**Semiclassical Analysis and Non-Self-Adjoint Operators**
**14 - 18 December, 2015**

**Anne-Sophie De Suzzoni: Turning invariant measures on the torus into invariant measures on the line by passing to the limit.**

In this talk, I will explain how to pass from the existence of a strongly invariant measure on the torus for an Hamiltonian equation to a weakly invariant measure on the real line by letting the size of the torus, L, go to infinity. We shall give a meaning to "weakly invariant". The strategy uses two main ingredients : first, what one might refer to as the Prokhorov-Skorohod machinery, which ensures that with quite rough, but uniform in L, estimates on invariant measures on the torus of size L, we get a weakly invariant measure on the line ; then, we use the Feynman-Kac theorem to get these estimates. These ideas apply to a general range of equations. I will give an example with the Schrödinger equation and comment upon the result.

**Semyon Dyatlov: Spectral gaps via additive combinatorics.**

A spectral gap on a noncompact Riemannian manifold is an asymptotic strip free of resonances (poles of the meromorphic continuation of the resolvent of the Laplacian). The existence of such gap implies exponential decay of linear waves, modulo a finite dimensional space; in a related case of Pollicott–Ruelle resonances, a spectral gap gives an exponential remainder in the prime geodesic theorem.

We study spectral gaps in the classical setting of convex co-compact hyperbolic surfaces, where the trapped trajectories form a fractal set of dimension $2\delta + 1$. We obtain a spectral gap when $\delta = 1/2$ (as well as for some more general cases). Using a fractal uncertainty principle, we express the size of this gap via an improved bound on the additive energy of the limit set. This improved bound relies on the fractal structure of the limit set, more precisely on its Ahlfors-David regularity, and makes it possible to calculate the size of the gap for a given surface.

**JeanPierre Eckmann: Classical Hamiltonian Systems, Driven out of Equilibrium, a Review.**

I will try to explain, starting from the historical papers, what has been understood (and not understood) from a rigorous point of view, about classical Hamiltonian systems. For example a chain of massive bodies, or a coupled set of rotators, which are driven out of equilibrium by stochastic forces acting on the ends of the chain.

**Jeremy Faupin: Scattering theory for Lindblad operators.**

In this talk, I will consider a quantum particle interacting with a target. The target is supposed to be localized and the dynamics of the particle is supposed to be generated by a Lindbladian acting on the space of trace class operators. I will discuss scattering theory for such models associated to a Lindblad operator. First, I will consider situations where the incident particle is necessarily scattered off the target, next situations where the particle may be captured by the target. An important ingredient of the analysis consists in studying scattering theory for dissipative operators on Hilbert spaces.
This is joint work with Marco Falconi, Juerg Froehlich and Baptiste Schubnel.

**Alain Grigis : Resonance widths for general Helmholtz resonators with straight neck.**

We prove an optimal exponential lower bound on the width of the resonance associated to the first eigenvalue of the cavity for a general Helmholtz resonator with straight neck, in any dimension.
This is a joint work with Thomas Duyckaerts and André Martinez.

**Colin Guillarmou: Invariant distributions and injectivity of X-ray transform for Anosov flows.**

For Anosov flows with an invariant smooth measure, we discuss the cohomological equation and the regularity issues by using new methods coming from microlocal analysis, and by introducing a new operator giving a sense to integrals of a function over flow orbits. We use this to show injectivity of X-ray transform for surfaces S with Anosov geodesic flows and existence of invariant distributions on the unit tangent bundle of S with prescribed pushforward to S.

**Maxime Ingremeau: Distorted plane waves in chaotic scattering.**

Distorted plane waves, also called Eisenstein functions, are a family of eigenfunctions of a Schrödinger operator which are not square integrable. We will study the behaviour in the semiclassical limit of distorted plane waves on manifolds that are Euclidean near infinity, under the assumption that the underlying classical dynamics has a hyperbolic trapped set, and that some topological pressure is negative ; we will emphasize the case of manifolds of nonpositive curvature.

