Frontiers of Operator Dynamics
September 28 - October 2, 2015

Jon Aaronson: Distributional limits of positive, ergodic stationary processes & infinite ergodic transformations.

Every random variable on the positive reals occurs as the distributional limit of the partial sums some positive, ergodic stationary process normalized by a 1-regularly varying normalizing sequence (& the process can be chosen over any EPPT).I'll try to explain this and (time permitting) some consequences for infinite ergodic theory.
Joint work with BenjaminWeiss.

Wolfgang Arendt: Convergence of orbits and compactness.

We describe situations where the orbits of a bounded operator or a semigroup converge to a one-dimensional projection. And in fact, for many models this is the desirable situtation one expects from the physical situation. A concrete example is the heat equation with non-local boundary conditions, which have not obtained the attention they merit, so far. Of special interest for operator theory is an unususal way to produce compactness, which we will explain. Another example are composition operators on Hardy spaces, whose spectrum is a very classical subject of operator theory.
References: W. Arendt, S. Kunkel, M. Kunze: Diffusion with nonlocal boundary conditions, preprint 2015 arXiv:1409.5689 [arxiv.org]

Idris Assani: Convergence of nonconventional ergodic averages

We show that J. Bourgain double recurrence theorem can be extended to a Wiener-Wintner double recurrence theorem. Some consequences are derived like an extension of B. Host and B. Kra norm convergence result of weighted Furstenberg averages. Another consequence is an answer to a question of B. Weiss on a nilsequence extension of this Wiener Wintner double recurrence theorem. A third consequence is an extension of T. Tao norm convergence result for commuting transformations. (Part of this work was done with D. Duncan and R. Moore).

Catalin Badea: Sets of integers determined by operator-theoretical results.

Following the natural tendency of turning nice theorems into definitions (example: Poincaré recurrence theorem/sets of recurrence), we will introduce and discuss in this talk several sets of integers determined by properties with an operator-theoretical flavour. In particular, the role of Jamison and Kazhdan sets will be emphasized.
This talk is based on jointwork with Sophie Grivaux.

Charles Batty: Rates of decay associated with operator semigroups.

General theory of operator semigroups provides abstract results which can be used to obtain optimal rates of decay or convergence in many evolution equations or dynamical systems. I will describe the abstract results, and indicate how they are obtained and how they can be applied in examples.

Frédéric Bayart: Common hypercyclic vectors for high dimensional families of operators

Let $(T_\lambda)_{\lambda\in\Lambda}$ be a family of operators acting on an F-space X, where the parameter space Λ is a subset of $\mathbb{R}^d$. We give sufficient conditions on the family to yield the existence of a vector $x \in X$ such that, for any $\lambda \in \Lambda$, the set $\{T_\lambda^n x;\ n \geq 1\}$ is dense in X. We obtain results valid for any value of $d \geq 1$ whereas the previously known results where restricted to $d = 1$. Our methods also shed new light on the one-dimensional case.

Vitaly Bergelson: Potpourri of Open Problems and Conjectures in Linear Dynamics and Ergodic Theory.

We will formulate and discuss various problems and results at the junction of Ergodic Theory and Linear Dynamics.

Alexander Borichev: Random and pseudo-random Taylor series.

We study the asymptotical behaviour of entire functions represented by random and pseudo-random Taylor series. It turns out that the angular zero distribution is determined by certain autocorrelations of the coefficient sequence. This is a joint work with Alon Nishry and Mikhail Sodin.

Alexander Bufetov: Markov operators, reverse martingles and ergodic theorems for group actions

The first part of the talk will be devoted to the method of Markov operators in the study of ergodic theorems for measure-preserving actions of free semigroups and groups. Applications (obtained in joint work with Klimenko and Khristoforov) will be given to Cesaro convergence of spherical averages for measure-preserving actions of Markov groups, a class that includes Gromov hyperbolic groups.

In the second part of the talk, we will consider actions of inductively compact groups (such as e.g. the infinite-dimensional unitary group) with a quasi-invariant measure, discuss the problem of ergodic theorems and ergodic decomposition for such actions (the reverse matringale convergence theorem plays an essential role) and consider the specific example of the infinite-dimensional analogue of the canonical invariant measure on the Grassmann manifold.

