Descriptive geometry: its role in history of mathematics and in teaching

Workshop, CIRM LUMINY, 18-22 January 2016

ABSTRACTS

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Descriptive geometry in 19th-century Spain

Elena Ausejo
University of Zaragoza (Spain)

The faithful translation into Spanish of Gaspard Monge's *Géométrie Descriptive* (1799) was published as early as 1803 for the training of building engineers at the recently established School of Roads and Waterways (1802), an institution conceived according to the model of the French school of *Ponts et Chaussées*. Both this pioneer translation and the school were the result of the work of Spanish engineers Agustín de Betancourt (1758–1824) and José María Lanz (1764 – 1839), under the influence of Monge and Hachette at the École Polytechnique since its very beginning. Further results of this collaboration were the translation into Spanish of Francoeur's *Traité de mécanique élémentaire* (1803), and the publication of Betancourt and Lanz's *Essai sur la Composition des Machines* (1808) by the École Polytechnique.

The introduction of descriptive geometry as a basic subject in the syllabus of this first engineering school marked a trend that expanded and consolidated with the development of the institutional framework for higher education of civil engineers in new schools for mining (1835), forestry (1846), mechanical (1851), and agricultural (1855) engineering. As of 1814, descriptive geometry was also taught at the military College of Artillery, and from 1819 onwards at the military Academy of Engineers by Mariano Zorraquín, author of the first Spanish textbook on *Analytical-Descriptive Geometry* **–**a work under the declared influence of Monge, Lacroix, Biot, Puissant, Hachette, Garnier and Boucharlat. Later, in 1857, descriptive geometry was included in the syllabus of the new Master's Degree in Mathematics and Physics at the also new Faculties of Science.

As regards textbooks, three French works prevailed throughout the third quarter of the 19th century: Leroy's *Traité de géométrie descriptive* (1846), Olivier's *Cours de géométrie descriptive* (1843-44, Spanish translation by U. Mas Abad, 1879), and Adhémar's *Traité de géométrie descriptive* (1841). It is also worth mentioning that Cirodde's *Lecciones de Geometría con unas nociones de la descriptiva* (Spanish translation by Manuel María Barbery, 1858, 22 reprints), was listed among official textbooks for military engineers and science faculties (1864 official lists), and was also used in secondary education. Moreover, six textbooks on descriptive geometry were produced by Spanish military men between 1846 and 1875; these were basically original works based on a careful selection of different French sources –Leroy and Olivier for Bielsa's book (1846), La Gournerie and Adhémar for Álix' introduction of axonometric perspective (1866), still undetermined for Rodríguez Arroquía's introduction of theory of dimensioning (1850).

In the last quarter of the 19th century civil engineers gradually tended to transfer mathematical disciplines –among them, descriptive geometry– from their syllabi to the highly competitive entrance examination. This policy raised tensions and frictions between civil engineers and mathematicians, as it diverted students from the Faculties of Science to a selected network of private preparatory schools. However, twelve new textbooks on descriptive geometry were published, all but one authored by military men; among the latter, two of them were navy officers, who wrote on descriptive geometry when it was first included in the entrance examination to the Naval School (1869), and later in its syllabus (1895).

Bibliography

AUSEJO, Elena (2006) «Quarrels of a Marriage of Convenience: On the History of Mathematics Education for Engineers in Spain». *International Journal for the History of Mathematics Education, 2(1),* 1-13.

CAMPO Y FRANCÉS, Ángel del (1996) «La descriptiva de Monge en la Escuela de Caminos». In: Gaspard Monge, *Geometría descriptiva*. Madrid, Colegio de Ingenieros de Caminos, Canales y Puertos (Facsímil de la edición de Madrid, Imprenta Real, 1803), pp. 9-54.

COMAS ROQUETA, Joaquín (2015) *La enseñanza de las matemáticas en la armada española en el siglo XIX*. Tesis Doctoral, Universidad de Zaragoza.

MARTÍNEZ GARCÍA, Mª Ángeles (2004) *Las matemáticas en la ingeniería: Las matemáticas en los planes de estudios de los ingenieros civiles en España en el siglo XIX.* Zaragoza, Universidad de Zaragoza Seminario de Historia de la Ciencia y de la Técnica de Aragón.

MONGE, Gaspard (1803) *Geometría descriptiva. Lecciones dadas en las Escuelas Normales en el año tercero de la República, por Gaspar Monge, del Instituto Nacional. Traducidas al castellano para el uso de los estudios de la Inspección General de Caminos*. Madrid, Imprenta Real.

VEA MUNIESA, Fernando & VELAMAZÁN GIMENO, Mª Ángeles (2011) «La formación matemática en la ingeniería». In: Silva Suárez, Manuel (ed.) *Técnica e ingeniería en España, Vol. 6: El ochocientos: de los lenguaje al patrimonio*, pp. 299-344.

VELAMAZÁN, Mª Ángeles (1993) «Nuevos datos sobre los estudios de Geometría superior en España en el siglo XIX: la aportación militar». *Llull, Revista de la Sociedad Española de Historia de las Ciencias y de las Técnicas, 16(31),* 587-620.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Descriptive Geometry in the German Speaking Countries
1815 – 1915

Nadine Benstein, Sebastian Kitz & Klaus Volkert
University of Wuppertal (Deutschland)

We start our presentation with some remarks concerning the term "German speaking countries". This is necessary because in the 19th century there was no homogeneous state called Germany but a lot of different territories, of which each had its own educational system. Some details on the educational systems – e.g. the creation of polytechnic school, the higher schools – will be given in part one and two of our contribution. Then, we shortly discuss the meaning of the term "descriptive geometry" (*darstellende Geometrie*), which, in Germany, often had a much broader sense than its French or English counterparts. In the German language, "*darstellende Geometrie*" meant the general science of projections including central and parallel projections and much more. This is related to the idea to use "*darstellende Geometrie*" as the King's road to projective geometry, which we will discuss in more detail in part three of our contribution.

During the 19th century, descriptive geometry found its way into secondary school education in the territory associated with Germany. Dealing with the German school system during that period, it is important to differentiate between the higher and the lower school systems. The latter was the *Volksschule*, the educational institution visited by the vast majority of students (at times nearly 97%; cf. Klein & Schimmack 1907). The former, the higher school system, was divided into the *Gymnasium*, the *Realgymnasium* (founded in 1859 in Prussia), the *Oberrealschule* (1878), and the *Realschule* in its different forms. These school forms differed in their educational focus and purposes, namely the extent of traditionally humanistic or realistic contents and subjects. The *Gymnasium*, with its roots in the 18th century, aimed at traditional humanistic education including the studies of Latin and Greek. The *Realgymnasium* and the *Oberrealschule* developed out of the different types of *Realschulen* that were founded at the beginning of the 20th century in order to adapt secondary education to the new requirements of technical developments, but could not achieve the same status as the existing *Gymnasien* yet (cf. Jeismann 1987). The *Realgymnasium* combined the education in Latin with the teaching of natural sciences and modern languages, and the *Oberrealschule*, especially founded as a preparation for polytechnic schools or technical colleges (cf. Lietzmann 1949), resigned traditional humanistic education in favor of realistic and scientific contents.

From a scientific perspective, the higher school forms, *Realgymnasium* and *Oberrealschule*, are most relevant concerning the realization of the mathematical discipline, since a scientific treatment of elements of descriptive geometry could be attained in these institutions. Nevertheless, from a sociological point of view, the *Volksschulen* have to be considered in order to explore the dissemination of the elements of descriptive geometry among the German population during the relevant time span and, furthermore, the lower education system had always been known for its pedagogical and didactical considerations and innovations (cf. Breidenbach 1958).

According to Paul Zühlke, who published a survey about the status of descriptive geometry as a school subject at the *Realanstalten* (the higher school forms except for the *Gymnasium*) of the German Empire in 1911, it had been established in southern Germany higher schools by 1840 (cf. Zühlke 1911). In northern Germany, namely Prussia, this had not happened until its establishment as obligatory matter in the Prussian curricula of 1901 – even in the *Gymnasium*. By 1911, elements of descriptive geometry were integrated in all states of the German Empire (cf. ibid.). Yet, it has to be noticed that these elements were not always obligatory but sometimes optional, and that their treatment was not scheduled throughout all classes in every state. Furthermore, it has to be considered that the contents of descriptive geometry were not always taught in mathematics lessons, but often within drawing subjects (e.g. linear drawing). Both components – mathematics and drawing – are important for the training of spatial abilities (visual perception and visual thinking). In the Prussian curricula from 1901 for all higher school forms, vividness and insights rather than spatial abilities were demanded in connection with geometry. Interestingly, in the regulations for the *Volksschule* from 1872, the aim of training the visual perception ability was concretely mentioned in connection with the treatment of the depiction of three-dimensional objects.

In connection with descriptive geometry as a school subject, the insufficient qualification of teachers for descriptive geometry was often criticized (cf. Baur 1958, Zühlke 1911). A possible explanation for this matter might be that, not until 1905, the training of teachers was integrated into the technical universities (cf. Papperitz 1899, Lexis 1905), where descriptive geometry was integrally taught, including intensive drawing tutorials (cf. Papperitz 1899). The responsibility for the teacher training for the higher school forms was born by the universities. Problematic about that was, that, in the exam regulations for future teachers, descriptive geometry was partially embedded in the optional field of applied mathematics (at least in northern Germany) (cf. Fladt 1958), so that not every prospective teacher came in contact with it. Early exceptions were the University of Munich and the ETH in Zurich. Teacher trainees for the *Volksschule* attended separate seminars and institutions (cf. Klein & Schimmack 1907, Thiele 1938).

The establishment of descriptive geometry in tertiary education goes back to the beginning of the 19th century. At this time, there existed different types of *Fachschulen* (professional or technical schools or colleges) in Germany which, within the time of 2 or 3 years, trained their students in specific technical disciplines (e.g. hydraulic engineering, the traffic system, or the building industry). The majority of their graduates worked as tradesmen in a private business afterwards. Indeed, the state also needed a small number of engineers, who, initially, were trained at the aforementioned *Fachschulen*. When it became apparent that the provided education at these schools did not meet the newly emerged technical requirements and that, in the course of industrialization, a greater amount of engineers was needed, polytechnic schools were founded, which shared characteristics with universities and were in charge of the training of engineers from then on (cf. Lexis 1904).

The first polytechnic school in Germany was founded in 1825 in Karlsruhe and laid the focus (following the example set by the *École polytechnique*) on the one hand onto mathematics, descriptive geometry, and natural sciences, and on the other hand onto an extensive treatment of modern technique (cf. Obenrauch 1897). The field of descriptive geometry was represented by Guido Schreiber (1799-1871), who also published the first profound German textbook about descriptive geometry (*Lehrbuch der Darstellenden Geometrie nach Monge's Géométrie descriptive*). Thereby, he oriented himself towards the edition of 1811, while refraining from the separation of three-dimensional geometry and descriptive geometry, as Monge had done in tis edition, to avoid that the latter was reduced to a "naked and dry lesson of projections".

In later textbooks on descriptive geometry, also projective geometry was treated, which had been emerging in Germany during the 1830s. However, these two disciplines stayed widely separated. The first actual conflation of them was succeeded by Wilhelm Fiedler (cf. Wiener 1884).

At the end, of our talk we will turn to a specific case study – dealing with Wilhelm Fiedler (1832 – 1912) and his work. Fiedler never attended a university because he did not have the financial means to do so. Through his ability in drawing and his scientific interests, he got a job as a teacher of descriptive geometry at a *Gewerbeschule* (first in Freiberg then in

Chemnitz). In 1858, he received a doctoral degree by the University of Leipzig under the guidance of A. F. Möbius. Later, he changed to the polytechnic school in Prague (1864) and then to that in Zürich (1867). In Zürich, he was a professor for descriptive geometry and projective geometry (called "*Geometrie der Lage*" following von Staudt); there, he was strongly supported by Carl Culmann (1821 - 1881), the father of geometric statics, who was influenced by French ideas (he received part of his education at Metz [perhaps he even knew Poncelet's successor at the *Ecole d'application* at Metz, Felix Michot]). In Zürich, Fiedler was also responsible for the department of future teacher training.

