

MURRAY A. MARSHALL, A BRIEF MATHEMATICAL OBITUARY

BY SALMA KUHLMANN

Murray Marshall have been updating his homepage at the University of Saskatchewan and his CV (see below) regularly. These contain links to the pdf files of his publications, as well as a listing of supervised graduate students. According to these valuable sources of information, Murray classified his own research contributions into two main areas: 1. Positive Polynomials and Sums of Squares and 2. Orderings, Valuations and Quadratic Forms. In this brief synopsis, we will focus on our joint contributions to the multi-dimensional and infinite moment problem.

In the winter of 1999-2000 Murray gave a series of lectures in our Algebra Seminar at the University of Saskatchewan. We cite the following from the Preface of his little blue book:

The goal of these lectures was to provide an elementary introduction (at a level suitable for first year graduate students) to recent developments in real algebraic geometry related to Schmüdgen's solution of the Moment Problem in 1991.

The little blue book (Positive Polynomials and Sums of Squares, Dipartimento di Matematica Dell' Universita Di Pisa, Dottorato di Ricerca In Matematica 2000) not only served for training our graduate students, but also provided later the core material for his - by now famous and widely used - monograph in the AMS Math. Surveys and Monographs series.

After that memorable seminar, we decided to surprise Murray by organizing a Colloquiumfest at the U of S in the honour of his 60th birthday [24. 03. 2000] and invited Konrad Schmüdgen as a key note speaker. Schmüdgen stated two conjectures: the solvability of the Moment Problem for cylinders with compact base in any dimension, and the saturation of the preorder associated to the two dimensional strip. It was in the summer of 2000 in Saskatoon that, challenged by Schmüdgen's conjectures, our long and fruitful collaboration was born. By the end of the summer we had solved the first conjecture (the second was established by Murray alone several years later).

Murray's interest in the Moment Problem only grew thereafter. While co-supervising Msc student Wei Fan in 2004 we discovered N.I. Akhiezer's book on The Classical Moment Problem. [Murray cherished this book

so much, that he re-assigned readings from it to our current Msc Student Ayoola Isaac Jinadu, more than ten years later]. Murray thus discovered very early A.E.Nussbaum's and T. Carleman's contributions to the Moment Problem. He seemed fascinated by the challenge of circumventing analytic and operator theoretic proofs and providing instead algebraic ones (based on his ideas using appropriate localizations of polynomial algebras). This is evident in his most recent publications in Math. Scandinavica (part II is to appear posthumous).

In the last five years, Murray became excited about the relationship between the topological closure of quadratic modules with respect to various locally convex topologies on the one hand, and the solvability of the Moment Problem for continuous linear functionals on the other. Our intense collaboration together with Mehdi Ghasemi - one of Murray's favourite PhD students - culminated in the discovery of applications to the infinite dimensional Moment Problem (to appear), and in particular to symmetric algebras of locally convex topological vector spaces (submitted, together with Maria Infusino). In this context, Murray became increasingly intrigued by the question of determinacy of the solving measure, as witnessed by the last note that he sent us about this issue (in preparation).

There are still many unsolved open questions related to our most recent collaboration with Murray. We will continue "digging" until we understand. It is appropriate to end by citing from an e-mail message that Murray sent us on April 10 2015 in discussing referee comments on our paper (I removed references for confidentiality):

Finally, I should say that I have no idea how the result in [x] is related to the [y] result in our other paper (although this, of course, is another matter). It seems though that the result in [x] is for general nuclear spaces, whereas the result of [y] is for nuclear spaces of a very special sort. I hope eventually to understand all these things. Murray

Addendum on applications to polynomial optimisation

by Mehdi Ghasemi

Application of the moment problem to polynomial optimization is one of the most beautiful and useful achievements of the area which has been developed extensively in past two decades. Murray has worked on this problem and published several articles. Since 2012, Murray and Ghasemi introduced a new approach and published a series of articles based on geometric programming to provide a lower bound for the global minimum of a polynomial where no other methods are able to do so. His latest work in this series, including a collaboration with J.B Lasserre, generalizes the geometric programming method to constrained polynomial optimization problems and the last one, which deals with optimization over semialgebraic sets is still under review.