concept of dissipativity in the sense of Willems for nonautonomous linear-quadratic (LQ) control systems. A nonautonomous system of Hamiltonian ODEs can be associated with such an LQ system, and the analysis of the corresponding symplectic dynamics provides valuable information on the dissipativity properties. The presence of exponential dichotomy, the occurrence of weak disconjugacy, and the existence of nonnegative solutions of the Riccati equation provided by the Hamiltonian system are closely related to the presence of (normal or strict) dissipativity and to the definition of the (normal or strong) storage functions.

This is a joint work with: Roberta Fabbri, Russell Johnson, Sylvia Novo and Rafael Obaya.

Sergio Oliva: *Human mobility in epidemic models and non-local diffusions*

Following Brockmann, where human mobility is introduced in a simple SIR model, we get a Reaction Diffusion equation with fractional power diffusion. The first interesting mathematical and epidemiological question is how to characterize the existence of positive equilibrium in these equations. We also present a correlation network between occurrences of reported cases of dengue between cities in the state of Rio de Janeiro-Brazil.

Hildebrando Rodrigues: *Synchronization and applications.*

The object of this lecture is to study some applications of Synchronization to nonlinear models, most of them chaotic, in the areas of Engineering, Physics, Biology, etc. Special emphasis will be given to Communication Systems where chaotic models will be used to codify and to decode signals. Some simulations and some videos will be presented. Also some mathematical methods that are used to prove synchronization will be discussed.

Kunimochi Sakamoto: *Diffusion in the bulk and non-diagonal flux on the boundary lead to Turing type instabilities.*

Turing's diffusion-induced-instability for reaction-diffusion systems requires the presence of a substantial difference in the diffusion rates between *activator* and *inhibitor* species. This "restriction" universally applies to reacting and diffusing systems in which no interactions are involved *on the boundary* of domain, i.e., under *no flux boundary conditions.* In many cell-biological systems, however, relevant reagents have nearly equal diffusivities, and hence, the traditional Turing-mechanism may not be applied to such systems in order to account for the emergence of spatial or temporal inhomogeneities out of uniform states.

Levine and Rappel may be the first to point out that a mechanism of diffusion induced destabilization is still operative in diffusive systems with equal diffusivity, provided that suitable interactions on the boundary (cell membrane) are taken into consideration. They proposed a hypothetical toy model of reaction-diffusion equations with boundary flux-interaction. Together with linear stability analysis, numerical simulations are performed on this model to display diffusion-induced-instabilities of steady and oscillatory types under the equal diffusivity of reagents.

The problem to be discussed in this talk is slightly different from, but related to, the model treated by Levine and Rappel.

Let $\Omega \subset \mathbb{R}^m$ $(m \geq 1)$ be a bounded domain with smooth boundary $\partial\Omega$. We consider the following system of diffusion equations under the *Robin type boundary conditions*

$$\partial_t \boldsymbol{u} = D \triangle \boldsymbol{u} \quad \text{in } \Omega, \qquad D\partial_{\boldsymbol{n}} \boldsymbol{u} = J\boldsymbol{u} \quad \text{on } \partial\Omega, \tag{1.1}$$

for the vector of N-species $\boldsymbol{u}(t,x) = (u_1(t,x), \ldots, u_N(t,x))$.

Based on my previous work, this talk illustrates several characteristic features of stability results for (1.1), as summarized as follows.

(1) If the mass transfer matrix J is stable (respectively, unstable), then (1.1) *tends to be* stable (respectively, unstable).
(2) More precisely, for a *symmetric* matrix J, (1.1) is stable if and only if J is stable. In this case, (1.1) is a gradient system and the associated eigenvalue problem is variational, implying that the eigenvalues are real numbers.
(3) If J is stable (but not necessarily symmetric) and D is close to a scalar matrix (nearly equal diffusivities), then (1.1) is stable.
(4) If J is unstable (but not necessarily symmetric) with a positive eigenvalue and D is close to a scalar matrix (nearly equal diffusivities), then (1.1) is unstable.

In (3) and (4) above, the condition that *D must be close to a scalar matrix* is in general indispensable.

