Model Theory, Difference
Differential Equations and Applications
April 7 -10, 2015

Daniel Bertrand : Galois groups of logarithmic equations : the abelian case.

The Galois groups of logarithmic equations on algebraic D-groups provide an approach to Lindemann-Weierstrass problems. In this joint work with A. Pillay, I will describe how to compute them for the "pure" D-groups attached to D-modules and to abelian varieties. Applications to the image of Manin maps will also be discussed.

Elisabeth Bouscaren : From Manin Mumford to Mordell-Lang via Model Theory.

We will explain how to derive the Mordell-Lang Conjecture for abelian varieties over function fields from the Manin-Mumford conjecture, using model theory but bypassing the Trichotomy theorem in Zariski Geometries. We will focus on the characteristic p case. The proof makes essential use of the model theory of groups of finite (relative) Morley Rank and goes through a result of quantifier elimination for the induced structure on divisible subgroups of the group of $K-$rational points of abelian varieties, when K is a non perfect separably closed field of finite degree of imperfection.
(Joint work with Franck Benoist and Anand Pillay) (2014)

Artem Chernikov: Graph regularity and incidence phenomena in distal structures.

In recent papers by Alon et al. and Fox et al. it is demonstrated that families of graphs with a semialgebraic edge relation of bounded complexity have strong regularity properties and can be decomposed into very homogeneous semialgebraic pieces up to a small error (typical example is the incidence relation between points and lines on a real plane, or higher dimensional analogues). We show that in fact the theory can be developed for families of graphs definable in a structure satisfying a certain model theoretic property called distality, with respect to a large class of measures (this applies in particular to graphs definable in arbitrary o-minimal theories and in p-adics).
(Joint work with Sergei Starchenko.)

James Freitag : On Differential Chow Varieties.

The set of positive cycles of a given variety of some fixed dimension and degree forms an algebraic variety, called a Chow variety. This talk is about establishing the differential algebraic analog. Several years ago, Li, Gao, and Yuan defined the differential Chow form, and began the study differential Chow varieties, whose existence they were only able to establish in very special cases. We answer the natural question left open by their work by establishing the existence of differential Chow varieties. This is joint work with Tom Scanlon and Wei Li.

Ehud Hrushovski : Proper intersections, pseudo-finite fields.

I'll discuss some old and new results on properness of intersections and modularity in structures related to pseudo-finite fields.

Holly Krieger: A case of the dynamical André-Oort conjecture.

In the past few years, conjectures have been made and partial results achieved on unlikely intersections in complex dynamics, following the program initiated by Baker-DeMarco. I will explain their dynamical generalization of the famous André-Oort conjecture on CM points in moduli spaces of abelian varieties. Ghioca, Nguyen, Ye, and myself recently proved the first complete case of this conjecture, for pairs of unicritical polynomials, and I will discuss our result and the connection to invariant subvarieties of P1 x P1, and the structure of the Mandelbrot set.

François Loeser: Trace formulas in non-archimedean geometry.

The Grothendieck-Lefschetz trace formula provides a cohomological interpretation for the number of points of varieties over finite fields. Using motivic integration, I proved with Denef a trace formula that was later generalized by Nicaise and Sebag to varieties over the field of complex Laurent series. I will outline new proofs of these results, that are based on non-archimedean geometry instead of resolution of singularities. This is joint work with E. Hrushovski.

Angus Macintyre: On quotients of models of Peano arithmetic by principal ideals.

Zilber raised a general question about quotients of models of true arithmetic by principal ideals. Can such a quotient ring interpret arithmetic? The answer is no, even in the Peano arithmetic setting. There are various cases, depending on the nature of m, a generator of the ideal. When m is prime, Ax's work gives a clear picture of the definability theory of the quotient. When m is a possibly nonstandard power of a prime, the matter is trickier, and relates to NTP2. The general case involves an internal version of the Mostowski-Feferman-Vaught theorem on products. This is joint work with Paola D'Aquino.

Alice Medvedev: Groups of finite rank in ACFA.

The basic finite groups in ACFA are defined by "$x \in A$ and $(x, \sigma(x)) \in B$" for some algebraic group A and some subgroup B of $A \times \sigma(A)$ projecting dominantly onto both A and $\sigma(A)$. In this talk, we describe some approaches to actually computing the rank of the group from the data A, B. We are especially interested in computing the (usually low) ranks of groups defined by "$x \in A$ and $(x, \sigma^N(x)) \in B$" for large N but fixed A and B.

Samaria Montenegro : Model theory of pseudo real closed fields.