Fiedler's basic aim was to use "*darstellende Geometrie*" as a preparation for projective geometry, thus, providing the first with a scientific aura borrowed from the second. He was convinced that "*darstellende Geometrie*" was also an excellent instrument to train spatial intuition. Fiedler presented his ideas in his textbook *Die darstellende Geometrie in organischer Verbindung mit der Geometrie der Lage* (1871, ^{2}1875, ^{3}1883 – 1888 in three volumes), which was translated into Italian (*Trattato della geometria descrittiva*) by A. Sayno and A. Padova in 1874.

This is an interesting work for several reasons. First, there is no tendency to separate descriptive and metric geometry (remember Poncelet and von Staudt), secondly he uses nowhere axioms but there is the idea – inspired by Steiner – of an organic construction of a whole based on some simple ideas. Last but not least, Fiedler's book is extremely rich concerning concrete examples, constructions, and so on. This underlines the idea that "*darstellende Geometrie*" can be seen as a source of problems and an inspirations for projective geometry.

Whereas it is sometimes said – e.g. by Chr. Wiener – that Fiedler created a specific German way of treating descriptive geometry, Fiedler's access was not successful in practice. In particular, his teaching was criticized for being too difficult and too abstract for future engineers. Nevertheless, it remains an interesting way of dealing with descriptive geometry. We will show that with the help of some concrete details (e.g. Fiedler's way of dealing with the cross ratio and its invariance under projections, his was of defining "*projektive Verwandtschaft*" inspired by Möbius).

Today, Fielder is known mainly because he translated (and reworked) several books by G. Salmon (in particular on conic sections and on the analytic geometry of space). Even today, we speak in German of “Salmon-Fiedler”. Fielder ran a huge correspondence with a lot of collegues[1].

References

Baur, Arnold (1958). “Die Darstellende Geometrie an den Höheren Schulen Norddeutschlands”. In: *Der Mathematikunterricht 4*, 1. 17-30

Benstein, Nadine (2015). *Darstellende Geometrie und Raumanschauung*. Unpublished master thesis, Bergische Universität Wuppertal.

Breidenbach, Walter (1958). *Raumlehre in der Volksschule.* 6th ed. Hannover: Hermann Schroedel Verlag KG.

Fiedler, Wilhelm (1871, ²1875, ³1883). *Die darstellende Geometrie in organischer Verbindung mit der Geometrie der Lage*. Leipzig: Teubner.

Fiedler, Wilhelm (1881). “Zur Geschichte und Theorie der elementaren Abbildungs-Methoden“. In: *Vierteljahreszeitschrift der Naturforschenden Gesellschaft in Zürich* 27, 125-175.

Fiedler, Wilhelm (1905). “Meine Mitarbeit an der Reform der darstellenden Geometrie in neuerer Zeit“. In: *Jahresbericht der Deutschen Mathematiker-Vereinigung* 14, 493-503.

Jeismann, Karl-Ernst (1987). “Das höhere Knabenschulwesen”. In: Jeismann, Karl-Ernst & Lundgreen, Peter (Eds.). *Handbuch der deutschen Bildungsgeschichte Band III 1800-1870: Von der Neuordnung Deutschlands bis zur Gründung des Deutschen Reiches.* München: Beck.

Klein, Felix & Schimmack, Rudolf (1907). *Vorträge über den mathematischen Unterricht an den höheren Schulen.* Leipzig: Teubner.

Lietzmann, Walter (1949). *Schulreform und mathematischer Unterricht*. Heidelberg: Quelle & Meyer.

Lexis, Wilhelm (Ed.) (1904). *Das Technische Unterrichtswesen*. In: *Das Unterrichtswesen im Deutschen Reich*, Vol. IV. Berlin: A. Asher & Co.

Maurer, Bertram (1998). *Karl Culmann und die graphische Statik*. Berlin-Diepholz-Stuttgart: IGN.

Papperitz, Erwin (1899). *Die Mathematik an den Deutschen Technischen Hochschulen*. Leipzig: Verlag von Veit & Comp.

Scholz, Erhard (1989). *Symmetrie. Gruppe. Dualität*. Basel-Boston-Berlin: Birkhäuser.

1 Fiedler’s correspondence with L. Cremona will be published soon by M. Menghini and others.

Schreiber, Guido (1828). *Lehrbuch der Darstellenden Geometrie nach Monge's Géométrie descriptive*. Karlsruhe u.a.: Herder.

Schubring, Gert (1988). "Ein früher 'Aufruf: Rettet die mathematisch-naturwissenschaftliche Bildung!': Die Denkschrift Schellbachs von 1860". In: *Der Mathematikunterricht 31*,1. 30-72.

Thiele, Gunnar (1938). *Geschichte der Preußischen Lehrerseminare Erster Teil: Allgemeine Voraussetzungen zur Geschichte der Preußischen Lehrerseminare*. Berlin: Weidmannsche Verlagsbuchhandlung.

Timerding, Heinrich E. (1909). "Über Ursprung und Bedeutung der darstellenden Geometrie". Speech at the public award presentation at the Technische Hochschule zu Braunschweig. Braunschweig: 26/11/1909.

Voss, Aurel (1913). "Wilhelm Fiedler". In: *Jahresbericht der Deutschen Mathematiker Vereinigung* 22 , 97-112.

Wiener, Christian (1884). *Lehrbuch der Darstellenden Geometrie, 1. Band.* Leipzig: Teubner.

Zühlke, Paul (1911). *Der Unterricht im Linearzeichnen und in der darstellenden Geometrie an den deutschen Realanstalten. Abhandlungen über den mathematischen Unterricht in Deutschland*, Bd. III, Heft 3.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Descriptive geometry in France: from the École polytechnique to the secondary school through the École centrale des arts et métiers

Évelyne Barbin
Laboratory LMJL UMR 6629, Université de Nantes (France)

Monge introduced the term "descriptive geometry" in a text of 1793 on "the studies in the schools for artists and workers of various kinds". In 1794-1795, he taught the descriptive geometry in the *École normale de l'An III*, devoted to future teachers, and in the *École Centrale des Travaux Publics*, which will become the *École polytechnique*. The famous textbook *Géométrie descriptive*, edited in 1799, comes from a transcription of his oral lessons given in the *École normale*. Jean Nicolas Pierre Hachette was the "assistant" of Monge in the *École polytechnique* in 1794 and he became professor of descriptive geometry there until his dismissal in 1816. He published *Supplément de la Géométrie descriptive de M. Monge* (1812), *Éléments de géométrie à trois dimensions* (1817) and his *Traité de géométrie descriptive* (1822). Charles François Antoine Leroy succeeded to Hachette as professor of descriptive geometry in the *École polytechnique* in 1816, where he stayed until 1849.
Rapidly the teaching on descriptive geometry decreased in the *École polytechnique*, but the subject appeared in the entrance program to this school in 1813 and arrived in the preparatory grades of private *Collèges* and *Lycées*. This led to the publication of textbooks devoted explicitly to the candidates to the *École polytechnique* and other schools of government. The authors were former students or examiners of the *École polytechnique*, and taught in the *Lycées* and *Collèges*. The descriptive geometry was taught also to artists and engineers, and, in this context, Louis Léger Vallée and Joseph Adhémar introduced novelties in their textbooks on descriptive geometry, like the rabattement and the decomposition of projections. From 1823 to 1841, Adhémar wrote many textbooks on descriptive geometry intended to artists and civil engineers, especially for the *Corps des Ponts et chaussées*.
In 1843, Théodore Olivier introduced the novelties of Vallée and Adhémar, under the name of "methods", and the famous "method of changes of projection planes" in his *Cours de géométrie descriptive*. He was professor in a rival school of *École polytechnique*, namely the *École Centrale des Arts et Manufactures*. This private school was created in 1829 to form

civil engineers and directors for industries and to develop the applications of the new sciences. It was an initiative of an industrial man, Alphone Lavallée, later joined by Olivier, the chemist Jean-Baptiste Dumas and the physicist Eugène Péclet. The *École Centrale* became a state school in 1857. The methods introduced in the textbook of Olivier met many criticisms in the period 1851-1856, which followed the reform of 1850 on the entrance to the *École polytechnique*. But they were promoted also in many textbooks, often titled "Elements of descriptive geometry" and intended for candidates to the *École polytechnique* and *Grandes écoles* or for students of other schools, like the *École des arts et métiers*. The authors referred to Olivier explicitly, like C. Bertaux-Villain (1847), Henri Édouard Tresca (1852), L. E. Aubré (1853) or M. Sasiar (1859-1860).

In 1853, Antoine Amiot, a former student of the *École normale supérieure*, teacher of the Lycée Saint-Louis and of the *École des beaux-arts*, published his *Leçons nouvelles de géométrie descriptive*, where he adopted the Olivier's methods. These *Leçons* are intended for the students of the preparatory grades for *École polytechnique*, but also for the *École normale supérieure*. Now, the future secondary teachers received teaching on descriptive geometry, and, in consequence, they will be prepared to teach it in *Lycées*. Indeed, descriptive geometry became a part of secondary teaching in grades of « elementary mathematics » in 1865, and the curricula followed the organization of Amiot's textbook. From 1863, many textbooks are written for the upper grade of the secondary school, like those of C. Briot and C. Vacquant (1869) or J. Dufailly (1869), and also for the *enseignement spécial*, like the one of E. Rouché (1875).

In this study, we will examine the process of changes in the teaching of descriptive geometry in the context of the engineer or artist schools, and the role of the creation of the *École centrale des arts et métiers*.

References

Adhémar, Joseph (1832). *Cours de mathématiques à l'usage de l'ingénieur civil. Géométrie descriptive*.

Adhémar, Joseph (1833). *Traité élémentaire de Géométrie descriptive*. Paris: Dépée Frères.

Adhémar, Joseph (1841). *Traité de géométrie descriptive*. Paris: Carilian-Gœury, Bachelier, Mathias.

Amiot, Antoine (1853). *Leçons nouvelles de géométrie descriptive*. Paris: Guiraudet et Jouas.

Aubré, L.E, Cours de Géométrie descriotive à l'usgage des élèves des Écoles Impériales, d'Arts et Méteirs, Châlons sur Marne: Barbat.

Barbin, Évelyne (2015). Top-down: the Role of the Classes préparatoires aux grandes Écoles in the French Teaching of Descriptive geometry (1840-1910). Bjarnadottir K., Furinghetti F., Pritz J., Schubring G., *Proceedings of the 3d Conference on the History of mathematics Education*, 49-64.

Barbin Évelyne (2015), Descriptive geometry in France: History of *Élémentation* of a Method (1795-1865), *International Journal for the History of Mathematics Education*, to appear.

Belhoste, Bruno (2001). La préparation aux grandes écoles scientifiques au XIX[e] siècle : établissements publics et institutions privées, *Histoire de l'éducation*, 90, 101-130.

Belhoste, Bruno (2012). Anatomie d'un concours: l'organisation de l'examen d'admission à l'École polytechnique de la Révolution à nos jours, *Histoire de l'éducation*, 94, pp. 141-176.

Bertaux-Levillain, C. (1847). *Éléments de géométrie descriptive*. Paris: Carilan-Gœury and Valmont.

Briot, Charles, Vacquant, Charles (1869). *Éléments de géométrie descriptive*. Paris: Hachette.

Hachette, Jean Nicolas (1817). *Éléments de géométrie à trois dimensions*. Paris: Courcier.

Hachette, Jean Nicolas (1822). *Traité de géométrie descriptive*. Paris: Corby.

Hottin, Christian, Georgeon-Liskenne Anne, Schöppel Claudia (2001). Mythologie centralienne, mythologie polytechnicienne. *Histoires de l'architecture*, 1, 61-74.

Leroy, Charles François Antoine, *Traité de géométrie descriptive* (1834). Paris: Carillan-Gœury & Anselin.

Monge, Gaspard (1992), *Leçons,* in Dombres, J. (dir.) *L'École normale de l'an III. Leçons de mathématiques*. Paris: Dunod.

Monge, Gaspard (1799). *Géométrie descriptive*. Paris: Baudoin.

Olivier, Théodore (1843). *Cours de géométrie descriptive*. Paris: Carilian-Gœury and Dalmont.