Siniša Slijepčević: *Entropy of Lagrangian systems and invariant sets of extended gradient systems.*

We consider in parallel dynamical properties of finite dimensional Lagrangian systems, and properties of formally gradient dynamics of its action functional. The considered example is a family of Arnold's-like Lagrangians in two and a half degrees of freedom, and the associate extended (or formally-) gradient system - a pair of coupled reaction-diffusion equations considered on an unbounded domain. (The techniques, though, seem to be applicable in much more general cases).

For Lagrangian systems, we consider the problem of constructing "diffusion" orbits of infinite length, and positive entropy invariant measures.

Our aim is towards developing a quantitative version of the variational approach to Arnold's diffusion by Bernard, Cheng, Kaloshin, Yan and Zhang. We construct diffusion orbits and positive entropy measures whenever there is no topological obstruction to diffusion, and estimate the diffusion time and the topological entropy in terms of a certain bound Δ_1 on the variation of the action, obtained by an application of the infinite-dimensional Morse-Sard theorem. The method seems optimal: for small values of the Arnold's parameter μ, we get topological entropy to be $O(\mu)$, which corresponds to optimally "fast" diffusion orbits which change momentum by $O(1)$ for the time $O(\mu\,|\log\mu|)$.

The construction method is new, and relies on constructing lots of uniformly-local invariant sets for the associated evolutionary equations on an unbounded domain. We apply an abstract theory partly jointly developed with Thierry Gallay, and construct invariant sets by a precise study of the interplay of the "energy" dissipation and flux, energy in this case being the Lagrangian action.

Joan Solà-Morales: *Convergence to steady-state and boundary layer profiles in a linear chromatography system.*

These are results of an ongoing joint work with J. Menacho. They are mostly contained in a paper of the same title appeared in SIAM J. Appl. Math. 75 (2015), no. 2, pp.

745–761. We study the hyperbolic system of equations of the so called Linear Transport Model in a True Moving Bed chromatography device with four ports. By using methods based on a suitable energy-functional we show that all solutions approach exponentially a unique steady-state solution. Then, with the use of Asymptotic Analysis techniques we calculate the limit profiles of these steady-state solutions when the mass transfer coefficient between the liquid and solid phases tends to infinity. Along this singular limit sharp boundary layers appear near some ports. We are able to obtain explicit and simple formulas for these limit profiles.

Hans-Otto Walther: *Local invariant manifolds for delay differential equations with states in the Fréchet space* $C^1((-\infty,0],\mathbb{R}^n)$.

Differential equations with variable time lags, state- or time-dependent, which are not necessarily bounded require initial data on $(-\infty,0]$. Banach spaces of such data would exclude certain solutions with rapid growth at $-\infty$. This suggests to try the spaces of all C^k-maps $(-\infty,0]\to\mathbb{R}^n$, with compact-open topologies. Results for the case of bounded delay indicate that differentiability of solutions with respect to initial data can be expected in $C^1=C^1((-\infty,0],\mathbb{R}^n)$.

Indeed, equations $x'(t)=f(x_t)$ with $f:C^1\supset U\to\mathbb{R}^n$ slightly better than being continuously differentiable (in the weak sense of Michel and Bastiani, which is appropriate in Fréchet spaces) define a semiflow of continuously differentiable solution operators on the Fréchet manifold $X_f=\{\phi\in U:\phi'(0)=f(\phi)\}$. The smoothness hypothesis allows for examples which involve unbounded delay - but implies that f is of *locally bounded delay* in an abstract sense. In case of examples like $x'(t)=g(x(t-d)),\ d=d(x(t))$ the latter property comes down to $d:\mathbb{R}\to[0,\infty)$ being locally bounded.

At stationary points of the semiflow there are local invariant manifolds. These are found not by adapting the Lyapunov-Perron- or Hadamard approach but by means of embeddings and transversality. For this *locally bounded delay* is instrumental.

[1] H. O. Walther, *Semiflows for differential equations with locally bounded delay on solution manifolds in the space* $C^1((-\infty,0],\mathbb{R}^n)$. Topological Methods in Nonlinear Analysis, to appear.
[2] H.O. Walther, *Local invariant manifolds for delay differential equations with state space in* $C^1((-\infty,0],\mathbb{R}^n)$. Preprint, 2016.

