Moduli Spaces in Geometry

October 26-30, 2015, CIRM, Luminy, France

Abstracts

Marian Aprodu
Title: Cayley-Chow forms of K3 surfaces and Ulrich bundles
Abstract: (joint work with Gavril Farkas and Angela Ortega) An Ulrich bundle on a projective variety is a vector bundle that admits a completely linear resolution as a sheaf on the projective space. Ulrich bundles are semi-stable and the restrictions to any hyperplane section remain semi-stable. This notion originates in classical algebraic geometry, being related to the problem of finding, whenever possible, linear determinantal or linear pfaffian descriptions of hypersurfaces in a complex projective space. Generally, the existence of an Ulrich bundle has nice consequences on the equations of the given variety, specifically, the Cayley-Chow form is the determinant of a matrix of linear forms in the Pluecker coordinates. We prove existence of stable rank-two Ulrich bundles on polarized K3 surfaces with a mild Brill-Noether condition. As a consequence, we obtain an explicit pfaffian representation of the associated Cayley-Chow form.

Philip Boalch
Title: Non-perturbative symplectic manifolds and non-commutative algebras
Abstract: From a moduli-theoretic viewpoint the irregular Riemann-Hilbert correspondence can be viewed as a machine that takes as input a simple "additive" symplectic/Poisson manifold and it outputs a more complicated "multiplicative" symplectic/Poisson manifold. In the simplest nontrivial example it converts the linear Poisson manifold $\mathrm{Lie}(G)^*$ into the dual Poisson Lie group G^* (which is the Poisson manifold underlying the Drinfeld-Jimbo quantum group). This talk will firstly describe some more recent (and more complicated) examples of such "nonperturbative symplectic/Poisson manifolds:, i.e. symplectic spaces of Stokes/monodromy data or "wild character varieties". Then the natural generalisations ("fission algebras") of the deformed multiplicative preprojective algebras that occur will be discussed, some of which are known to be related to Cherednik algebras.

Jim Bryan
Title: Curve counting on Abelian surfaces and threefolds and Jacobi forms
Abstract: We explain how generating functions for curve counting problems on Abelian surfaces and threefolds are given by certain nice Jacobi forms. A new computational technique mixes motivic and toric methods and makes a connection between the topological vertex and Jacobi forms.

Ionut Ciocan-Fontanine
Title: Wall-crossing in quasimap theory
Abstract: Quasimap theory is concerned with virtual counting of maps from curves to certain GIT quotients. In fact, one has a family of curve counting theories

(with Gromov-Witten theory included among them), with parameter the linearization in the GIT problem. It may be viewed as one possible mathematical incarnation of the "A-twist" of the physicists' GLSM in the geometric phases - the linearization is the so-called FI parameter of the GLSM. I will present a wall-crossing formula at the level of virtual classes in all genera, as the size of the linearization changes. I will also discuss some of its numerical consequences and their relevance to mirror symmetry. This is joint work with Bumsig Kim.

Gavril Farkas
Title: The uniformization of the moduli space of abelian 6-folds
Abstract: The general principally polarized abelian variety of dimension at most five is known to be a Prym variety. This reduces the study of abelian varieties of small dimension to the beautifully concrete theory of algebraic curves. I will discuss recent breakthrough on finding a structure theorem for principally polarized abelian varieties of dimension six as Prym-Tyurin varieties associated to covers with E_6-monodromy, and the implications this uniformization result has on the geometry of the moduli space A_6. This is joint work with Alexeev, Donagi, Izadi and Ortega.

Daniel Greb
Title: Higgs sheaves on singular spaces and uniformisation for varieties of general type
Abstract: I will explain how to extend the notion of Higgs sheaf to varieties with singularities as they appear in the Minimal Model Program. After establishing fundamental properties, in particular a restriction theorem for Higgs sheaves à la Metha-Ramanathan and Langer, I will present an application to higher-dimensional uniformisation theory: the Miyaoka-Yau inequality holds for minimal varieties of general type, and in case equality is attained, the canonical model of the variety X under discussion can be realized as a quotient of the unit ball (which is the period domain for the variation of Hodge structure induced by a natural Higgs sheaf on X) by a discrete subgroup of $\mathrm{PSU}(n-1,1)$.