Ralp Chill: Quantified versions of Ingham's Tauberian theorem

We review proofs of Ingham's Tauberian theorem for Laplace and Fourier transforms. One version of this theorem states that if the Laplace transform of a bounded, uniformly continuous function defined on the positive reals extends analytically across the imaginary axis, then the function itself must vanish at infinity. In 2008, Batty and Duyckaerts stated a quantified version of this theorem, relating the growth of the analytic extension of the Laplace transform on the imaginary axis with the decay of the original function at infinity. Their proof is based on Korevaar's esthetic, complex variable proof of Ingham's theorem. Tauberian theorems with rates have found applications in semigroup theory, for example in the qualitative study of semigroups arising from wave equations. In this talk we show that Ingham's original and very short proof allows us to recover quantified versions, too, and we discuss several variants of quantified Ingham theorems.
This is joint work with David Seifert.

Jean-Pierre Conze: Almost mixing of all orders $\mathbb{Z}^2$-actions, Ledrappier's example and the CLT

In 1978, F. Ledrappier gave an example of a $\mathbb{Z}^2$-subshift which is 2-mixing, but not 3-mixing. Ledrappier's system is an example of a $\mathbb{Z}^2$-action by algebraic automorphisms on a compact abelian group which can be viewed as "almost mixing" of all orders (cf. L. Arenas-Carmona, D. Berend and V. Bergelson (2008)).

Based on the results of these latter authors and on methods previously applied to automorphisms of connected compact abelian groups, the talk will focus on the central limit theorem for Ledrappier's system.

(joint work with Guy Cohen, Ben-Gurion University)

Fabien Durand: Eigenvalues of minimal Cantor systems

Given a dynamical system (X, T, μ), where μ is a T-invariant measure, its Koopman operator is defined on $L^2(\mu)$ by $U(f) = f \circ T$. In this talk we present results concerning the existence of continuous eigenfunctions for U. For many families of dynamical systems we are able to characterize those eigenvalues having a continuous eigenfunction by means of the combinatorics of the systems given by Kakutani-Rolhin partitions. Roughly speaking, a necessary and sufficient condition for $\lambda = \exp(2i\pi\alpha)$ to have a continuous eigenfunction is that the combinatorics of the systems provide a good diophantine approximation of α.

Tanja Eisner: Weighted ergodic theorems

We present an overview on good weights for the pointwise ergodic theorem.

El Houcein El Abdalaoui: On the Banach spectral problem in ergodic theory and the polynomialflatness problem with connection to the Mahler problem and related topics.

We establish that the following three problem are equivalent: the spectral Banach problem, the Mahler problem in the class of Newman polynomials and the flatness polynomial problem. Applying some combinatorial ideas on the Singer and Sidon sets combined with Marcinkiewicz-Zygmund inequalities in the H^p interpolation theory, we prove that the L^4 classical strategy fails and we get a positive answer in some class of Newman polynomials. This gives that there exist a rank one map acting on a space of infinite measure with simple Lebesgue spectrum.

Romuald Ernst: Hypercyclic scalar sets

A well-known result of F. León Saavedra and V. Müller states that if an operator T is hypercyclic then all unimodular multiples of T are hypercyclic too. This result follows easily from the fact that a vector x is hypercyclic for T if and only if the set $\mathbb{T}x$ has dense orbit under T where $\mathbb{T}$ denotes the unit circle in $\mathbb{C}$. An important tool in the proof is the group-structure of the set $\mathbb{T}$ and even if this fact is crucial in the proof of León and Müller, it is not clear whether it is really important or not. In this talk, I will be interested in giving a characterization of those scalar sets $\Gamma \subset \mathbb{C}$ satisfying the following property:

For every bounded operator $T : X \to X$, for every $x \in X$, $\overline{Orb(\Gamma x; T)} = X$ if and only if x is hypercyclic for T.

This is a joint work with S. Charpentier (Marseille) and Q. Menet (Mons).