POSTERS

Jorge Gonzalez: *Parametrization method for stable/unstable manifolds of periodic points for maps.*

The Parameterization Method is a general functional analytic framework for studying invariant manifolds of dynamical systems. We develop a version of the method for stable/unstable manifolds associated with periodic points of discrete time dynamical systems. The novelty of our approach is that by introducing new variables we are able to avoid computing compositions of the map. We describe the method in general and implement the method for some one and two dimensional manifolds in some two and three dimensional dynamical systems.

The rigorous validations of our numerical computations are established using functional analysis techniques and relying on the Radii Polynomial Theorem. This computer-assisted tool is a Newton-Kantorovich type argument tailored to the field.

This is a joint work with J.D Mireles James.

Jonathan Jaquette: *Stability and uniqueness of slowly oscillating periodic solutions to Wright's equation.*

Jones conjecture states that when $\alpha > \pi/2$, then there exists an unique slowly oscillating periodic solution (SOPS) to Wrights equation

$$y'(t) = -\alpha y(t-1)[1 + y(t)].$$

While the existence of SOPS has been known since 1962, the conjecture has yet to be proven in whole. To prove uniqueness, it is sufficient to prove that all SOPSs to Wrights equation are asymptotically stable. By developing asymptotic bounds on the behavior of SOPS and then estimating their Floquet multipliers, in 1991 Xie was able to prove the conjecture for $\alpha > 5.67$. We sharpen Xies result by using rigorous numerics to estimate the Floquet multipliers of SOPS to Wrights equation. In this manner we are able to make progress on a long standing conjecture in delay differential equations.

This is a joint work with Konstantin Mischaikow and Jean-Philippe Lessard.

Shane Kepley: *Rigorous integration of material surfaces.*

The evolution of a particle advected by an analytic vector field can be expressed as a Taylor series on some interval in time. It is reasonable to expect that a higher dimensional smooth manifold of initial conditions should be analytic in both space and time. We show how the evolution of such a surface can be computed as a sequence of multi-variate Taylor coefficients embedded in an appropriate Banach algebra. The computation is made rigorous by defining a suitable Newton-like operator on this algebra and proving it is a contraction. An important feature of our method is that non-autonomous flows are no harder to integrate than autonomous flows. We will illustrate the method with a 1-dimensional non-autonomous example.

John Jinho Kim: *Numerical computation of a connecting orbit based on the principle of Wazewski.*

The principle of Wazewski states that if there is a bounding region for the differential equation $\dot{x} = f(x)$ that satisfies certain hypotheses, then there exists a solution inside the region all the time. This has been a major tool to show the existence of traveling waves in many areas of mathematics such as predator-prey models, chemical reactions, and thermodynamics.

The purpose of this research is twofold. The first part is to develop a shooting method based on the principle of Wazewski to numerically compute the heteroclinic and homoclinic solutions of the dynamical system, and determine the speed of convergence in terms of eigenvalues and the angles of intersection between stable and unstable manifolds of the equilibria.

The second part is to prove an inverse of the principle of Wazewski for some specific cases. Given the differential equation $\dot{x} = f(x)$ in $\mathbb{R}^3$, if there is a heteroclinic orbit formed by the intersection between the unstable manifold of E^- and the stable manifold of E^+, then the trapping region containing such an orbit can be constructed.

This poster is based on the thesis in North Carolina State University under direction of Professor Xiao-Biao Lin.

Anna Kostianko: *Inertial manifolds for the 3D modified Leray-α (ML-α) model with periodic boundary conditions*

The existence of an inertial manifold for the ML-α model (which is one of the truncated versions of the Navier-Stokes equations) with periodic boundary conditions in three-dimensional space is proved by using the so-called spatial averaging principle. This method was introduced by G. Sell and J. Mallet-Paret in order to treat reaction-diffusion equations on a 3D torus. The novelty of this work lies in the application of the spatial averaging principle to the system of equations, which cannot be done in general situation but appears to be possible in our case, because the spatial averaging of the derivative of the non-linearity of ML-α model is identically zero. Furthermore, the adaptation of classical Perron method for constructing inertial manifolds is proposed for the particular case of zero spatial averaging.