**Oana Ivanovici: Dispersion estimates for the wave and the Schrodinger equations outside strictly convex obstacle.**

We consider the wave and the Schrodinger equations with Dirichlet boundary conditions outside a (general) strictly convex obstacle in $R^d$ : if d=2 or d=3 we show that the dispersive estimates do hold like in the flat case, while for d>3, we show that there exist strictly convex obstacles for which a loss occur with respect to the boundary less case (such an optimal loss is obtained by explicit computations). This is a joint work with Gilles Lebeau.

**Frédéric Klopp: Stark-Wannier ladders and cubic exponential sums.**

The talk is devoted to Stark-Wannier ladders i.e. the resonances of a one dimensional periodic operator in a constant electric field. These periodic sequences of points in the lower half of the complex plane have been conjecture to be very sensitive to the number theoretical properties of the electric field. Computing the asymptotics of the reflection coefficients in the case of a simple 1-periodic potential, we relate the resonances to cubic exponential sums in which the frequency is computed from the electric field. In the case of rational frequency, we derive "large imarginary part" asymptotics for the resonances.
The talk is based on joint work with A. Fedotov (St Petersburg).

**Rémi Léandre: Malliavin Calculus of Bismut type for an operator of order four on a Lie group.**

This talk enters in the philosophy of the beautifull probabilistic index theory of Bismut. With path integrals, we see the formulas which are simpler to check by the theory of parabolic equation. In this framework, we had translated in semi-group theory the Malliavin Calculus of Bismut type for diffusion (Path integrals are given by the theory of stochastic differential equations). We continue in this direction by giving in this talk an adaptation of the Malliavin Calculus of Bismut type for a four order operator on a Lie group. Path integrals are not well defined as measures. Since we have done an adaptatation of Wentzel-Freidlin estimates for a four order operator with the classical normalization of W.K.B. semi-classical analysis (valid here only for the semi-group!), we can do the marriage of these large deviation estimates and the Malliavin Calculus in order to get Varadhan estimates for the involved heat-kernel as application.

**André Martinez: Estimates on the molecular dynamics for the predissociation process.**

We study the survival probability associated with a semiclassical matrix Schrödinger operator that models the predissociation of a general molecule in the Born- Oppenheimer approximation. We show that it is given by its usual time-dependent exponential contribution, up to a reminder term that is exponentially small (in the semiclassical parameter) with arbitrarily large rate of decay. The result applies in any dimension, and in presence of a number of resonances that may tend to infinity as the semiclassical parameter tends to 0.
This is a joint work with Ph. Briet.

**Shu Nakamura: High energy asymptotics of the scattering matrix for Schroedinger and Dirac operators.**

We consider short-range perturbations of elliptic operators on $R^d$ with constant coefficients, and study the asymptotic properties of the scattering matrix as the energy tends to infinity. We give the leading terms of the symbol of the scattering matrix. The proof employs semiclassical analysis combined with a generalization of the Isozaki-Kitada theory on time-independent modifiers. We also consider scattering matrices for 2 and 3 dimensional Dirac operators.
(joint work with Alexander Pushnitski (King 's College London)

**Galina Perelman: Near soliton dynamics for the energy critical NLS.**

We consider the focusing energy critical nonlinear Schrodinger equation in Rd with radial initial data close to a ground state. We show that for d sufficiently large and during their lifespan, the solutions stay close in the energy space to the ground state family, are global and scatter? to a member of this family.

**Vesselin Petkov: Location and Weyl formula for the eigenvalues of non self-adjoint operators.**

We study the location of the complex eigenvalues of two non self-adjoint operators. The first one is the generator of a semigroup of contractions related to the wave equation with dissipative boundary conditions. The second one is a matrix non self-adjoint operator related to the interior transmission eigenvalues (ITE). The (ITE) are related to the inverse scattering problems for the reconstruction of the form of non convex obstacles by the far filed operator. We show that the eigenvalues of these non self-adjoint operators lie in parabolic neighborhoods of the real axis or the imaginary one. We obtain also a Weyl formula with remainder for the counting function of complex (ITE). The proof is based on a new approach working without an application of Tauberian theorems. The remainder depends on the eigenvalue-free region. The results for the (ITE) are obtained in a joint work with G. Vodev.