Rouché, Eugène (1875). *Éléments de géométrie descriptive*. Paris: Delagrave.

Sakarovitch, Joël (1994), La géométrie descriptive, une reine déchue, Belhoste, B., Dahan-Dalmedico A., Picon A. (ed), *La Formation polytechnicienne, 1794-1994.* Paris: Dunod.

Sakarovitch, Joël (1998). *Épures d'architecture*. Berlin: Birkhäuser.

Sasiar, M. (1859-1860). *Géométrie descriptive, École navale Impériale*. Brest: Typographie et Litographie E. Roger.

Tresca, Henri-Edouard (1852). *Traité élémentaire de géométrie descriptive rédigé d'après les ouvrages et les leçons de Th. Olivier.* Paris: Hachette.

Vallée, Louis Léger (1819). *Traité de géométrie descriptiv*e. Paris: Courcier.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

The teaching of Descriptive Geometry in Egypt (1837-1902)

Pascal Crozet, Laboratory SPHERE, University Paris Denis Diderot

Between the Arabic translation of Emile Duchesne's textbook in 1837 and the takeover of secondary and higher Education by the British in the 1890s, Descriptive Geometry has been taught in a sustained and regular way in Egyptian engineering and military Schools. This paper will focus on the main features of this period, marked by the French influence. The trajectory of the actors and the place of the discipline in the curricula will be discussed, as well as the evolution of the textbooks, from the translation of books like those by Olivier, Leroy or Gerono, to the composition of original treatises (Aḥmad Nağīb, Ṣābir Ṣabrī, ...).

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Charles Potier : the Origines of the Descriptive geometry in Russia

Dmitri Gouzevitch et Irina Gouzevitch

In 2013, we published the biography of Charles Potier, the third director of the Institute of the Corps of Engineers of Ways of Communication of Saint-Pétersburg (ICEWC; 1834-1836) [1]. Together with three other polytechnicians – Alexandre Fabre, Pierre-Dominique Bazaine et Maurice Destrem – he was invited to Russia in 1810. Potier was appointed to the newly created ICEWC. During the academic year 1810/11, Fabre taught there, for the first time in Russia, a course of descriptive geometry. Potier took it over the next year, 1811/12. In their teaching, they used as a basic source a manuscript copy of the work of Gaspard Monge.
From the summer 1812 to the spring 1815, the four polytechnicians lived in exil, in Siberia. Once back to Saint-Petersburg, Potier was appointed professor of the course of descriptive geometry. He occupied this chair during three years and a half, till the spring 1818, and he managed to extend the course by adding such sections as « La théorie des projections », « La coupe des pierres», « La perspective aérienne ». The teaching was held in French.
During this period, Potier wrote three handbooks of descriptive geometry and of its applications [2; 4; 5]. These works have been immediatly translated into Russian by his pupil and assistant Jakov Sevast'janov, and published in Russian version only [3; 6], or as a bilingual French-Russian edition. The fourth work which completes this collection is the *Cours de constructions = Курс построения* [7] which was then considered as an application of the descriptive geometry. Sevast'janov, who relayed Potier as professor of this discipline, wrote his own textbooks [8]. In spite of this, the Potier's books continued to be published during at list three more decades [9; 10], in Russia as well as abroad. In 1817, his book was published in France by the Ecole polytechnique [11], which could be explained by the lack of textbooks in this matter after a long series of wars. Contrastingly, the fact that the Société d'encouragement pour l'instruction élémentaire in Liège reedited Potier's textbook in 1842 [12], whereas the literature dealing with the descriptive geometry was no more lacking, testifies of its great didactic potential.

Potier’s work was extremely clear and concise, containing only few necessary illustrations and presented according to the axiomatic principle. Simply structured, it included three parts: definitions, preliminary theorems and applications. The author examined there the most frequent positions necessary to solve the practical problems. This approach allowed to evoid any complex constructions which overdid the drawing (épure) while the reasoning could be applied to any other example. To simplify the construction and the reading of the drawings, Potier used the dotted lines to mark the plans, and the methods of analysis to explain the graphic constructions. A limited number of drawings in the book aimed to stimulate the autonomous work of the students.

Potier enriched the theory of formation and construction of surfaces of a new type – the left cylinder and the left plan (or, in modern terms – the regulated surfaces of Catalan: cylindroids and conoides) applicable to the projects of ships, of the machines’ working elements, to the works of fortification and of ways of communication, and the le cylinder and the cone of revolution (envelopes) useful for the construction of shadows and of perspectives. The axiomatic presentation is also typical of the other Potier’s textbooks. Among his pioneering achievements – the application of the descriptive geometry to the aerial perspective, much earlier than Vallée (1821) and Hachette (1822).

The impact of Potier's works on the later development of the descriptive geometry is little studied. We can take the measure of it thanks to some illuminating examples.

The Parisian edition of his textbook appears among 10 main monographs of descriptive geometry quoted by Hachette in the foreword of his fundamental *Traité de géométrie descriptive* which made an assessment of realizations in the field for 1822 and which was reedited in 1828. Even in 1916, the Russian geometer Rynin quotes Potier’s works as referential. These works laid the foundations of the Russian scientific school of descriptive geometry, which counts some prominent scholars, mostly his ancient students and continuators: Ja. Sevast’janov, A. Reder, N. Durov, N. Makarov, V. Kurdjumov, N. Rynin. Very rapidly, the courses of descriptive geometry became part of the curriculum of the Russian high technical schools such as the Centrale School of military engineering, the Corps of cadets of mines, the Technological Institute, the Institute of Civil Engineers, the Artillery School, the Corps of the Cadets of Navy, the Pedagogical Institute, the University of Kazan, and so on. In most cases, they used Potier’s textbooks, often in Sevast’janov’s Russian version.

An « illustre inconnu » in France, Potier found in Russia the recognition of his work. The historiography of the Russian mathematics and of their teachings takes them into account.

There is however an aspect of the history which was highlighted during the preparation of his biography. It seemed, indeed, that Potier's works are at the origin of the scientific schools of descriptive geometry in Poland and in the German lands

Already in 1817, the *Traité de géométrie descriptive* by Potier was published in Polish version in Vilnius (Wilno) and in Warsaw [13]. His translator was Polish mathematician Gregor Hreczyna. Like Sevast'janov later on, he had to elaborate, on this occasion, the national Polish terminology of descriptive geometry. So, the work of Potier gave an impetus to the introduction of the descriptive geometry in Poland, and its book was of use as base for the teaching of this discipline at the University of Vilnius.

In Germany, Potier's works were mediated by the baron Hungern-Sternberg, native of Baltic States, military engineer and captain of the Russian service. In 1828, he published in Leipzig un a work in German titled «Projectionslehre» (« Treaty of geometrical projections ») and structured upon the plan proposed by Potier in his *Traité* of 1816. The German title is original: the baron tried to find an equivalent to the term « descriptive geometry » considering that his own would be more convenient for the Nordic languages. But his term did not acclimatize, especially as Poncelet had already created its projective geometry.

This brief overview allows us to glimpse Charles Potier's significant role in the broadcasting of the descriptive geometry and its applications in Europe. A role all the more particular as Potier never positioned himself as mathematician but only as a practical engineer who, having left the chair of descriptive geometry in the ICEWC, never touched the mathematical problems again. Simply, he was Polytechnician, pupil of the great masters and the responsible man. That is why, having received for task to create a school course to teach a new science to the future Russian engineers, he settled it at the highest scientific and didactic level. La noblesse polytechnicienne oblige!

References

1. Gouzévitch D., Gouzévitch I., N. Eliseev. *Karl Ivanovič (Charles Michel) Potier : 1785-1855: Troisieme directeur de l'Institut du Corps des ingénieurs des voies de communication*. SPb. : PGUPS, 2013. 492 p.
2. Potier Ch. *Traité de Géométrie descriptive...* . SPb. : Chez Pluchart, 1816. XII, 129 p., 2 f.
3. Pot'e K.I. *Osnovaniâ načertatel'noj geometrii...* / Per. Sevast'ânova. SPb., 1816. X, 119, [13] p.
4. Potier Ch. *Traité de la coupe de pierres... = Načal'nye osnovaniâ razrezki kamnej...* / Per. Sevast'ânova. SPb., 1818. 80 p.

5. Potier Ch. *Application de la Géométrie Descriptive à l'art du dessin...* SPb.: Chez A. Pluchart, 1817. 96 p.

6. Pot'e K.I. *Priloženie načertatel'noj geometrii* k risovaniû… / Per. Sevast'ânova. SPb., 1818.. VI, 90, [7] c.

7. Potier Ch.M., Bazaine P.D. *Cours de construction*. SPb., 1816 (1816/17?); Potier Ch.M., Bazaine P.D., Déviatine A. *Cours de construction*. SPb., 1819.

8. Sevast'ânov Â.A. *Priloženie načertatel'noj geometrii k risovaniû; Teoriâ teney, linejnaâ perspekstiva, optičeskie izobraženiâ.* 2 t. SPb., 1830; Idem. *Priloženie načertatel'noj geometrii k vozdušnoj perspektive, k proèkcii kart i k gnomonike*. 2 т. SPb., 1831 ; Idem. *Osnovaniâ načertatel'noj geometrii*. Izd. 2-e, ispr. i dop.SPb., 1834. III, XIV, 204 p. ; Idem. Kurs plotničnago iskusstva. SPb., 1840. 33 f.

9. Pot'e K.I. *Osnovaniâ načertatel'noj geometrii...* / Per. Sevast'ânova. 2-e izd. SPb., 1834. VI, 105, XI p.

10. Pot'e K.I. *Osnovaniâ načertatel'noj geometrii: Priloženiâ načertatel'noj geometrii k risovaniû*. SPb.: IKIPS., [18]49. 32 f.

11. Potier Ch. *Traité de Géométrie discriptive*. Paris: Firmin Didot, 1817. 96 p.

12. Potier. Traité de Géométrie descriptive. Nouv. éd. augmentée et publiée par la Société d'encouragement pour l'instruction élémentaire à Liège. Liège: Dessain, 1842. 96 p.

13. Potier Ch. *Wykład geometryi rysunkowey dla użycia uczniow Instytutu Drog Kommunikacyynych* / Przez Potier; gprzetł. G. A. Hreczyna. Wilno; Warszawa; 1817. [12], 95 s.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Descriptive Geometry in England: a drawing or a mathematical technique?

Snezana Lawrence, Bath Spa University (Great Britain)

This paper will investigate the translation versions and the reception that Descriptive Geometry had in the 19th century England. It will examine and juxtapose the ‘original’ technique with the various inventions and techniques that sprang in Britain throughout the 19th century, mostly with the aim of replacing the ‘French’ version.
Descriptive Geometry was considered to be too abstract for the teaching in technical schools and colleges in England at the time of its translation in the early 19th century. A few editions published for the military schools were never re-issued, and the architectural and engineering schools were not established until the 1830s, by which time an English equivalent of a technique was adopted. Although in the syllabi of engineering schools for example, the title sometimes remained to refer to ‘descriptive geometry’, the technique that was actually taught was a poorer version of the original as developed by Monge.
Most importantly, the perception that descriptive geometry is a drawing rather than a mathematical technique, remains to be widely held in England to the present time. This talk and forthcoming publication will explore this in detail.
Bibliography is extensive, and so only a very narrow selection is given before the presentation.