Carlos Quesada Gonzalez: *Higher order parabolic problems in uniform spaces*

We consider $2m$-th order parabolic problems in $\mathbb{R}^N$ where the main operator is a power of the Laplacian. The equations we will consider have linear or nonlinear perturbations of the form $h(x; D^a u)$ with $a < 4$ and h in the uniform Lebesgue spaces $\dot{L}^p_U(\mathbb{R}^N)$ to be described.

We also introduce the uniform Bessel spaces $\dot{H}^{\gamma,q}_U(\mathbb{R}^N)$ for $1 < q < \infty$. Then, we want to find the range of γ such that, for $u_0 \in \dot{H}^{4\gamma,q}_U(\mathbb{R}^N)$, $1 < q < \infty$, the problem is well posed. We prove that $\gamma \in I$, where I is an interval depending only on a, p, q and N.

Furthermore the solution is given by an analytic semigroup $S(t)u_0$ and satisfies smoothing estimates between the space of initial data an the space of the solutions.

Christian Reinhardt: *Validated computation of unstable manifolds for parabolic PDEs.*

This poster presents a method for computing polynomial approximations together with explicit a-posteriori error bounds of unstable manifolds associated to equilibrium solutions of parabolic PDEs posed on compact domains with suitable boundary conditions. These polynomials have a finite number of variables, even though they map into an infinite dimensional state space. The approach is based on the so-called parametrization method and builds on explicit knowledge of the spectral stability data at the equilibrium solution that we also obtain via validated numerical methods. We implement the method numerically, and by combining analytical a-posteriori error estimates with careful management of floating point round-off errors we obtain mathematically rigorous bounds on the truncation and discretization errors associated with our polynomial approximation. One of the motivations for the development of this method is the validated computation of connecting orbits for PDEs. We illustrate the approach with applications to Fisher's equation. Also we comment on the challenges associated to the generalization to non-compact domains.

This is joint work with Jason Mireles-James and Jan Bouwe van den Berg.

Dario Valdebenito: *Quasiperiodic solutions to nonhomogeneous elliptic equations in* $\mathbb{R}^d$.

We establish sufficient conditions to obtain solutions of

$$-\Delta u + u_{yy} + a_1(x)u + bf(x,u) = 0 \ \text{ in } \ \mathbb{R}^{N+1} ,$$

which are quasiperiodic in $y \in \mathbb{R}$ and decaying in $x \in \mathbb{R}^N$, uniformly in y. Here $f(x,u) = \mathcal{O}(u^2)$ as $u \to 0$. Such solutions are found using a center manifold reduction and KAM theory. These conditions are satisfied for a large class of sufficiently smooth radial potentials a_1 and radial nonlinearities f.

Manuel Villanueva Pesqueira: *Homogenization in thin domains with oscillatory boundaries.*

We analyse the behaviour of solutions to Poisson's equation with Neumann boundary conditions posed in thin domains with oscillatory boundaries. We consider thin domains with the following general structure

$$R^\varepsilon = \{f(x;y) \in \mathbb{R}^2 \ : \ 0 < x < 1 \ ; \ -\varepsilon H_\varepsilon(x) < y < \varepsilon G_\varepsilon(x)g\}$$

where $G_\varepsilon, H_\varepsilon : \mathbb{R} \to \mathbb{R}$ are two bounded positive functions which oscillate as ε tends to zero. We are particularly interested in understanding how the geometry of the domain affects the behaviour of the solutions. Hence, we obtain the homogenised limit problem as the thickness tends to zero and provide some corrector results for different kind of oscillatory behaviours. In particular, we study the periodic case assuming very mild hypothesis on the regularity of the thin domain and, beyond the periodic case, we consider locally periodic oscillatory boundaries and thin domains where the top and the bottom boundary present oscillations with different profile and different order of frequency.

Joseph D Walsh: *Blending Continuous and Discrete Methods to Compute Optimal Transport Partitions*

Optimal transportation can be used to partition a continuous region based on a set of discrete points, but such partitions are difficult to determine exactly, and they have been nearly as hard to approximate numerically. We propose a new numerical technique, combining discrete and continuous methods, that generates faster and more accurate results than existing techniques. By considering the optimal transportation problem as an optimal coupling problem, we developed a semi-stable descent method and combined that with a network technique that exploits region continuity to reduce complexity.