WORKSHOP : ZERO ENTROPY SYSTEM

CIRM, LUMINY, 27 NOVEMBER-1ST DECEMBER 2017

Abstracts

Dou Dou

Entropy dimension for zero entropy systems

Abstract: Entropy dimension is a class of entropy type quantities which measure the intermediate complexity for zero entropy systems. In this talk, I will introduce some related results and progresses on this theme.

Sébastien Ferenczi

Rigidity for square tiled interval exchange transformations

Abstract: We look at interval exchanges defined as first return maps on the set of diagonals of a flow of direction α on a square-tiled surface: our main result is to show, by a combinatorial approach, that they are not rigid when the surface has at least one true singularity and α has bounded partial quotients.

Anna Frid

Sturmian numeration systems and decompositions to palindromes

Abstract: We extend classical Ostrowski numeration systems so that they better reflect the structure of the respective characteristic Sturmian word. In particular, this allows to prove that for any given K, in every Sturmian word there exists a prefix which cannot be decomposed as a concatenation of at most K palindromes.

Arek Goetz

Lecture 1. Piecewise Isometries - The beautiful. Charm, intrigue, and the unknown.

Abstract: We explore the vast possibilities in the investigation of a repeated application of a collection of isometries defined on mutually disjoint domains. We start by slowly analyzing the most simple examples in R1 and R2, and illustrate subareas of interest due to computer evidence of spectacular phenomena that remain at time a mystery. No background in piecewise isometries is needed to take an active part in the first lecture.

Arek Goetz

Lecture 2. Piecewise Isometric research - Quo vadis?

Abstract:

In the first elementary lecture we outlined various aspects and areas of piecewise isometries. In this lecture we illustrate some recurrence and invariance results in unbounded cone exchanges. We conclude with a discussion of possible directions where investigations may lead to new realistic discoveries and where it likely remain fruitless.

Pascal Hubert

Novikov's problem on foliations and Arnoux-Rauzy interval exchange transformations.

Abstract:

In this talk, I will discuss results with Avila, Dynnikov and Skripchenko about Arnoux-Rauzy iet and some foliations on surfaces arising from linear foliations of the 3-dimensional torus. I will explain how theses two subjects are related and I will mention some results on the dynamical behavior of most of these maps.

John Lowenstein

I. Pseudochaotic kicked-oscillator maps and parametric piecewise isometries

Abstract: I consider a one-dimensional harmonic oscillator kicked impulsively in 4:1 resonance with its natural frequency. The case of a kick amplitude varying sinusoidally with position has been studied extensively. The area-preserving Poincaré map produces chaotic orbits which typically tend to infinity diffusively, with distance from the initial point increasing as the square-root of the time. If, on the other hand, the kick amplitude is replaced by a periodic, piecewise linear (sawtooth) function of position, one can obtain, by varying the slope, a variety of different asymptotic behaviors, including what has been called pseudochaos. Here there exists a fractal set of orbits characterized by power-law behavior at infinity, but without the presence of true chaos.

After a brief review of the literature, I will introduce a parameter dependence of the piecewise linear kick amplitude which leads to a description of the dynamics in terms of a parametric piecewise affine map on the square - the fundamental domain of an infinite crystalline array. The map is seen to be conjugate to a parametric piecewise isometry on a rhombus, of a sort studied recently by Franco Vivaldi and myself. I will concentrate on an infinite sequence of 'octagonal? models, whose spatial scale factors are increasing powers of a fundamental unit $\sqrt{2}-1$. The periodic islands cover the rhombus with full Lebesgue measure, but their complement, the so-called exceptional set, is nontrivial. For each model, I will describe the dynamics of the residual set (boundary avoiding members of the exceptional set) in terms of symbolic codes and odometer-like updating, and derive an exact formula for its Hausdorff dimension.

John Lowenstein

II. Global scaling in a one-parameter family of pseudochaotic kicked-oscillator maps

Abstract: I will discuss the behavior on the infinite plane of the parametric kick-oscillator maps introduced in I. The key to understanding the dynamics is a factorization of the map into a local piecewise affine map on a square and a global map on the 2D integer lattice. I will describe the global renormalization process, which, thanks to the 4-fold rotations of the plane, is quite distinct from that of the local map. Self-similarity now requires synchronization of the local return-map periods with the four-fold rotations, leading to a new global scale factor. The latter will be shown to enter into the asymptotic large-time behavior of individual orbits. For the infinite family of octagonal models introduced in I, the orbits initiated in the residual set typically have sub-diffusive behavior at infinity.

Paul Mercat

Rauzy fractals, one dimensional Meyer sets, β-numeration and automata

Abstract: Given a fixed point of a substitution associated to a Pisot number β, we can associate a Meyer subset of $\mathbb{R}$ which is invariant by multiplication by β. For any Pisot number β, we give a constructive necessary and sufficient condition for a β-invariant one dimensional Meyer set to come from a substitution. This permits to construct substitutions whose Rauzy fractal approximate any compact subset of $\mathbb{R}^n$ containing 0. And this permits for example to compute a substitution whose Rauzy fractal is an intersection, an union, a complementary or a Minkowski sum of two Rauzy fractals, when it is possible. The main tool used is a description of Meyer sets by regular languages and beta-numeration. Using this tool, we can also show that the Rauzy fractal of the substitution $a \mapsto b, b \mapsto c, c \mapsto ab$ is a countable union of Hokkaido tiles that is the Rauzy fractal of the substitution $a \mapsto ab, b \mapsto c, c \mapsto d, d \mapsto e, e \mapsto a$ union a part of dimension less than two.