Bibliography

BARNARD Henry. National Education in Europe; being an account of the organization ... of public schools ... in the principal states ... Case, Tiffany & Co.: Hartford, 1854

BINNS William S. An Elementary Treatise on Orthographic Projection; being a new method of teaching the science of mechanical and engineering drawing, etc. E. & F. N. Spon: London, 1857

BINNS William S. A Course of Geometrical Drawing, containing practical geometry, including the use of drawing instruments, etc. John Weale: London, 1860

BINNS William S. An Elementary Treatise on Orthographic Projection and Isometrical Drawing, etc. Longman & Co.: London, 1864

BINNS William S. A Course of Geometrical Drawing, containing practical geometry, including the use of drawing instruments, etc. Simpkin & Marshall: London; John Heywood: Manchester, 1869

BINNS William S. The Second Course of Orthographic Projection, etc. E. & F. N. Spon: London, 1869

BOOKER Peter Jeffrey. "Gaspard Monge, The Father of Descriptive Geometry and Founder of the Polytechnic School." Paper presented to the Newcomen Society for the Study of the History of Engineering and Technology on 1st November 1961. In The Engineering Designer: London, 1961

BOOKER Peter Jeffrey. A History of Engineering Drawing. Chatto & Windus: London, 1963

BRADLEY Thomas. Practical Geometry, Linear Perspective, and Projection, etc. Baldwin & Cradock: London, 1834

BRADLEY Thomas. Lecture On Practical Plane and Descriptive Geometry, mechanical and machine drawing, and building construction, etc. G. E. Eyre & W. Spottiswoode: London, 1860

BRADLEY Thomas. Elements of Geometrical Drawing or, Practical Geometry, Plane and Solid, including both orthographic and Perspective Projection. Published for the Committee of Council on Education. Chapman and Hall: London, 1861

LAWRENCE, S. (2010) "Alternatives to Teaching Space – Teaching of Geometry in 19th century England and France". In Franco-British Interactions in Science Since the Seventeenth Century. R. Fox and B. Jolly (eds.) London: College Publications.

LAWRENCE, S. (2007) Descriptive Geometry in England – a Historical Sketch. in Proceedings of the 5th European Summer University on the History and Epistemology in Mathematics Education. Evelyne Barbin, Nada Stehlikova, Constantinos Tzanakis, (eds). Prague 2007.

LAWRENCE, S. (2003). History of Descriptive Geometry in England. In The Proceedings of the First International Congress on Construction History, Madrid, 2003. 14 pages.
http://en.structurae.de/refs/items/index.cfm?ID=r0004552

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Descriptive Geometry in Serbia: small places, big ideas, and the global phenomena

Snezana Lawrence, Bath Spa University (Great Britain)

Serbia was, from the late 13th to the late 19th century, a non-entity both geographically and culturally. With the rise of the nation states and the fall of the Ottoman Empire, the cultural and intellectual life of the Serbian people was reignited and reinvented at the very outset of the 19th century. This was done initially from across the border, and through Matica Srpska, the oldest surviving Serbian cultural and scientific institution founded in Pest (eastern part of Budapest) in 1826. The first translation of descriptive geometry into Serbian was done by Atanasije Nikolić (1803-1882), an officer of the Austro-Hungarian army, and the native of Sombor (now northern Serbia, then the Austro-Hungarian Empire).

Nikolić was a prolific author of mathematical textbooks, first to be published in the native language, and an energetic leader of educational reform. He held the first chair of mathematics at the first Lyceum in Serbia (Kragujevac), and tried to establish a Serbian vocabulary for all mathematical terms from Latin and modern languages into Serbian.

The talk and resulting publication will investigate how a small nation got to know descriptive geometry (as a global phenomena) before the *Elements* of Euclid, and how the technique played an important role in the development of applied mathematics at the schools and universities across the country, to remain a popular subject to the present time.

Initial bibliography is based on original research done by the author and noted below, and will also include some additional material from the archives in Serbia.

Bibliography

LAWRENCE, S. (2005) Balkan Mathematics before the WWI. BSHM Bulletin: Journal of the British Society for the History of Mathematics. Oxford: Taylor & Francis.2005: 4, 22-36.

LAWRENCE, S. and KASTANIS, N. (2005) Serbian Mathematics Culture of the 19th century. HPM Newsletter, 15-24.

LAWRENCE, S. (2008) “A Balkan Trilogy: Mathematics in the Balkans before World War I”. In The Oxford Handbook of the History of Mathematics. Robson, E. & Steddall, J (eds), Oxford University Press, Oxford.

LAWRENCE, S. (2008) "Mihailo Petrovic": contribution to the website dedicated to the celebration of the First Century of the International Commission on Mathematics Instruction 1908-2008, http://www.icmihistory.unito.it/portrait/petrovic.php, ed. Livia Giacardi and Fulvia Furinghetti.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Descriptive Geometry in Denmark

Jesper Lützen
Department of Mathematical Sciences, University of Copenhagen (Denmark)

Teaching and research of descriptive geometry in Denmark was from the start connected to the Polytechnical High School in Copenhagen. This school was founded in 1829 after HC. Ørsted and a commission of university professors had modified a plan originally sent to the King in 1829 by the mathematician and astronomer G. F. Ursin. Ursin's plan was to create a polytechnic school modelled after the German Gewerbeschulen but the university commission increased the intended academic level. Ørsted had already tried to upgrade the level of the natural sciences at the university without much success and he now saw the chance to connect the new high school to the University as a kind of faculty of natural sciences. Ørsted and the commission explicitly modelled their plan of the school on the Paris École Polytechnique. They emphasized the need for laboratory facilities also for the students, and syllabus included both differential calculus, including differential equations and analytic geometry. However strangly enough descriptive geometry was not mentioned explicitly neither in the original plan nor in the first rules og the Polytechnic Highschool. However the rules mention laboratories in geometrical drawing, and apparently descriptive geometry was taught in this connection.
The first year after the opening of the school descriptive geometry and drawing was taught together in connection with the course on machines, and that proved to be a bad idea. To be sure, the Professor of machines, GF. Hetsch, had written a book on geometric drawing together with Ursin for use in the teaching of architects at the arts academy, so as pointed out by Wagner (1998, p. 313)he was probably the most able teacher of the subject in the kingdom. However, according to Steen (1879 p. 10) his assistant, cabinet maker Olsen did not possess of the necessary theoretical knowledge about geometry. The situation was improved in the summer of 1830 when Lieutenant (later Colonel) Keller was appointed to take care of the teaching of descriptive geometry. He had just arrived from Paris where he had learned the subject. The same year Keller was also nominated teacher of descriptive geometry at the Royal Military High school that admitted its first students that year. He taught the two courses in parallel for 31 years and wrote a new book on the subject. He seems to have been a

successful teacher but it is debated in the secondary literature whether his classes were too theoretical for the polytechnicians.

He was succeeded by CJL. Seidelin who taught even longer, namely from 1861 to 1903. His text book on descriptive geometry appeared in three editions in this period. According to Lundbye, Seidelin was not a good teacher. Still his position that had only been part time assistant position, was upgraded to a docentship in 1891 and further to a professorship in 1894. That placed the subject on the same level as analytic geometry and mechanics that had been taught by titular professors since the opening of the school.

When Keller took over the descriptive geometry course, it was separated from the drawing class and lab-work. This separation between theory and practice was a matter of debate, and during a few years in the early 1900s the beginning of the descriptive geometry course was taught in connection with the drawing classes. However it was no success and the decision was reversed after a few years.

Seidelin was succeeded by Johannes Petersen (Hjelmslev) and when he became professor at the University in 1917 Tommy Bonnesen took over his professorship at the Polytechnic High School. Hjelmslev and Bonnesen were both very active in the debates concerning the teaching of geometry in the Danish schools, Hjelmslev advocating for his so-called geometry of reality. Moreover, they both became internationally famous, Bonnesen for his textbook written together with Fenchel about convexity.

In the talk I hope to be able to develop on this story. In particular I hope I can answer (some of) the following questions:

How does the different Danish textbooks on descriptive geometry compare with each other and with foreign text books? What characterizes their methods? Can one see a general development? Did Hjelmslev's ideas on geometry of reality influence his presentation of projective geometry? Was there a clear distinction between descriptive geometry and projective geometry, or was descriptive geometry almost synonymous with synthetic geometry?

Were there debates at the Polytechnic High School concerning the purpose, content and methods of the course on descriptive geometry, in particular concerning the balance between theory and practice?

What was the situation at the Military High School?

The analysis course was common to the university students and the polytechnic students until around World War II. They also took part in the drawing course. Did the university students also follow the course on descriptive geometry?

When did the course on descriptive geometry stop?

The Technical University of Denmark (the successor of the Polytechnic High School) has a well-organized archive and a scholarly staffed center for the history of technology (http://www.historie.dtu.dk/Omos). They have been a great help in my preliminary investigations and will continue to help me with this research that they consider to be a desirable complement to their main interests in the history of technology.

Literature

Harnow, H. *Den danske Ingeniørs Historie 1850-1920*. Systimes teknologihistorie 2005.

Lundbye, J.T. *Den Polytekniske Læreanstalt 1829-1929*, Gad, København

Steen, A. *Polyteknisk Læreanstalts første halvhundrede år 1829-1879*, Bianco Luno, København 1878

Wagner, M. F. *Det Polytekniske Gennembrud*. Aarhus Universitetsforlag 1998.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Luigi Cremona and Wilhelm Fiedler:
Central projection as a link between projective and decriptive geometry.

Marta Menghini, Dipartimento di Matematica, Università di Roma Sapienza (Italy)

The object of this talk is placed in the period of the Unification of Italy, in 1860s and 1870s, when the new country builds up its educational systems. Particularly the technical instruction is influenced by French and German models. We will explore the links between the work of Luigi Cremona and that of Wilhelm Fiedler in the field of descriptive geometry, analyzing their books, their papers, and their correspondence.
Like Cremona, Fiedler sees a symbiosis between descriptive and projective geometry.
It is Fiedler who succeeds in framing central projection into descriptive geometry, establishing a biunivocal correspondence between a fifure and it representation through central projection.
The book of descriptive geometry by Fiedler (1874), written for the *Technische Hochschulen* of Germany, was explicitly translated into Italian and adapted for use at secondary school level: in the Technical Institutes of the Italian Kingdom. In the same kind of school also Cremona's Projective Geometry (1871) was used. In the following years, the book of Cremona was translated into German, French and English to be used in higher education. According to Fiedler, the main scope of the teaching of descriptive geometry is the scientific construction and development of "Raumanschauung". Fiedler reinforced this point of view in a paper translated and published in the *Giornale di Matematiche*. In the period 1822-1888 a correspondence between Fiedler and Cremona takes place, where Fiedler shows to be interested in Italian secondary Technical Education.
This talk will evidence the opinion that both geometers had about technical istruction at school and University level, their vision about the educational role of descriptive geometry, their use of central projection, and the links with Monge's original conception.
The link between projective and descriptive geometry epytomizes, in that times, the link between pure mathematics and its applications. The relation between the two topics will last only a few years at secondary school level, but some more at university level. Important mathematicians will have a role at the moment of the creation of the Faculty of Archiitecture.

References

Dell'Aglio, Luca , Emmer, Michele & Menghini, Marta (2001). Le relazioni tra matematici e architetti nei primi decenni della Facoltà di Architettura: aspetti didattici, scientifici e istituzionali. In *La Facoltà di Architettura dell'Università di Roma La Sapienza dalle origini al duemila (a cura di V. Franchetti Pardo)*, Roma : Gangemi Editore, 55-72

Fiedler, Wilhelm (1874). *Trattato di geometria descrittiva*. Firenze: Successori Le Monnier.

Knobloch, Eberhard & Reich, Karin (eds), to appear. 50. Letters from Wilhelm Fiedler (1862-1888). In Israel (ed) De Diversis Artibus, Correspondence of Luigi Cremona (1830-1903), BREPOLS: Turnhout.

Menghini, Marta "The Role of Projective Geometry in Italian Education and Institutions at the End of the 19th Century". *International Journal for The History of Mathematics Education*, 1(2006): 35-55.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Developments arising from geometry as a language for technology and architecture in the late Modern European space: Descriptive geometry in 19th century Spain

A. Millan Gasca
Roma Tre University, Rome (Italia)

The 19th century evolution of the geometrical tools of the engineer and the architect (drawing and representation, the training of space vision, design, and "the art of avoiding calculation") had both an influence on the cultural background of these professionals or civil servants and on the evolution of mathematics and the "mathematical world". This evolution is part of the constitution of the "Europe of sciences", the European space (Blay, Nicolaidis 2001) combining national scientific development and international dialogue and networking.
Engineers and architects (often military personnel) have had an important part in the tension versus innovation characteristic of the European scene since the late Middle Ages; the level of mathematical culture in the European countries in the Modern Age depended strongly on the demand of mathematical information from this professional entourage – together with accounting and banking (Rossi 1970, Knobloch 2004). In the age of liberalism both professions (with some conflicts in architecture) evolved into scientific-learned trustworthy experts serving both the state or the private companies (Buchanan 1989, Picon 1992, Porter 1995, Nicolaidis, Chatzis 2000. Millán Gasca 2005).
Monge's descriptive geometry and the scientific formation model of the École polytechnique was the input of an evolution regarding the geometrical tools for design and technical calculation as well as technical education, in response to the evolution of technology and thanks to the dynamism derived from its spreading among almost every European country and cultural area. Thus, the creation of the Federal Institute of Technology (ETH) in Zurich in 1855, and Culmann's development of the Mongian idea of "drawing as the language of the engineer" leading to graphical calculus and statics (Scholz 1994) radically renewed the geometrical instruments of the technological professions. This innovation, together with Reye's and Fiedler's combination of descriptive and projective geometry as a basis to graphical methods in engineering education at the ETH (Knobloch 2011, Hensel 1989), spread in North and Mediterranean Europe. The spirit of emulation and national pride

suggested to import new trends in research and educational innovations (Hormigón, Millán 1991, Millán Gasca 2011).