**Claude-Alain Pillet: Nonequilibrium statistical mechanics of harmonic networks.**

We consider a general network of harmonic oscillators driven out of thermal equilibrium by coupling to several heat reservoirs at different temperatures. The action of the reservoirs is implemented by Langevin forces. Assuming the existence and uniqueness of the steady state of the resulting process, we construct a canonical entropy production functional S(t) which satisfies the Gallavotti-Cohen fluctuation theorem. More precisely, we prove that cumulant generating function of S(t) has a large-time limit e(a) which is finite on a closed interval centered at a=1/2, infinite on its complement and satisfies the Gallavotti-Cohen symmetry e(1-a)=e(a) for all a. It follows from well known results that S(t) satisfies a global large deviation principle with a rate function I(s) obeying the Gallavotti-Cohen fluctuation relation I(-s)-I(s)=s for all s. We also consider perturbations of S(t) by quadratic boundary terms and prove that they satisfy extended fluctuation relations, i.e., a global large deviation principle with a rate function that typically differs from I(s) outside a finite interval. This applies to various physically relevant functionals and, in particular, to the heat dissipation rate of the network. Our approach relies on the properties of the maximal solution of a one-parameter family of algebraic matrix Ricatti equations. It turns out that the limiting cumulant generating functions of S(t) and its perturbations can be computed in terms of spectral data of a Hamiltonian matrix depending on the harmonic potential of the network and the parameters of the Langevin reservoirs. This makes our approach well adapted to both analytical and numerical investigations. This is joint work with Vojkan Jaksic and Armen Shirikyan.

**Duong H. Phong: Non-linear partial differential equations in complex geometry.**

Starting with the search for Kähler-Einstein and Hermitian-Einstein metrics, the last few decades have seen an ever increasing interaction between complex geometry and the theory of partial differential equations. In this talk, we survey some recent developments, including elliptic equations such as Hessian equations and Strominger systems and parabolic equations such as the Ricci flow. The emphasis will be on open problems, and the corresponding a priori estimates.

**Sylvia Serfaty: Mean Field Limits for Ginzburg-Landau Vortices.**

Ginzburg-Landau type equations are models for superconductivity, superfluidity, Bose-Einstein condensation, etc. A crucial feature is the presence of quantized vortices, which are topological zeroes of the complex-valued solutions. We will present a new result on the derivation of a mean-field limit equation for the dynamics of many vortices starting from the parabolic Ginzburg-Landau equation or the Gross-Pitaevskii (=Schrodinger Ginzburg-Landau) equation.

**Petr Siegl: Convergence of pseudospectra, constant resolvent norm and Schrödinger operators with complex potentials.**

We establish the convergence of pseudospectra for closed operators acting in different Hilbert spaces and converging in the generalized norm resolvent sense. As an assumption, we exclude the case that the limiting operator has constant resolvent norm on an open set. We extend the class of operators for which it is known that the latter cannot happen by showing that if the resolvent norm is constant on an open set, then this constant is the global minimum. Finally, we discuss the spectral and pseudospectral approximations of Schrödinger operators with complex potentials by domain truncation.
This is joint work with S. Bögli and C. Tretter

**Petar Topalov: Generic non-selfadjoint Zakharov-Shabat operators.**

In this talk I will study within a family of non-selfadjoint operators depending on a parameter in a real Hilbert space, those with (partially) simple spectrum. As a case study we consider the Zakharov-Shabat operators appearing in the Lax pair of the focusing NLS on the circle. The main result is that the set of such operators is path connected and dense.
(This is a joint work with P. Lohrmann and T. Kappeler.)

**Andras Vasy: The Feynman propagator and its positivity properties.**

In this talk, partially on joint work with Jesse Gell-Redman, Nick Haber and Michal Wrochna, I will explain the properties of the Feynman propagator, i.e. the inverse of the wave operator on 'Feynman function spaces', in various settings, concentrating on Lorentzian scattering metrics (a.k.a. asymptotically Minkowski-like spaces). I will also explain its positivity properties, and the connection to spectral and scattering theory in Riemannian settings.