Milton Minervino

Tree substitutions and Rauzy fractals

Abstract: Rauzy fractals are well-known geometrical representations of Pisot substitutive dynamical systems. We show how to construct a tree substitution generating a self-similar tree which fills at the limit the Rauzy fractal. By describing the contour of this tree we compute an interval exchange transformation of the circle covering the original substitution.
(Joint work with Thierry Coulbois).

Hitoshi Nakada

On Cruz and da Rocha's idea for a piecewise rotation map of the circle and its application

Abstract: We introduce a tower representation of a piecewise rotation map on the circle based on the idea by Cruz and da Rocha (2005). This tower is constructed by a fixed discontinuous point and an associated critical iterate of the given map. As an easy application, we have "d+2 distance theorem" for a piecewise rotation map with d discontinuous points instead of "three distance theorem" for an irrational rotation of the circle.

The main part of this talk is a construction of a translation surface from the tower. We will see that for any given singularity order vector and any marked singularity, there exists a piecewise rotation of the circle such that one of discontinuous points and one of its associated critical iterate generate a translation surface which has the given sigularity orders with the given marked singularity.
(joint work with Kae Inoue)

Olga Paris-Romaskevich

Generic behaviour of tiling billiards

Abstract: Take your favorite triangle, and consider a tiling of the plane by the tiles congruent to this triangle. Define a billiard in this tiling in a following way. A particle goes in a straight line till the moment when it reaches an edge of some triangle. Then it reflects across this edge into the neighbouring triangle with a refractive index equal to -1. This system was defined by Davis-DiPietro-Rustad and St Laurent. We are interested in the study of dynamics of this system. We prove that often (often will be precised in the talk) this system has a very simple behaviour : the orbits are either closed or their symbolic dynamics is governed by an irrational rotation of the circle. Although, for some very specific cases of parameter values the trajectories of this system have fractal behaviour. In one of the cases one can recover the Rauzy fractal. The study of

tiling billiards systems is closely related to the study of interval exchange transformations with flips and to translation flows on non-oriented flat surfaces. I will discuss these connections and formulate some open questions.
This is work in progress, joint with Pascal Hubert.

Pedro Peres
Embeddings of Interval Exchange Transformations into Planar Piecewise Isometries

Abstract: Although piecewise isometries (PWIs) are higher dimensional generalizations of one dimensional interval exchange transformations (IETs), their generic dynamical properties seem to be quite different. In this talk I will consider embeddings of IET dynamics into PWI with a view to better understanding their similarities and differences. I will introduce a family of PWIs with apparent abundance of invariant nonsmooth fractal curves supporting IETs. I will recall some notions used in the study of IETs, introduce some new tools and describe how they can be used to establish some results on the existence of embeddings of IETs into PWIs.
(Joint work with P. Ashwin, A. Goetz and A. Rodrigues).

Mao Shinoda
Uncountably many ergodic maximizing measures for dense continuous functions

Abstract: The purpose of ergodic optimisation is to describe maximising measures which maximise the space average of a function. In the context of thermodynamic formalism they appear as the zero temperature limit of equilibrium measures. Uniqueness of maximising measure for generic continuous function is proved by Jenkinson. On the other hand I will present in this talk the existence of uncountably many ergodic maximising measures for dense continuous functions.

Serge Troubetzkoy
Irrational billiards

Abstract: I will give a survey of known results on billiard dynamics in irrational polygons and then present a new result: the billiard map in a Baire typical is topologically weakly mixing.

Franco Vivaldi
Nonlinear rotations on a lattice

Abstract: We consider a prototypical two-parameter family of invertible maps of $\mathbb{Z}^2$, representing rotations with decreasing rotation number. These maps describe the dynamics inside the island chains of a piecewise affine discrete twist map of the torus, in the limit of fine discretisation. We prove that there is a set of lattice points of full natural density which, depending of the parameter values, either are all periodic or all escape to infinity. The proof is based on the analysis of an interval-exchange map over the integers, with infinitely many intervals.

Ren Yi

Self-induced rectangle exchange maps

Abstract: Rectangle exchange maps (REMs) are higher dimensional generalizations of interval exchange maps which have been well-studied for more than 40 years. We study REMs that arise from cut-and-project schemes on lattices associated to cubic Pisot numbers. We prove that these REMs are minimal and self-induced. Moreover, these REMs are parametrized by matrices in SL(3,Z) whose leading eigenvalues are Pisot numbers. Via matrix products, we identify a family of renormalizable REMs whose parameter space is a four-dimensional Cantor set. We give a symbolic encoding of the dynamical system on the parameter space.
This is joint work with Ian Alevy and Richard Kenyon.