If France – as a matter of fact the cradle of these developments – showed reluctant, as Darboux complained in his letters to Cremona (Dell'Aglio forthcoming), Italy newly founded or reorganized Schools of engineers (in Milan and in Rome) and Cremona's books took over, contributing to their diffusion and development outside the German speaking area. Moreover, Cremona attention to applications was combined with attention to the cultivation of science for its own sake (because science was for him the "religion of the century"), and his research stimulated a new interest in algebraic geometry (birrational transformations and the theory of surfaces) and in synthetic methods among professors, engineers, and military personnel involved in geometrical education (Israel, forthcoming). In the 80's and 90's new developments regarded pioneer work in graph theory (Reye's "configurations") developed also in connection with logistics (Gropp 2004, Knobloch et al 1994).

The Cremonian balance between the attention to descriptive geometry and engineering and architectural applications of geometry and the development of mathematical research with new geometrical results was not always easy to keep, as we will see considering the case of Spain. I will describe the situation in Barcelona (where a debate on the growing abstraction and depart from intuition and the physical world of geometry took place) and in Madrid, where in 1892 Torroja y Caballé, who in his 1870 short treatise mentioned Fiedler and La Gournerie textbooks was praised by the State education national council for having broken the "old and paltry forms of descriptive geometry" adopting new, larger ones more suitable for "the fair aspirations of pure science" (see Millán Gasca 1990, 1991). The French geometrical tradition (descriptive and projective geometry), and then geometrical "purism" in Italy and Germany aroused admiration, but the new university ethos of research spread very slowly, because of the importance given to technological education rather than to liberal education, including science education (Millán Gasca 2012). Significantly enough, Torroja's sons were an outstanding structural engineer (Eduardo, see Levi et al 2000) and a pioneer of photogrammetry (José María, see Muro Morales et al 2002). The example of Spain is useful to gain a better understanding of the emergence of late modern European outlook on the relationship between pure science and technology, analyzing the case of descriptive geometry in this framework: the tension between pure science and technology which is still present in the European Union scientific research policy.

References

National Spanish Archive, Archivo General de la Administración, Legajo 1470-76 (E. y C.)

E. AUSEJO, M. HORMIGÓN, A. MILLÁN, M. A. VELAMAZÁN 1990, "Mathematics and engineering at the rise of capitalism in Spain", in *The interaction between technology and science*, B. Gremmen ed., Wageningen, Wageningen Agricultural University: 199-209.

M. BLAY, E. NICOLAÏDIS, eds. 2001 *L'Europe des sciences. Constitution d'un espace scientifique*, Paris, Seuil.

R. A. BUCHANAN 1989, *The engineers: a history of the engineering profession in Britain, 1750-1914*, London, Kingsley.

L. CARLEVARIS, L. DE CARLO, R. MIGLIARI, eds. 2012 *Attualità della geometria descrittiva*, Roma, Gangemi editore.

L. CREMONA 1872 *Le figure reciproche nella statica grafica*, Milano, Hoepli (3a ed. Milano, Hoepli 1879).

1873 *Elementi di Geometria Projettiva, ad uso degli Istituti Tecnici del Regno d'Italia*, vol. I, Roma/Torino/Milano/Firenze, Paravia (English translation *Elements of Projective Geometry*, Oxford, Clarendon Press, 1885).

1874 *Elementi di calcolo grafico. Ad uso degli istituti tecnici del regno d'Italia,* Torino, Paravia (Engl. translation with Cremona 1872, *Graphical Statics. Two treatises on the Graphical Calculus and Reciprocal Figures in Graphical Statics,* Oxford, Clarendon Press, 1890).

L. DELL'AGLIO forthcoming (ed.) "Letters from Gaston Darboux (1870-1898)" in in *Correspondence of Luigi Cremona (1830-1903), conserved in the Department of Mathematics, "Sapienza" Università di Roma*, G. Israel, gen. ed., Turnhout, Brepols.

W. FIEDLER 1867 "Das Polytechnikum in Mailand (Augsburger)", *Allgemeine Zeitung*, 24.2, Nr. 55: 894-895.

1871 *Die darstellende Geometrie: ein Grundriß für Vorlesungen an Technischen Hochschulen und zum Selbststudium*, Leipzig, Teubner.

J. DE LA GOURNERIE 1860 *Traité de géométrie descriptive*, Paris, Mallet Bachelier.

I. GRATTAN-GUINNESS 2002 "The end of dominance: The diffusion of French mathematics elsewhere, 1820-1870", in *Mathematics unbound: The evolution of an international mathematical research community, 1800-1945*, K. Parshall, A. Rice, eds., Providence, R.I., American Mathematical Society: 17-44

H. GROPP 2004, "*Reseaux réguliers* or regular graphs. Georges Brunel as a French pioneer in graph theory", *Discrete mathematics*, 276 (1): 219-227.

O. VON GRUBER 1924 *Einfache und Doppelpunkteinschaltung im Raume*, Jena, Fischer.

S. HENSEL 1989 "Zu einigen Aspekten der Berufung von Mathematikern an die Technischen Hochschulen Deutschlands im letzten Drittel des 19. Jahrhunderts", *Annals of Science*, *46*: 387-416.

M. HORMIGÓN, A. MILLÁN 1992, "Projective geometry and applications in the second half of the nineteenth century", *Archives Internationales d'Histoire des Sciences*, *42* (129): 269-289.

G. ISRAEL, forthcoming, "Luigi Cremona", in *Correspondence of Luigi Cremona (1830-1903), conserved in the Department of Mathematics, "Sapienza" Università di Roma*, Turnhout, Brepols.

E. KNOBLOCH 2004, "Mathematical methods in preindustrial technology and machines", in Lucertini, Millán Gasca, Nicolò, eds.: 3-20.

2013 "Luigi Cremona and his German correspondents", unpublished lecture, *International Conference Mathematical Schools and National Identity (16th-20th cent.)*, Turin, October 10-12, 2013.

E. KNOBLOCH, J. NIEHANS, A. HOFMANN, A., R. D., TEOCHARIS 1994, *Wilhelm Launhardts "Mathematische Begrüngung der Volkswirtschaftslehre". Vademecum zu einem Klassiker der Theorie der Raumwirtschaft*, Düsseldorf, Verlag Wirstschaft und Finanzen GmbH.

F. LEVI, M. A. CHIORINO, C. BERTOLINI CESTARI 2000 *Eduardo Torroja. From the philosophy of structures to the art and science of building*, Milano, Franco Angeli.

G. LORIA 1921 *Storia della geometria descrittiva*, Milano, Hoepli.

M. LUCERTINI, A. MILLÁN GASCA, F. NICOLÒ, eds. 2004, *Technological concepts and mathematical methods in the evolution of modern engineering systems*, Basel, Birkhäuser.

A. MILLÁN GASCA 1990a *Estudio de la obra de Julio Rey Pastor en el contexto del desarrollo histórico de la geometría proyectiva*, Zaragoza, University of Zaragoza, unpublished Ph.D. Thesis.

1990b "Methods of Synthetic Geometry in the Second Half of the Second Century", in *International Symposium Structures in Mathematical Theories (San Sebastián, 25-29 September 1990)*, A. Díaz et al. (eds) San Sebastián, Universidad del País Vasco: 283-288.

1991 "Los estudios de geometría superior en España en el siglo XIX", *Llull, Revista de la Sociedad Española de Historia de las Ciencias y de las Técnicas, 14* (26): 117-186.

1992 "I corrispondenti spagnoli di Luigi Cremona", in: *La corrispondenza di Luigi Cremona (1830-1930), volume 1*, Millán, A. (ed) Roma, Università di Roma La Sapienza/Facoltà di Scienze: 165-172.

2003 "Early approaches to the management of complexity in engineering systems", in *Determinism, Holism, and Complexity*, V. Benci et al. (eds.) New York/Boston/Dordrecht/London/Moscow, Kluwer Academic/Plenum Publishers: 349-357.

2005 *Fabbriche, sistemi, organizzazioni. Storia dell'ingegneria industriale*, Milan, Springer.

2011 "Mathematicians and the nation in the second half of the nineteenth century as reflected in the Luigi Cremona correspondence", *Science in context*, 24, 1: 43-72.

2012 "La matematica nella "sfida della modernità" della Spagna liberale e il ruolo del modello italiano", in *Europa matematica e Risorgimento Italiano*, L. Pepe, ed., Bologna, CLUEB: 81-90.

C. M. DE MOŸ 1888 *Estática gráfica. Primera parte: Geometría Proyectiva.* Barcelona.

S. MUNDÍ I GIRÓ 1884 *Apuntes de geometría de la posición tomados de las explicaciones del Dr. D. Santiago Mundí, catedrático de dicha asignatura en esta Universidad, por D. Julio Enamorado, D. Arturo Ydrach, D. Luis Cuello y D. Arturo Vidal.* Barcelona (self published treatise).

J. I. MURO MORALES, L. URTEAGA, F. NADAL 2002, "La fotogrametría terrestre en España (1914-1958)", *Investigaciones geográficas*, 27: 151-172.

E. NICOLAÏDIS, K. CHATZIS, K., eds. 2000 *Science, technology, and the 19th century state*, Athens, Institut de Recherches Néohelleniques.

A. PICON 1989 "Les ingénieurs et la mathématisation. L'exemple du génie civil et de la construction", *Revue d'Histoire des Sciences*, 42 (1-2):155-172;

PICON, A., 1992, *L'invention de l'ingénieur moderne. L'Ecole des Ponts et Chaussées 1747-1851*, Paris, Presses de l'Ecole Nationale des Ponts et Chaussées.

PORTER, T. M., 1995, *Trust in numbers*, Princeton (N. J.), Princeton University Press.

T. REYE 1866, *Die Geometrie der Lage*, Hannover, Rümpler, (2nd ed. 1868)

P. ROSSI 1970, *Philosophy, Technology and the Arts in the Early Modern Era, 1470-1700*, New York, Harper &Row.

J. SAKAROVITCH 1996 *Epures d'architecture. De la coupe des pierres à la géométre descriptive XVI^{e}-XIX^{e}*, Basel, Birkhäuser.

E. SCHOLZ 1994 Graphical statics, in *Companion Encyclopedia of the History and Philosophy of the mathematical sciences*, I Grattan-Guinness, ed., London, Routledge: vol. 2, 987-993.

R. TATON 1954, *L'histoire de la géométrie descriptive*, Paris, Palais de la Découverte.

E. TORROJA 1879 *Axonometría o perspectiva axonométrica: sistema general de* *representación geométrica que comprende, como casos particulares, las perspectivas caballera y militar, la proyección isográfica y otros varios*, Madrid (Imp. de Fortanet).

1884 *Geometría descriptiva. Programa y resumen de las lecciones de Geometría descriptiva esplicadas en la Universidad Central por D. Eduardo Torroja*, tomo I, Madrid (manuscript treatise).

1899 *Tratado de Geometría de la Posición y sus aplicaciones a la geometría de la medida,* Madrid (Establecimiento tipográfico de G. Juste).