Balint Farkas: A Bohl–Bohr–Kadets type theorem characterizing Banach spaces not containing c_0

We prove that a separable Banach space E does not contain a copy of the space c_0 of null-sequences if and only if for every doubly power-bounded operator T on E and for every vector $x \in E$ the relative compactness of the sets $\{T^{n+m}x - T^n x : n \in \mathbb{N}\}$ (for some/all $m \in \mathbb{N}$, $m \geq 1$) and $\{T^n x : n \in \mathbb{N}\}$ are equivalent. With the help of the Jacobs–de Leeuw–Glicksberg decomposition for strongly compact semigroups the case of (not necessarily invertible) power-bounded operators is also handled.

Krzysztof Fraczek: Birkhoff and Oseledets genericity along curves

I am going to present some recent results (with Ronggang Shi and Corinna Ulcigrai) on Birkhoff and Oseledets genericity along certain curves in the space of affine lattices and in moduli spaces of translation surfaces. They are illustrated some applications to dynamical billiards, mathematical physics and number theory.

Eli Glasner: A universal hypercyclic representation

For any countable group, and also for any locally compact second countable, compactly generated topological group, G, there exists a "universal" hypercyclic representation on a Hilbert space, in the sense that it simultaneously models every possible ergodic probability measure preserving free action of G. I will discuss the original proof of this theorem (a joint work with Benjy Weiss) and then, at the end of the talk, say some words about the development of this idea and its applications as expounded in a subsequent work of Sophie Grivauux.

Alexander Gomilko: Rates in mean ergodic theorems: inverse results

Let $(T(t))_{t\geq 0}$ be a bounded C_0-semigroup on a Banach space X, with generator $-A$. It is a trivial that for x from the range of A one has

$$\|C_t(A)x\| = \|\frac{1}{t}\int_0^t T(s)x\,ds\| = \mathrm{O}(1/t), \qquad t\to\infty.$$

On the other hand, conversely, by a well-known theorem due to Butzer and Westphal if X is reflexive then from

$$\|C_t(A)x\| = \mathrm{O}(1/t), \qquad t\to\infty,$$

it follows that x belongs to the range of A. We present a number of generalizations of this converse result saying that certain decay of $C_t(A)x$ implies that x is from the range of $g(A)$, where $g(A)$ is an appropriate function of A. Moreover, we show that such implications are optimal in a natural sense.

This is joint work with Yu. Tomilov (Warsaw).

Karl Grosse-Erdmann: Upper frequently hypercyclic operators.

Recent work of Bayart-Ruzsa, Bès-Menet-Peris-Puig, and Menet have clarified the link between frequently hypercyclic operators and chaotic operators. But these papers also show that Shkarin's notion of upper frequently hypercyclic operators is more interesting than previously thought. We investigate these operators in greater detail. This is joint work with Antonio Bonilla.

Markus Haase: On some operator-theoretic aspects of ergodic theory.

I will describe the main features and methods of a strictly operator-theoretic/functional-analytic perspective on structural ergodic theory in the spirit and in continuation of a recent book project (with T.Eisner, B.Farkas and R.Nagel). The approach is illustrated by a review of some classical results by Abramov on systems with quasi-discrete spectrum and by Veech on compact group extensions (joint work with N.Moriakov).

Alexey Klimenko: On asymptotics of Arnold tongues for some families of flows on 2-tori.

Recently it was discovered that there are families of vector fields of 2-tori such that only domains where rotation number is integer have nonempty interior, while for other rational values this does not hold. We show how to estimate asymptotics of boundaries for these tongues, thus proving that the width of tongues vanishes infinitely many times.

Eva Kopecká: Strange products of orthogonal projections

Let X and Y be two closed subspaces of a Hilbert space. If we send a point back and forth between them by orthogonal projection, the iterates converge according to von Neumann to the projection of the point onto the intersection of X and Y.

If H is an infinite dimensional Hilbert space, there exist three orthogonal projections X_1, X_2, X_3 onto closed subspaces of H, $z_0 \in H$ and a sequence of indices $k_1, k_2, \dots \in \{1, 2, 3\}$ so that the sequence of iterates defined by $z_n = X_{k_n} z_{n-1}$ does not converge in norm.

We will explain how this implies that in every infinite dimensional Hilbert space there exist three orthogonal projections X_1, X_2, X_3 onto closed subspaces of H such that for *every* $0 \neq z_0 \in H$ there exist $k_1, k_2, \dots \in \{1, 2, 3\}$ so that the sequence of iterates $\{z_n\}_{n=0}^{\infty}$ does not converge in norm.