The notion of PAC fields has been generalized by S. Basarab and by A. Prestel to ordered fields. Prestel calls a field M pseudo real closed (PRC) if M is existentially closed (in the language of rings) in every regular extension L to which all orderings of M extend. Thus PRC fields are to real closed fields what PAC fields are to algebraically closed fields. In this talk we will present some results in the model theory of bounded PRC fields (i.e., with finitely many algebraic extensions of degree m, for each m>1). We fix such a field M, and add to the language of rings constant symbols for an elementary substructure; Th(M) is then model complete, M admits only finitely many orders, and these orders are definable. As conjectured by A. Chernikov, I. Kaplan and P. Simon, we show that Th(M) has NTP2, and in fact is strong of burden the number of definable orders on M. This also allows us to explicitly compute the burden of types, and to describe forking. Moreover, we show that a PRC field which has NTP2 must be bounded. [These results were independently obtained by W. Johnson for some particular PRC fields M]. Other results of independent interest are some amalgamation results, and elimination of imaginaries for bounded PRC fields. Some of these results generalize to bounded PpC fields, using the same kind of techniques.

Rahim Moosa: Nonstandard compact complex manifolds with a generic automorphism.

If CCM denotes the theory of compact complex spaces in the langauge of complex-analytic sets, then the theory of models of CCM equipped with an automorphism has a model companion, denoted by CCMA. The relationship to meromorphic dynamical systems is the same as that of ACFA to rational dynamical systems. I will discuss recent joint work with Martin Bays and Martin Hils that begins a systematic study of CCMA as an expansion of ACFA. Particular topics we consider include: stable embeddedness, imaginaries, and the Zilber dichotomy.

Joël Nagloo: On the non-generic Second Painlevé equation.

In this talk we will look at the problem of existence of algebraic relations over $\mathbb{C}(t)$ between solutions of the second Painlevé equation $P_{II}(\alpha):\ y'' = 2y3 + ty + \alpha$ for algebraic $\alpha \notin 1/2 + \mathbb{Z}$. In particular, I will discuss recent progress in showing that if $y_1, \ldots, y_n$ are distinct solutions, then $tr.deg\,(\mathbb{C}(t)(y_1, y_1', \ldots, y_n, y_n')/\mathbb{C}(t)) = 2n$, that is $y_1, y_1', \ldots,\ y_n, y_n'$ are algebraically independent over $\mathbb{C}(t)$.

Seiji Nishioka : Double-angle formulae and algebraic independence .

Looking at the double-angle formula of the cosine function, we find a polynomial in itself of degree 2. The exponential function also has a double-angle formula with a polynomial of degree 2, and Weierstrass function has one with a rational function of degree 4. Additionally, the independent variable has one with polynomial of degree 1. Since we obtained different numbers, 2, 4 and 1, we find that the exponential/cosine function, Weierstrass function and the independent variable are algebraically independent over the field of complex numbers. I will introduce the reason of the above argument and an application to the theory of transcendental numbers.

Anand Pillay: Galois groups of logarithmic equations : the semi-abelian case.

I discuss Galois groups for logarithmic differential equations on semiabelian varieties over $K = C(t)$. The field K generated over K by the kernel of the Manin map plays an important role.
(Joint work with D. Bertrand.)

Françoise Point: Transfer results in topological differential fields.

We consider certain classes of topological differential fields where one can axiomatize the class of its existentially closed members, for instance the class of ordered differential fields. We will consider the question of which properties transfer from the class of their existentially closed reducts (forgetting about the derivation), such as the NIP property, existence of "good" bounds for VC-density of definable sets, existence of a fibered dimension, density of definable types. We will give partial answers in particular in the case of closed ordered differential fields.
(Part of this work is joint with Quentin Brouette.)

Hiroshi Umemura : Quantum Picard-Vessiot theory.

Since the 19th century, we were interested in q-analogues of special functions such as q-hypergeometric function. The Galois group of q-hypergeometric function is an algebraic group and it is not a quantum group. It is natural to wonder why Galois group is not quantized when we consider the q-analoques. Can we expect a quantized Galois theory in which Galois group is a quantum group? The answer seems affirmative. As a first step we propose a quantum Picard-Vessiot theory over a constant base field.

Michael Wibmer: Etale difference algebraic groups.

Difference algebraic groups, i.e, groups defined by algebraic difference equations occur naturally as the Galois groups of linear differential or difference equations depending on a discrete parameter. If the linear equation has a full set of algebraic solutions, the corresponding Galois group is an étale difference algebraic group. Like étale algebraic groups can be described as finite groups with a continuous action of the absolute Galois group of the base field, étale difference algebraic groups can be described as certain profinite groups with some extra structure. I will present a decomposition theorem for étale difference algebraic groups, which shows that any étale difference algebraic group can be build from étale algebraic groups and finite groups equipped with an endomorphism.