R. S. TURNER 1971 "The growth of professional research in Prussia, 1818 to 1848: Causes and context", *Historical studies in the physical sciences* 3: 137-182.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

History of descriptive geometry teaching in the Czech Lands before the war II

RNDr. Vlasta Moravcova, University of Prague (Czech Republic)[2]

Introduction

The development of industry in Europe caused the progress of technical sciences and their education during the 19th century. Several polytechnical schools were established in the manner of the École Polytechnique (1806 in Prague, 1808 in Naples, 1811 in Graz, 1815 in Vienna, 1817 in Rome, 1821 in Berlin etc.). Descriptive geometry was gradually becoming one of the obligatory subjects of technical studies in many European countries, including Austro-Hungarian Empire, primarily in Cisleithania in which the Czech lands were embodied.

Descriptive geometry at secondary schools

Descriptive geometry (as an important part of technical education) appeared as a subject firstly at technical universities, but soon also at secondary schools, especially at real-schools[3] in 1850s.

Real-schools were instituted as six-year secondary schools in 1849. The number of classes was increased to seven in about 1870. Firstly the education was provided only in the German language, but from 1860s the Czech language was also used.

Descriptive geometry was being integrated into education gradually and without a given curriculum in 1850s and 1860s. The curriculum of descriptive geometry for real-schools was firstly determined in 1874 and modified in 1898, 1909 and 1933. Initially it contained orthogonal projection onto two planes (Monge's projection) and central projection (especially linear perspective). The other projection methods like orthogonal axonometry and oblique projection were introduced in the first half of the 20th century, but all parts were reduced and simplified as well.

The first graduation exams at some real-schools were held in 1869 (in connection with extension to seven classes); since 1872 these exams were set by law (before that the exams could be taken only at grammar schools). Descriptive geometry exam was one of the obligatory parts at real-schools, it had a written form and lasted five hours.

---

2 *Gymnasium Na Pra□ a□ ce, Prague 3* and *Department of Mathematics Education, Faculty of Mathematics and Physics, Charles University in Prague, Prague 8.*

3 Real-school (from German: *die Realschule*) was a special kind of secondary school with the emphasis on mathematics and natural sciences as a contrary to grammar school (gymnasium), where more lessons of Greek, Latin etc. were taught. These real-schools were constituted in Austria in 1849 by the Exner-Bonitz reform.

The high level of descriptive geometry at real-schools before the World War II can be supported with several extant materials found in libraries or archives like graduation exam exercises, students' drawings, school notes etc.

The development of the descriptive geometry curriculum can be also observed through Czech textbooks for real-schools. The first one was published in 1862 and even though it contained many mistakes, it was very important for Czech descriptive geometry because of the formation of the Czech terminology. Problems with the terminology were worked out by Vincenc Jarolímek (1846–1921) in his textbook *Deskriptivní geometrie pro vyšší školy reálné* (Praha, the 1st part 1875, the 2nd part 1876, the 3rd part 1877). This one was supplemented with the German-French-Czech dictionary containing all used terms. Moreover, all topics were organized very clearly and logically and the signage was similar to the contemporary one. The textbook was so good that there was a high demand for it abroad. It was translated into Bulgarian and published in Plovdiv in 1895.

Descriptive geometry was also taught at real-grammar schools[4] and at secondary industrial schools before the World War II.

Real-schools ceased to exist during the World War II (most of them were transformed into grammar schools) and since then importance of descriptive geometry at secondary schools has been decreasing continuously.

Descriptive geometry at technical universities

The greatest emphasis was put on descriptive geometry at technical universities, which were intended to prepare students for practical jobs in technical professions like an architect, a building constructor, an engineer etc. Descriptive geometry lessons appeared firstly at the Technical University in Prague[5] in 1830s. The department of descriptive geometry was established there in 1850. The first professor was Rudolf Skuherský (1828–1863). He created the first descriptive geometry syllabus and in 1861 started to lecture in the Czech language (the lessons were initially organized in German). His name is also associated with an orthogonal projection method similar to axonometry.

Among other important Czech descriptive geometry lecturers we can mention František Tilšer (1825–1913), Karel Pelz (1845–1908), Vincenc Jarolímek (1846–1921), Bedřich Procházka (1855–1934), František Kadeřávek (1885–1961) and Josef Kounovský (1878–1949). The German lessons were taught by Wilhelm Fiedler (1832–1912), Karl Josef Küpper (1828–

4 Real-grammar school (from German: *das Realgymnasium*) was a special kind of eight-year secondary school that was legalized in Austria in 1908 (schools of this kind were already established in the Czech lands since 1862) as a compromise between real-schools and classical grammar schools.

5 The Technical University in Prague was formally divided into the Czech and the German one in 1864. The separated Czech Technical University and the German Technical University were established in 1869.

1900), Eduard Janish (1868–1915) and Karl Mack (1882–1943). V. Jarolímek and B. Procházka published a suitable textbook for technical school students in 1909. Explanation of topics has logical structure and is focused on applications. The book is a continuation of Jarolímek's textbook for secondary schools.

Besides the Czech and German Technical University in Prague the German (1849) and Czech (1899) Technical University in Brno were established.

Technical university students graduated especially from real-schools. Syllabi of descriptive geometry lessons continued in the curriculum for real-schools. The characteristic feature of the descriptive geometry lessons was the high number of them and their difficulty. They were obligatory for all students except for chemists; moreover, descriptive geometry was one of the topics in state exams. The syllabi of particular universities were firstly different, but they were gradually being adjusted during the second half of the 19th century and at the beginning of the 20th century they were already similar at all technical universities.

Conclusion

The descriptive geometry education at real-schools and at technical universities in the Czech lands was at the advanced level in the second half of the 19th century and at the beginning of the 20th century (in comparison with the present day). Moreover, many first-rate geometers published significant scientific works on descriptive geometry with new results in this period.

Selected references

Bečvářová M.: *Česká matematická komunita v letech 1848 až 1918.* Edition Dějiny matematiky, vol. n. 34, Matfyzpress, Praha, 2008, 355 p.

Birk A.: *Die Deutsche Technische Hochschule in Prag 1806–1931.* J. G. Calve'sche Universitäts-Buchhandlung Robert Lerche, Prag, 1931, 177 p.

Černý J.: *150 let deskriptivní geometrie na pražské technice.* Pražská technika **5** (2004), 8–10.

Drábek K.: *125 let katedry matematiky a deskriptivní geometrie stavební fakulty ČVUT.* Dějiny věd a techniky **12** (1979), 33–45.

Entwurf der Organisation der Gymnasien und Realschulen in Oesterreich. Ministerium des Cultus und Unterrichts, Wien, 1849, 258 p.

Folta J.: *Vytváření ortografických názorných zobrazovacích metod a přínos R. Skuherského k jejich vypracování.* In *Sborník pro dějiny přírodních věd a techniky VII*, Nakladatelství Československé akademie věd, Praha, 1962, 27–61.

Folta J.: *Česká geometrická škola: historická analýza.* Academia, Praha, 1982, 90 p.

Lavička V.: *Deskriptiva ze stanoviště historicko-paedagogického.* F. & V. Hoblík, Pardubice, 1883, 156 p.

Loria G.: *Storia della geometria descrittiva dalle origini sino ai giorni nostri.* Ulrico Hoepli, Milano, 1921, 584 p.

Moravcová V.: *Descriptive Geometry Teaching in the Czech Lands.* Doctoral thesis, Faculty of Mathematics and Physics, Charles University in Prague, Prague, 2015, 496 p.

Obenrauch F. J.: *Geschichte der darstellenden und projectiven Geometrie mit besonderer Berücksichtigung ihrer Begründung in Frankreich und Deutschland und ihrer wissenschaftlichen Pflege in Österreich.* Carl Winiker, Brünn, 1897, 443 p.

Sklenáriková Z.: *Z dejín deskriptívnej geometrie v Rakúsko-Uhorsku.* In Bečvář J., Fuchs E. (ed.): *Matematika v proměnách věků II*, edition Dějiny matematiky, vol. n. 16, Prometheus, Praha, 2001, 14–45.

Stark F.: *Die k. k. deutsche technische Hochschule in Prag 1806–1906.* Deutsche technische Hochschule, Prag, 1906, 518 p.

Šišma P.: *Matematika na německé technice v Brně.* Edition Dějiny matematiky, vol. n. 21, Prometheus, Praha, 2002, 322 p.

Velflík A. V.: *Dějiny technického učení v Praze, I. díl.* Nákladem sboru professorského c. k. české vysoké školy technické v Praze a České matice technické, Praha, 1906 and 1909 (2 parts), 632 p.

Velflík A. V.: *Dějiny technického učení v Praze, II. díl.* Česká matice technická, Praha, 1910 and 1925 (2 parts), 337 p.

Veselá Z.: *Vývoj české školy a učitelského vzdělání.* Masarykova univerzita v Brně, Brno, 1992, 147 p.

Annual reports of secondary schools, programmes of universities and other archival sources.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

The development of descriptive geometry in Greece

Christine Phili, National Technical University of Athens (Greece)

Introduction

It might be stressed that even captive Greece was not an intellectual desert. Before the proclamation of the war of Independence in 1821 and more specifically between 1770 and 1821 many mathematical textbooks were published in Greek in Europe, as for example E. Voulgaris, Elementary treatise of the Elements of Mathematics... Leipzig 1797 (partial translation of Johann Andreas von Segner's book Elementa Arithmeticae, geometricae et Calculi geometrici. Neue Auflage Halle, Magdeburg 1756; S.Assanis and I. Sparmiotis, Summary of Conic sections of Guidonis Grandi... Vienna 1800 (translation of Guido Grandi's book); Sectionum Conicarum Synopsis Firenze 1750; E. Voylgaris, Elements of Tacquet's Geometry (translation of A. Taquet's book Elementa geometricae. Antwerp 1654).

All these books are written in a difficult, archaic language or tend to be incomplete and unmethodical, and generally never proved their effectiveness as university handbooks. The main aim of all these books was the education of the Greeks, an indispensable condition for their revolt.

The Ionian Academy

The Ionian Academy, founded in 1823 by the generous Frederic North, 5th Earl of Guilford, and officially starting to operate in 17 May 1824, became the new cradle of Letters and Sciences in Corfu , and was the preparatory stage for the intellectual rebirth of Greece.Ioannis Carandinos (1784-1834), former student (auditeur libre) of the École polytechnique, had quickly understood the importance of methodical teaching and adequate handbooks. So he conceived an ingenious idea for the Academy's function and for improving Greek scientific prospects. He worked day and night translating French books into Greek.

Having rejected former treatises for his lectures, he began to translate "pilot books" for teaching and for students' preparation.

The mathematical level of his courses was very high through his training and through his published and unpublished books.As the lectures of descriptive geometry constituted a part of the curriculum , Carandinos taught according Monge's text , as he stressed in his autograph note. Carandinos, as he revealed in his letter addressed to the French Academy of Sciences(Letter of 1st September 1828, Academy of Sciences, Proceedings .... Vol IX session of 26.1.1829, p.184), "almost finished the translation of Monge's Descriptive Geometry".

The lack of financial support and Carandinos' illness stopped his dream, the creation of a complete series of contemporary mathematical books.His premature death was a misfortune for the Academy. Nevertheless, Carandinos gave a great impetus to mathematics, a long line of descendants was born from his enlightened teaching. Thanks to his translations, he contributed to the propagation of mathematical sciences in Greece and to the hellenization of mathematical terminology.

The Military School

French mathematics was becoming more and more familiar to those who could understand sufficently well. Ioannis Capodistrias (1776-1831), first governor of independent Greece opted for the establishment of the Military School in Nafplion(fisrt capital of Greece from 1829-1834) in 1828. This newly founded Military School was created following the model of the French Ecole Polytechnique, gave a prominent role to mathematics. H. Pauzié(1792-1848), organizer and director of the new institution, transfered the French mathematical program in Greece (it might be stressed that the former students of the École Polytchnique A. Th. Garnot and J.P. Peytie came also in Greece in order to organize this new establishment).

Carandinos efforts began to be justified and his dream became reality. His former student in the Ionian Academy, D. Despotopoulos was appointed to teach mathematics in the new institution. The textbooks were the books which already were Carandinos' choice namely Bourdon's, Legendre's and Monge's treatises. In the State Archives (doc. 54 January 1829 file 102), we discovered the mention that the students of the Military School should study descriptive geometry from Monge's treatise.