Michael Lin: Symmetrization of Markov operators

let P be a Markov operator with invariant probability m, ergodic on $L_2(S, m)$, and let $\{\xi_n\}$ be the Markov chain with state space S and transition operator P on the space of trajectories $(\Omega, \mathbb{P}_m)$ with initial distribution m. Important problems in the theory of Markov chains are those of limit theorems for for $\{f(\xi_n)\}$ (where $f \in L_2(S, m)$), like the CLT, the LIL, and other. many results are known when P is symmetric on $L_2(S, m)$. The symmetrization method is a way of obtaining limit theorems for the original chain by looking at the symmetrized operator $P_s := \frac{1}{2}(P + P^*)$. Limit theorems can be obtained for f in the linear manifold $\mathcal{H}_{-1} := \sqrt{I - P_s} L_2(m)$. For P symmetric, or even normal, the CLT was proved when $f \in \sqrt{I - P} L_2(m)$ (Kipnis-Varadhan, Gordin-Lifshitz); therefore one of our main goals is to compare $\mathcal{H}_{-1}$ with $\sqrt{I - P} L_2(m)$, and find condition for equality of these manifolds. We charcterize the sector condition for P on the real $L_2(m)$, which means

$$|\langle (I - P)g, h\rangle|^2 \leq K \langle (I - P)g, g\rangle \langle (I - P)h, h\rangle \qquad \forall g, h \in L_2(S, m) \quad \text{real},$$

by the numerical range of P on the complex $L_2(m)$ being in a sector with vertex at 1. If P has a real normal dilation which satisfies the sector condition, then $\mathcal{H}_{-1} = \sqrt{I - P} L_2(m)$.

Etienne Matheron: Some remarks regarding ergodic operators

Let us say that a continuous linear operator T acting on some Polish topological vector space is *ergodic* if it admits an ergodic probability measure with full support. This talk will be centred in the following question: how can we see that an operator is or is not ergodic? More precisely, I will try (if I'm able to manage my time) to talk about two "positive" results and one "negative" result. The first positive result says that if the operator T acts on a reflexive Banach space and satisfies a strong form of frequent hypercyclicity, then T is ergodic. The second positive result is the well-known criterion for ergodicity relying on the perfect spanning property for unimodular eigenvectors, of which I will outline a "soft" Baire category proof. The negative result will be stated in terms of a parameter measuring the maximal frequency with which (generically) the orbit of a hypercyclic vector for T can visit a ball centred at 0. The talk is based on joint work with Sophie Grivaux.

Quentin Menet: Linear chaos and frequent hypercyclicity

An operator T is said to be hypercyclic if there is some vector whose orbit visits (infinitely often) each non-empty open subset U of X. If T possesses in addition a dense set of periodic points then T is said to be chaotic. In 2004, Frédéric Bayart and Sophie Grivaux introduced the notion of frequent hypercyclicity which relies on the existence of some vector whose orbit frequently visits each non-empty open subset U of X. The goal of this talk will be to investigate the links between these two notions. In particular, we answer one of the main current questions in linear dynamics by showing that there exists a chaotic operator on ℓ^1 which is not frequently hypercyclic.

Vladimir Müller: Mean ergodic theorem for polynomial subsequences

Let T be a power bounded Hilbert space operator and let p be a polynomial satisfying $p(\mathbf{N}) \subset \mathbf{N}$. Then the Cesaro sums $N^{-1}\sum_{n=1}^{N} T^{p(n)}$ converge in the strong operator topology.This generalizes known results for unitary operators and Hilbert space contractions. The method can be used also for other polynomial-type sequences and for bounded strongly continuous semigroups of operators. (joint work with A.F.M. ter Elst)

Marina Murillo Arcila: Frequently hypercyclic translation semigroups.

Frequent hypercyclicity for translation C_0-semigroups on weighted spaces of continuous functions is studied. The results are achieved by establishing an analogy between frequent hypercyclicity for the translation semigroup and for weighted pseudo-shifts and by characterizing frequently hypercyclic weighted pseudo-shifts on spaces of vanishing sequences. Frequently hypercylic translation semigroups on weighted L_p-spaces are also characterized.