Richard Hain
Title: Mixed Motives associated to Elliptic Curves
Abstract: The absolute Galois group of the rational numbers acts on the various flavours (profinite, prounipotent, pro-ℓ) of the fundamental group of a smooth projective curve over the rationals. The image of the corresponding homomorphism normalizes the image of the profinite mapping class group in the automorphism group of the geometric fundamental group of the curve. The image of the Galois action modulo these "geometric automorphisms" is independent of the curve. A basic problem is to determine this image. This talk is a report on a joint project with Francis Brown whose goal is to understand the image mod geometric automorphisms in the prounipotent case. Standard arguments reduce the problem to one in genus 1, where one can approach the problem by studying the periods of iterated integrals of modular forms and their relation to multiple zeta values.

Tamas Hausel
Title: Toric non-abelian Hodge theory
Abstract: We will overview some conjectures on the mixed Hodge structure of

character varieties in the framework of non-abelian Hodge theory on a Riemann surface. Then we introduce and study toric analogues of these spaces, in particular we prove that the toric character variety retracts to its core, the zero fiber of the toric Hitchin map, that its cohomology is Hodge-Tate and satisfies curious Hard Lefschetz, as well as the purity conjecture. We will indicate how these shed light on the P=W conjecture in the toric case as well as for general character varieties. This is based on joint work with Nick Proudfoot.

Jochen Heinloth
Title: Some results on the cohomology of moduli spaces of Higgs bundles
Abstract: Hausel, following Sen conjectured that the intersection form on the cohomology of certain moduli spaces of Higgs bundles should vanish. We confirm this, by a deformation argument. This also shows that the 0-fiber (i.e. the global nilpotent cone) does not contribute appear as support of the cohomology sheaves for the Hitchin fibration. As a byproduct we get combinatorial information on the number of irreducible components of the global nilpotent cone.

Jacques Hurtubise
Title: Monopoles on Sasakian 3-folds
Abstract: Analogously to the case of the product of a surface and a circle, the analysis of singular monopoles on a regular Sasakian 3-fold (also a circle bundle) has a complex geometric interpretation, this time associated to a Gauduchon structure on the complex surface given by product with a circle. A key ingredient is simply monodromy around the circle; this time, however, the non-triviality of the circle bundle gives rise to a holomorphic object rather reminiscent of a gerbe. (joint work with Indranil Biswas)

Marcos Jardim
Title: Torsion free sheaves with zero dimensional singularities
Asbtract: We study stable rank 2 torsion free sheaves on projective space whose singular set has dimension zero. We provide conditions for such sheaves to fill out entire irreducible components of the Gieseker-Maruyama moduli space $M(n)$ of semistable rank 2 torsion free sheaves with zero first and third Chern classes and second Chern class equal to n, giving various examples. In fact, we show that the number of such irreducible components becomes arbitrarily large as n grows. We also provide two applications. First, we describe the three irreducible components of $M(2)$, recovering results of Le Poitier; two of these components consist of torsion free sheaves with zero dimensional singularities, while the other one is the instanton component. Second, we prove that M(3) has a total of eight irreducible components, five of which consisting of torsion free sheaves with zero dimensional singularities. Joint work with Dimitri Markushevich and Alexander Tikhomirov.

Ludmil Katzarkov
Title: Kähler metrics on categories
Abstract: In this talk we will introduce a new approach to stability conditions on categories.

Bruno Klingler
Title: An André-Oort conjecture for variations of Hodge structures
Abstract: The classical André-Oort conjecture describes the distribution of CM-points on Shimura varieties. More generally, one can consider a similar question for any smooth family of algebraic varieties over a complex smooth quasi-projective base S (or more generally any variation of Hodge structure over S). In this talk I will describe the corresponding conjecture and partial results towards it.

Alexander Kuznetsov
Title: Geometry and moduli spaces of Gushel-Mukai varieties
Abstract: A Gushel-Mukai variety is a Fano variety of coindex 3, Picard number 1, and degree 10. I will discuss classification of these Fano varieties, their moduli spaces, and their relation to EPW sextics. This is a joint work with Olivier Debarre.

Adrian Langer
Title: Bogomolov's inequality and its applications
Abstract: I will review recent results concerning appearance of Bogomolov's inequality for Higgs sheaves, flat bundles etc. Some of the results work in positive characteristic (also in the logarithmic case), giving interesting insight into invariants of algebraic varieties in positive characteristic. Some other results, due to T. Mochizuki (for good filtered flat bundles), still await its algebraic proofs and new applications.