The University of Athens

Capodistrias' assasination in 1831 marked the beginning of a new and difficult period in Greece. The Great Protective Powers (Great Britain, France, Russia) after many rejected offers of the throne of Greece among the royal courts of Europe, finally found a willing young prince , eager to accept the thorny crown, Othon of Bavaria (1815-1867).

In 1837, king Othon established the first university, the othonian university unique in the Orient. The new institution was organized on the German model where the ordo philosophicus comprises the three faculties of the department of philosophy. If the Greeks could have had the option of establishing the French model (where the Faculty of Letters is separated from that of the mathematical and physical sciences) the study of sciences would not have been neglected and these disciplines might have contributed to the country's growth. By construction and definition the University of Athens had a direct foreign influence and during its development incorporated both foreign and Greek experiences.

In the program of the first academic year 1837-38 is noted that the curriculum comprises i) the last five books of Legendre's Geometry ii) Legendre's rectilinear trigonometry iii) general properties of numbers iv) algebra v) Hachette's descriptive geometry.

Thus in a way, this program continued Carandinos' tradition in the Ionian Academy. Professor C. Negris, a former student of the École polytechnique (class of 1829), who taught mathematics in the newly established university, in his inaugural lecture makes no secret of his French background in mathematics.

It is also remarkable that in the very first years a special care will taken to teach students the principles of practical geometry such as lands surveying, leveling and of course the use of geometrical instruments. These last instructions were indispensable for their educational career but also for those who would follow a military career. The students had adopted that mathematics in their applied use are at the base of astronomy, mechanics, architecture , fortification , navigation etc,

The Polytechnic School of Athens

The construction of the royal palace in Athens revealed the lack of qualified Greek builders (stonemason, bricklayers etc), thus a noble Bavarian officer Friedrich von Zettner conceived the idea to establish a Technical school according to the model of the Royal School of Building (koeniglich Baugewerkeschule in Munich , established in 1826 following the model of the Ecole Polytechnique), as well as the technical school in Lyon , La Martinière .

In the first period this school, the School of Arts, functioned only on Sundays and on holidays in order to facilate "the students", who actually worked, to attend the lessons of : elementary mathematics, design and constructions. During its first reformation, in 1843, Othon invited many foreign professors as f. ex. the well known Danish architect Th. Hansen. Moreover its first director Lyssandros Kavtanzoglou, who studied in France and in Italy, modified the curriculum. Thus the studies reached the three years and new disciplines enriched the curriculum as physic, chemistry, grammar and stenography! (in order to compensate the lack of manuals). Thus in 1857 graduated the first architects and many qualified technicians in building constructions. In 1887 this School of Arts was promoted to a higher institution for building construction engineers and adopted the name of School of Industrial Arts. Almost the majority of the professors were professors of the university of Athens or of the Military School. The lectures of the descriptive geometry constitute a main part of the curriculum.

Literature

Ch. Phili, "La reconstruction des mathématiques françaises en Grèce: l'apport de Ioannis Carandinos (1784-1834)" dans L' Europe Mathématique: éd C. Goldstein J. Gray , J. Ritter. Paris 1996 pp. 305-319

Ch. Phili, "Mathematics and Mathematical Education in the University of Athens from its foundation to the beginning of the XXth century" Archives Internationales d' Histoire des Sciences vol. 51 2001 pp. 74-98

Ch. Phili, "1837-1937: ein Jahrhundert der hoeheren Institutionen in Griecheland" Oesterreichisches Symposium zur Geschichte der Mathematik. Oesterreichische Geselschaft fuer Wissenschaftgeschichte . Miesenbach 2004 pp. 70-87.

Ch. Phili, "Ioannis Carandinos (1784-1834): L'initiateur des mathématiques françaises en Grèce". Archives Internationales d' Histoire des Sciences. vol. 56 no 156-157 Juin - Décembre 2006 pp. 79-124.

Ch. Phili, Mathematics in the Ionian Academy (to appear).

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

Notes on the teaching of Descriptive Geometry in Portugal

Eliana Manuel Pinho, José Carlos Santos and João Pedro Xavier (Portugal)

Descriptive Geometry became a science throught the efforts of Gaspard Monge (1746 – 1818) and his teaching at the École Polytechnique (see [Mon99] and [Lor21, cap. IV]). The treatises of three French mathematicians from the next generation, Théodore Olivier (1793 – 1853), Louis Lefébure de Fourcy (1787 – 1869) and Charles-François-Antoine Leroy (1780 – 1854), had a great influence on the teaching of Descriptive Geometry in Portugal (see [Lor21, § XI.4]. It was above all Olivier's approach which had a great influence on three Portuguese Geometry teachers: Gustavo Adolfo Gonçalves e Sousa (1818 – 1899), Luís Porfírio da Motta Pegado (1832 – 1903) and Alfredo Augusto Schiappa Monteiro de Carvalho (1838 – 1919).
Gustavo Adolfo Gonçalves e Sousa was a Mathematics teacher at the Academia Politécnica do Porto since 1851 and later became the headmaster of the Instituto Industrial do Porto. Luís Porfírio da Motta Pegado, on the other hand, taught Descriptive Geometry for many years at the Escola Politécnica de Lisboa. He was the first Portuguese mathematician who published research articles about Descriptive Geometry; see [Mot67], [Mot75] and [Mot78]. Alfredo Augusto Schiappa Monteiro de Carvalho, who also taught at the Escola Politécnica de Lisboa, also published research concerning Descriptive Geometry; see [Car77].
In Polytechnical schools, Descriptive Geometry is taught since 1837, whereas in secondary schools, some elements of Descriptive Geometry appear since the mid-XIXth century; see [Pim03]. By the end of the XIXth century, it had become accepted in Portugal that Descriptive Geometry was an important object of study, with the publication of textbooks for secondary schools [PA81], treatises for the students of polytechnic schools [Mot99], and even popular diffusion editions as [Cor82] and [Cor85].
The aim of this work is to determine how the teaching of Descriptive Geometry in Portugal unfolded through the XIXth century and the early XXth century.

References

[Car77] AlfredoAugusto Schiappa Monteiro de Carvalho. “Mémoire de géométrie descriptive sur l’intérséction des surfaces du second ordre et des surfaces de révolution”. In: *Instituto de Coimbra* 2 (1875–77), pp. 21–24.

[Cor82] David Corazzi, ed. *Geometria no Espaço*. Casa Editora de David Corazzi, 1882.

[Cor85] David Corazzi, ed. *Geometria descriptiva: illustrada com 47 figuras e contendo além do programma official do Curso Geral dos Lyceus muitas indicações adaptadas ao Curso da Escola Polytechnica*. Casa Editora de David Corazzi, 1885.

[Lor21] Gino Loria. *Storia della Geometria Descrittiva dalle origini sino ao giorni nostri*. Ulrico Hoepli, 1921.

[Mon99] Gaspard Monge. *Géométrie Descriptive*. Baudouin, 1799.

[Mot67] Luís Porfírio da Motta Pegado. “O logar geometrico dos pontos que distam egualmente de dos rectas dadas e un parabloide hyperbolico isosceles”. In: *J. sc. Math. Astr.* (1867), pp. 188–197.

[Mot75] Luís Porfírio da Motta Pegado. “Seciones conicas do conoide circumscripto a una conica”. In: *J. sc. Math. Astr.* (1875), pp. 65–72.

[Mot78] Luís Porfírio da Motta Pegado. “Determinação dos eixos de sombra ou projecção oblíqua de um círculo”. In: *J. sc. Math. Astr.* (1877–78), pp. 217–228.

[Mot99] Luís Porfírio da Motta Pegado. *Geometria Descriptiva da Escola Polytechnica*. Tipografia da Academia Real das Sciências, 1899.

[Pim03] Joaquim A. B. Pimenta. “Desenho: Manuais do século XIX de autores portugueses”. Tese de mestrado. Faculdade de Letras da Universidade do Porto, 2003.

[PA81] Adriano Augusto de Pina Vidal and Carlos Augusto Morais de Almeida. *Elementos de geometria no espaço e de geometria descriptiva*. Academia Real de Sciencias, 1881.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

The early teaching of descriptive geometry in the United States (1817-1868)
Changing textbooks publishing, changing institutions and changing countries

Thomas Preveraud, Laboratory CRHIA, University of antes (France)

In the United States, descriptive geometry was a subject very few mathematicians, teachers or engineers knew about before 1820. Most of them were self-taught, as it was not introduced in any curriculum before 1817, especially since mathematics and science in general were not a leading subject of high education and its practice, teaching and diffusion remained modest and in the making during the first half of the century.

At that time, high education was mostly provided in colleges where algebra, geometry, trigonometry, surveying and some of the doctrine of fluxions were taught as subjects supposed to train students in the art of rigorously reasoning. Most of students would become teachers, lawyers, pastors, traders or just intended to pursue an intellectual, cultural and moral training. Engineers were not trained in colleges yet, but in West Point Military Academy, the only military engineering school of the country in the period 1802-1820.

The first course of descriptive geometry ever given in the United States was by French polytechnician Claude Crozet, the West Point professor of civil engineering between 1817 and 1823, who introduced the subject in West Point curriculum in 1817. For his teaching, he wrote *A Treatise on Descriptive Geometry* in 1821 as a textbook for his cadets. Soon, descriptive geometry became a taught subject in colleges, especially those which had already started to offer their students elective courses, or special engineer-training programs: Harvard (1825), Rensselear Institute (1830), University of South Carolina (1836), Kentucky University (1844), Dartmouth (1851), University of Mississippi (1854). By the end of the decade 1870-1880, descriptive geometry became a compulsory subject in most of colleges and universities curricula, even for students who did not plan to work in engineering. It was not seen anymore as a separate subject that has to be taught to future engineers but as sequel of the general mathematics curriculum.

Before descriptive geometry was added to colleges curricula and taught to all students, half of a dozen American textbooks on that specific subject were published between 1822 and 1860. After Crozet's work, none of them were specifically designed for engineering and military teaching. Instead, the authors intended to catch a wider audience including college students and general education. Because most of the written sources an American author could refer to at that time were French, the textbooks borrowed some material from their French peers in terms of presentation, mathematical language, figures and pedagogical methods.

In the communication we analyze the teaching methods used in the textbooks, taking into account the institutional context of publication. As descriptive geometry went gradually from a restrictive audience subject to a general interest subject, we wonder how the textbooks changed their presentation. We focus of the élémentation of the method of projection, initially introduced by Monge in *Géométrie descriptive* (1799) and continued in other French textbooks of the nineteenth century: how was it implemented for American less educated students? The communication points out how crucial the question was since it came to teach the subject to non-engineers. We also have in mind the problems exposed in French geometry descriptive textbooks and we analyze the way they were transformed, adapted, removed for a different and changing American audience. Last, we show that the writing of domestic descriptive geometry textbooks, though adapted from foreign textbooks and teaching methods, also clearly depended on the habits and the uses of the American mathematics publishing context: on that field, the internationalization of mathematics would wait.

Bibliography

ADHÉMAR Joseph A., *Cours de géométrie descriptive*, Paris, Imprimerie de Chaignieau fils aîné, 1823.

ALBREE Joe, ARNEY David C., RICKEY V. Frederick, *A Station Favorable to the Pursuit of Science: Primary Materials in the History of Mathematics at the United States Military Academy*, Providence, American Mathematical Society, 2000.

AMADIEU M.F., *Notions élémentaires de géométrie descriptive exigées pour l'admission aux diverses écoles du gouvernement,* Paris, Bachelier, 1838.

ARNEY Chris, *West Point's Scientific 200: Celebration of the Bicentennial. Biographies of 200 of West Point's Most Successful and Influential Mathematicians, Scientists, Engineers, and Technologists*, Lexington, Palmetto Bookworks, 2002.

BARBIN Evelyne, « Top-down: the role of the Preparatory Classes to the Grandes Écoles into the French System of Mathematical Curriculum (1870-1970) », dans Bjarnadottir K., Furinghetti F., Prytz J., Schubring G. (Eds.), *Proceedings of the Third International Conference on the History of Mathematics Education, Uppsala, Sweden, 25-28 september 2013* (p. 49-65), Uppsala, Uppsala University, 2015.