Pavel Nikitin: Two semigroups connected with the infinite symmetric group, their representations and random walk on graded graphs.

Representation theory of the infinite partial bijections semigroup and the infinite Brauer semigroup naturally leads to the two operations on Bratteli diagrams. This operations are connected with the random walk on these diagrams. We will show how to parametrize the extremal harmonic measures (and the corresponding semigroup characters) for the resulting class of diagrams. (joint work with A.Vershik)

Alfred Peris: On the shadowing property in linear dynamics

Anosov and Bowen introduced the so called shadowing property for a dynamical system (X, f), where (X, d) is a metric space. Given $\delta > 0$, a (finite or infinite) sequence $\{x_n\}_{n=0}^N$ is a δ-pseudo orbit if

$$d(f(x_{n-1}), x_n) < \delta, \qquad n = 1, \ldots, N.$$

Given $\varepsilon > 0$, we say that a true orbit $\{y_n\}_{n=0}^N$ (i.e., $f(y_{n-1}) = y_n$, $n = 1, \ldots, N$) ε-shadows $\{x_n\}_{n=0}^N$ if

$$d(x_n, y_n) < \varepsilon, \qquad n = 0, \ldots, N.$$

The dynamical system (X, f) has the (numerical) shadowing property if, for $(N < \infty)$ $N = \infty$, and for each $\varepsilon > 0$, there exists $\delta > 0$ such that every δ-pseudo orbit $\{x_n\}_{n=0}^N$ is ε-shadowed by a true orbit $\{y_n\}_{n=0}^N$. We will study the shadowing property, and related notions, for the dynamics of linear operators. Some connections with other known properties in linear dynamics will be established. This is part of joint work with S. Bartoll, F. Martinez and P. Oprocha.

Samuel Petite: Automorphism groups of subshifts with low complexity.

The study of automorphisms of a dynamical system (X, T), that is self homeomorphisms of X commuting with the map T, is a classical topic. Several questions are still open for subshifts. We show, in a joint work with F. Durand, S. Donoso ans A. Maass, that the automorphism group is virtually $\mathbb{Z}$ for minimal subshifts with affine complexity on a subsequence. This class includes the minimal substitutions, linearly recurrent subshifts and even some minimal subshifts with sub exponential complexity. Furthermore, we get that the automorphism group of an almost 1-1 extension of a nil translation is nilpotent.

Maria Roginskaya: r-Bohr sequences which are not (r+1)-Bohr

We construct some sequences which are recurrent for all products of r rotations, but not for some products of (r+1)-rotations. (joint with Sophie Grivaux)

David Seifert: Asymptotics of infinite systems of ODEs

Motivated by applications in control theory to models of vehicle platoons, we consider a special class of infinite systems of coupled ODEs. In many cases of interest, the solution $x(t)$, $t \geq 0$, of such a system approaches a steady state in the sense that $\|\dot{x}(t)\| \to 0$ as $t \to \infty$. We investigate *how fast* this convergence takes place and obtain (almost) optimal estimates by studying the associated semigroup and its generator on appropriate vector-valued sequence spaces. This talk is based on joint work with L. Paunonen.

Tom Ter Elst: On one-parameter Koopman groups

A unitary one-parameter C_0-group $(U_t)_{t \in R}$ in a standard Borel probability space $(X, , \mu)$ is called a Koopman group if for all $t \in R$ there exists a measurable $T_t \colon X \to X$ such that $U_t f = f \circ T_t$ for all $f \in L_2(X)$. We characterize Koopman one-parameter C_0-groups in the class of all unitary one-parameter C_0-groups on $L_2(X)$ as those that preserve $L_\infty(X)$ and for which the infinitesimal generator is a derivation on the bounded functions in its domain. This is joint work with M. Lemańczyk.

Rafael Tiedra: Commutator criteria for strong mixing

We present new criteria, based on commutator methods, for the strong mixing property of discrete flows $\{U^N\}_{N \in \mathbb{Z}}$ and continuous flows $\{e^{-itH}\}_{t \in \mathbb{R}}$ induced by unitary operators U and self-adjoint operators H in a Hilbert space $\mathcal{H}$. Among other examples, our results apply to skew products of compact Lie groups, time changes of horocycle flows and adjacency operators on graphs