Radu Laza
Title: Birational geometry of moduli spaces of K3 surfaces II
Abstract: I will continue the discussion from K. O'Grady's talk by providing additional details. I will close by exploring a possible connection between our work and asymptotic GIT for K3s and K-stability theory.

Chiu-Chu Melissa Liu
Title: On the remodeling conjecture for toric Calabi-Yau 3-orbifolds
Abstract: The remodeling conjecture proposed by Bouchard-Klemm-Marino-Pasquetti relates Gromov-Witten invariants of a semi-projective toric Calabi-Yau 3-orbifold to Eynard-Orantin invariants of the mirror curve of the toric Calabi-Yau 3-fold. It can be viewed as a version of all genus open-closed mirror symmetry. In this talk, I will describe results on this conjecture based on joint work with Bohan Fang and Zhengyu Zong.

Kieran O'Grady
Title: Birational geometry of moduli spaces of K3 surfaces I
Abstract: The moduli space of polarized K3 surfaces of a given degree is identified with a locally symmetric variety, and hence has a Baily-Borel compactification. On the other hand, GIT quotients of m-embedded polarized K3's provide different (projective) birational models of the Baily-Borel compactification. E. Looijenga has constructed a framework that allows to compare these compactifications. In this talk, I will discuss an enrichment of this picture, essentially a continuous interpolation between the GIT and BB models. While the discussion will be mostly

concerned with the case of hyperelliptic quartic K3 surfaces, we expect such an interpolation to hold quite generally. There is a strong analogy with the Hassett-Keel program that studies the birational geometry of the moduli space of curves. This is a report on joint work with R. Laza.

Dragos Oprea
Title: Segre classes and Hilbert scheme of points
Abstract: I will discuss joint work with Alina Marian and Rahul Pandharipande aimed at studying the tautological ring of the moduli space of K3 surfaces. The method involves equivariant localization of virtual fundamental classes of certain relative Quot schemes. We produce in this fashion relations intertwining the kappa classes and the Noether-Lefschetz loci. Along the way, we also obtain closed formulas for the top Segre classes of tautological bundles over the Hilbert schemes of points of a fixed K3 surface, confirming a prediction of Lehn.

Artan Sheshmani
Title: On the proof of S-duality modularity conjecture on quintic threefolds
Abstract: I will talk about joint work during the recent years with Amin Gholampour, Richard Thomas and Yukinobu Toda, on proving the modularity property of the generating series of certain DT invariants of torsion sheaves with two dimensional support in ambient threefolds. More specifically, I will talk about algebraic-geometric proof of S-duality conjecture in superstring theory made formerly by physicists: Gaiotto, Strominger, Yin, regarding the modularity of DT invariants of sheaves supported on hyperplane sections of the quintic Calabi-Yau threefold. Our strategy is to first use degeneration and localization techniques to reduce the threefold theory to a certain intersection theory over relative Hilbert scheme of points on surfaces and then prove modularity; More precisely, together with Gholampour we have proven that the generating series, associated to the top intersection numbers of the Hibert scheme of points, relative to an effective divisor, on a smooth quasi-projective surface is a modular form. This is a generalization of the result of Okounkov-Carlsson for absolute Hilbert schemes. These intersection numbers, together with the generating series of Noether-Lefschetz numbers, will provide the ingrediants to prove modularity of the above DT invariants over the quintic threefold.

Matei Toma
Title: Moduli spaces for slope-semistable sheaves on projective manifolds
Abstract: This talk reports on joint work with Daniel Greb. Let X be a complex projective manifold, In order to obtain reasonable moduli spaces for algebraic vector bundles on X, Mumford introduced the notion of "slope-stability" when $\dim(X) = 1$. His definition was later extended to higher dimensions. For $\dim(X) > 1$ Giesecker and Maruyama considered a new stability notion which allowed them to construct also in this case moduli spaces of semistable sheaves. Later Le Potier and Li gave a construction when $\dim(X) = 2$ of moduli spaces of slope-semistable sheaves. These spaces are homeomorphic to the compactifications of moduli spaces of Yang-Mills connections obtained by Donaldson and Uhlenbeck

in gauge theory. In this talk we present a construction of a moduli space of slope-semistable coherent sheaves in higher dimensions. We also explain the pathologies connected to change of polarization, which are encountered in this case.