BELHOSTE Bruno, *La formation d'une technocratie : L'École polytechnique et ses élèves de la Révolution au Second Empire*, Paris, Belin, 2003.

BRISSON Barnabé, *Géométrie descriptive par G. Monge, augmentée d'une théorie des ombres et de la perspective, extraite des papiers de l'auteur*, Paris, Courcier, 1820.

BRUBACHER John S., RUDY Willis, *Higher Education in Transition*, *A history of American colleges and universities, 1636-1968,* New-York, Evanston, London, Harper & Row, 1968 (1958).

CAJORI Florian, *The Teaching and History of Mathematics in the United States*, Washington, D.C., Bureau of Education, 1890.

CHURCH Albert E., *Elements of Descriptive Geometry*, New York, Barnes, 1864.

CHURCH Albert E., *Plates of Descriptive Geometry,* New York, Barnes, 1864.

CRACKER Theodore J., *West Point: A bicentennial history*, Lawrence, University Press of Kansas, 2002.

CROZET Claude*, A Treatise on Descriptive Geometry, for the use of the cadets of the United States Military Academy*, New York, Goodrich and Co., 1821.

DAVIES Charles, *Elements of Descriptive Geometry, with their Applications to Spherical Trigonometry, Spherical Projections and Wrapped Surfaces*, Philadelphia, Carey and Lea, 1826.

GRISON Emmanuel, « Claude Crozet (1789-1864) ou la carrière d'un polytechnicien aux États-Unis », *Bulletin de la SABIX*, No. 6, 1990, p. 3-9.

HACHETTE Jean, *Traité de géométrie descriptive*, Paris, Corby, Guillaume and Cie, 1822.

HAYWARD James, *Elements of Geometry upon the Inductive Method*, Cambridge, Ma, Hilliard and Brown, 1829.

HUNTER Robert F., DOOLEY Edwin L., Jr., *Claudius Crozet: French Engineer in America, 1790-1864*, Charlottesville, University of Virginia Press, 1992.

LACROIX Sylvestre-François, *Essai de géométrie sur les plans et les surfaces courbes*, Paris, Duprat, 1795 (1802 2nde éd.).

LAWRENCE Snezana, *Geometry of Architecture and Freemasonry in 19th century Britain*, Dissertation, Open University, Milton Keynes UK, 2002.

LEFÉBURE de FOURCY Louis, *Traité de géométrie descriptive*, Paris, Bachelier, 1830.

LEROY Charles F.A., *Traité de géométrie descriptive*, Paris, Carilian-Goeury et Anselin, 1834.

LUNDGREEN Peter, « Engineering Education in Europe and the USA, 1750-1930: The Rise to Dominance of School Culture and the Engineering Professions », *Annals of Science*, Vol. 47, 1990, p. 33-75.

MANSFIELD Edward D., « Military Academy at West Point », *American Journal of Education*, Vol. XIII, Hartford, Barnard, 1863, p. 17-48.

MONGE Gaspard, *Géométrie descriptive*, Paris, Baudoin, 1799.

OLIVIER Théodore, *Cours de géométrie descriptive*, Paris, Carilian-Goeury et Dalmont, 1843.

OLIVIER Théodore, *Atlas du Cours de géométrie descriptive*, Paris, Carilian-Goeury et Dalmont, 1844.

PARSHALL Karen H., ROWE David E., *The Emergence of the American Mathematical Research Community 1876-1900: J.J. Sylvester, Felix Klein, and E.H. Moore*, History of Mathematics, Vol. 8, Providence, American Mathematical Society, 1994.

REYNODLS Terry S., « The Education of Engineers in America before the Morrill Act of 1862», *History of Education Quaterly*, Vol. 32, No. 4, 1992, p. 459-482.

RICKEY V. Frederick, SHELL-GELLASCH Amy, « Mathematics Education at West Point: the First Hundred Years », *Convergence,* 2010 (en ligne à l'adresse http://www.maa.org/publications/periodicals/convergence/mathematics-education-at-west-point-the-first-hundred-years.).

SAKAROVITCH Joël, « La géométrie descriptive, une reine déchue », dans Bruno BELHOSTE, Amy DAHAN-DALMEDICO, Antoine PICON (dir.)*, La formation polytechnicienne*, Paris, Dunod, 1994, p. 77-93.

SAKAROVITCH Joël, « Théodore Olivier, Professeur de Géométrie descriptive », dans Claudine FONTANON, André GRELON (dir.), *Les professeurs du Conservatoire national des arts et métiers, dictionnaire bibliographique, 1794-1955*, Paris, INRP/CNMA, 1994, p. 326-335.

SNOW Louis F., *The College Curriculum in the United States,* London, Forgotten Books, 2012 (1907).

VALLÉE Louis-Léger, *Traité de la géométrie descriptive*, Paris, Courcier, 1819.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

The myth of the Polytechnic School

Gert Schubring (Rio de Janeiro/Bielefeld)

From 1806 on, polytechnic schools were founded in various European states. According to traditional historiography, and in particular to *Festschriften* of these institutions published at some anniversary, these schools followed the *École Polytechnique* in Paris as a model, with descriptive geometry as a key teaching discipline. A closer investigation shows, however, that these schools began at a rather low educational level, often as commercial schools and depending on the ministry of commerce, and with quite elementary teaching contents in mathematics. The only institution projected at the level of higher education in the first half of 19^{th} century Europe was a Polytechnic Institute in Berlin: its rationale should be mathematics teacher education. It was only in the second half of the nineteenth century that these institutions, after steady rise in status and level of formation provided, succeeded in attaining the level of higher education and in offering demanding mathematical courses. The question is hence why a structure, which proved so successful in France, was not viable in other countries – at least in the first half of the nineteenth century.

What is at stake proves to be the character of applications of mathematics as demanded and as practiced in the respective countries. The paper will study these differences in the level and degree of development of applications of mathematics in these countries on the one hand and, on the other hand, the demands there for such applications and, in particular, of geometrical knowledge needed and the functions being realized by descriptive geometry. This investigation will reveal a very specific and telling pattern enabling the functioning of the *École Polytechnique* in a rather unique way.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

The dissemination of Descriptive Geometry in Latin America

Gert Schubring, Vinicius Mendes, Thiago Oliveira, Luiz Carlos Guimarães
Universidade Federal do Rio de Janeiro (Brazil)

„Latin America“ is used here in two different meanings: firstly the entire region of middle and South America, with the countries speaking Spanish or Portuguese. But there is also the more restricted meaning of the Spanish speaking countries in this region.

Using the restricted meaning, one has to emphasize a decisive difference between Brazil and Latin America, as regards the development of mathematics in general and possible reception of descriptive geometry in particular. While various regions in Latin America were endowed with universities from the early colonial period on, universities in Brazil were founded only from the 1930s. There, from the end of the colonial period, in 1808, and in particular from the independence in 1822, higher education adopted the French model of *écoles spéciales*. The Military Academy and the later polytechnic schools were of higher education level and taught descriptive geometry.

The universities in Latin America, despite their different institutional character, seem to have cultivated mathematics not so differently from the Brazilian case, however: due to a missing context for pure mathematics, the respective faculties within these universities functioned to a considerable degree for the formation of engineers. Thus, it seems highly likely that descriptive geometry was taught here, too. Yet, nineteenth century history of university mathematics is very badly researched upon so far, so that we do not yet have sufficient information as regards the teaching of descriptive geometry and the textbooks used there. Research will be continued to reveal more information.

The situation is very different, however, for Brazil where there is considerable research. Three pertinent institutions were created there: the Military Academy in Rio de Janeiro, founded in 1811, forming engineers and teaching descriptive geometry from the beginning. This institution underwent several changes, and functioned later on as Polytechnic School in Rio de Janeiro. Later in the century, an *Escola de Belas Artes* were founded there, too, teaching also descriptive geometry for architecture students.

Higher education became more differentiated in the second half of the nineteenth century: In Ouro Preto (Minas Gerais), an *Escola de Minas* was founded in 1876, with a formation in descriptive geometry. And in 1893, in the city of São Paulo was founded the *Escola Politecnica de São Paulo*. While one knows the first textbook used in the Military Academy, a translation of Monge's textbook, one has course programs for the other two schools, but not yet sufficient knowledge about the used textbooks.

What is highly revealing is that there have been found entrance exams for the *Escola de Minas*, which required solving problems of descriptive geometry. This implies that there has been prior teaching of this matter at secondary schools. In fact, it was prescribed in the curriculum of the *Colégio Pedro II*, the leading secondary school in Brazil, from the 1890s.

We are confident to present a relatively complete picture of nineteenth century teaching of descriptive geometry and of its professors in Brazil – and some rather general information about Latin America - until the workshop.

Descriptive geometry: its role in history of mathematics and in teaching
Workshop, CIRM LUMINY, 18-22 January 2016

The Teaching of the Descriptive Geometry in Italy in the 19th Century

Roberto Scoth (Italy)

The spread of the descriptive geometry in Italy, began in the years of the French occupation (1796-1813). The creation of Italian small satellite states and the imposition in them of new educational systems similar to those in France, were decisive for the beginning of this process. After the Congress of Vienna (1818), during the years of the Restoration, the descriptive geometry has spread widely in the Italian states, mainly thanks to military schools and universities that had preserved the syllabuses of the French era.

In the same years in Italy the first technical-professional schools were founded. The onset of a phase of industrialization brought with it a substantial demand for skilled labour for employment in new processes of production. At first this new kind of education per se was regarded with scant interest on the part of the state, and was primarily concerned with the needs of the economic entities involved. Shortly after, during the 1830s and 1840s, state authorities became more interested in this new educational model. The schools for crafts and trades, run by private entities and aimed at artisans and labourers, were soon flanked by state technical schools, which provided alternatives to secondary classical education (*ginnasio-liceo*). The Descriptive geometry (theoretical and/or graphical) was taught in many of these schools, that have played a key role for the popularization of this subject.

After the unification and the creation of the Kingdom of Italy (1861) a new regulation for the universities and a new model of secondary education (classical and technical) were created. The teaching of the descriptive geometry was mandatory in the university courses of mathematics and engineering, and in technical secondary schools. In the early 19th century Italy, after France, was the most advanced country in the field of descriptive geometry, and this tradition had continued in the universities even after the unification. Many famous Italian mathematicians were professors of descriptive geometry, for example: Luigi Cremona, Gino Fano, Gino Loria, and later Beppo Levi e Federigo Enriques. On the contrary, in the secondary school this tradition did not continue. The descriptive geometry was taught only at the technical schools but over the years this subject was neglected and gradually eliminated from the mathematics syllabuses.

In my paper I intend to analyze the main aspects of the teaching of the descriptive geometry in Italy during the 19th century in the high education (universities and military schools) and in secondary schools. In particular I will analyze the secondary education during the period after unification, trying to find the causes of the lack of interest in teaching of descriptive geometry at this school level.

For example:

- technical education was considered on a lower level than the humanistic secondary education;
- descriptive geometry was not considered a true branch of mathematics but rather a "discipline for engineers";
- descriptive geometry is linked to the spatial intuition and to the graphical representation. On the contrary, the use of deductive reasoning, of the dogmatism and the recourse to rote learning were prevalent at the time, in the Italian secondary school;
- after the unification, Italian politicians govern the technical education with great difficulty: the structure and the syllabuses of technical institutes were changed eight times between 1861 and 1891 (the work proposed by Marta Menghini is related to the reforms of 1871/72).

Main bibliography

Loria G., 1921, *Storia della Geometria descrittiva. Dalle origini sino ai giorni nostri*, Milano, Hoepli.

Fiocca A., 1992, *La geometria descrittiva in Italia (1798-1838)*, «Bollettino di Storia delle Scienze Matematiche», XII, 2, pp. 187-249.

Scoth R., 2008, *Gli insegnamenti matematici nella legge Casati: il caso della Geometria Descrittiva*, «L'educazione Matematica», XXIX, 3, pp. 32-42.

Scoth R., *L'insegnamento della Geometria Descrittiva in Italia (1859-1923): da Casati a Gentile*, Tesi di Dottorato di Ricerca in Storia, Filosofia e Didattica delle Scienze, Università degli Studi di Cagliari, A. A. 2007/2008 (not published).