Universität Hamburg
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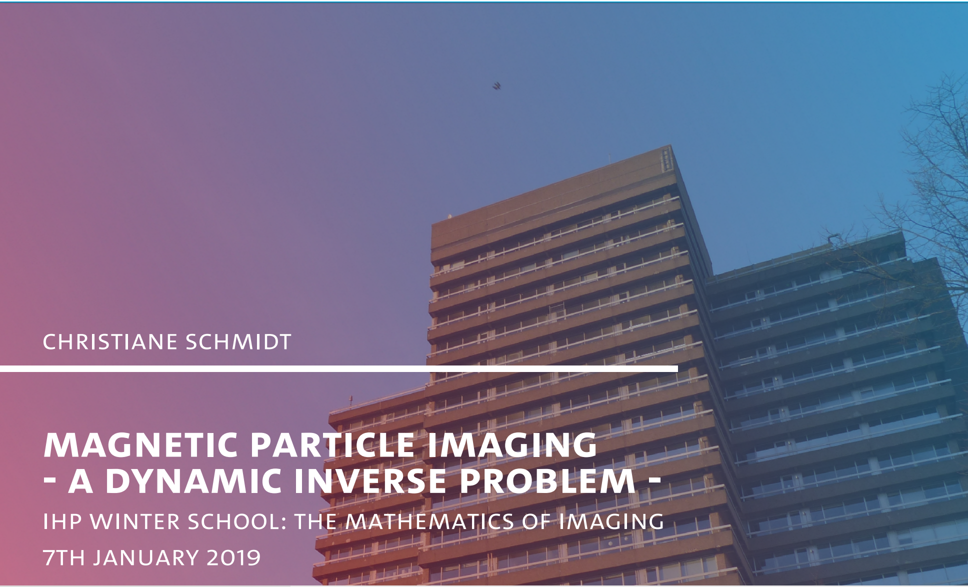
FAKULTÄT
FÜR MATHEMATIK, INFORMATIK
UND NATURWISSENSCHAFTEN
FACHBEREICH MATHEMATIK
OPTIMIERUNG UND APPROXIMATION



CHRISTIANE SCHMIDT

MAGNETIC PARTICLE IMAGING - A DYNAMIC INVERSE PROBLEM -

IHP WINTER SCHOOL: THE MATHEMATICS OF IMAGING

7TH JANUARY 2019

WHY ANOTHER IMAGING MODALITY?

- Do you know SPECT and PET?
  - \- ionizing $\rightarrow$ Cancer?!
  - \+ high resolution functional imaging

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- Do you know MRI?
  - \- slow
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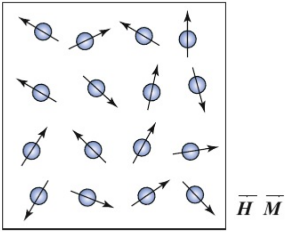
  - \+ non ionizing
- Let's take the best of both!

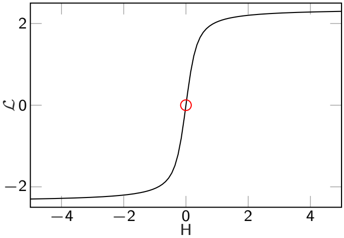
- **Magnetic Particle Imaging (MPI)**
  - \+ fast
  - \+ high resolution functional imaging
  - \+ non ionizing

MAGNETIC PARTICLE IMAGING

I) Magnetic Nanoparticles



Modelling the particle magnetization



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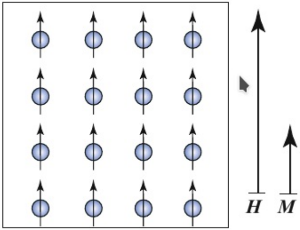

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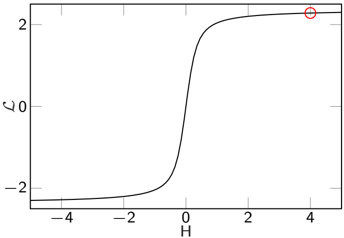
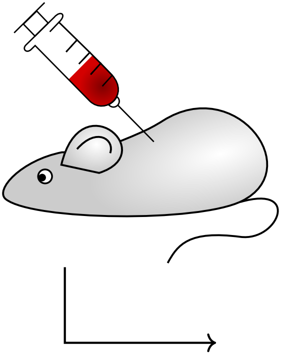
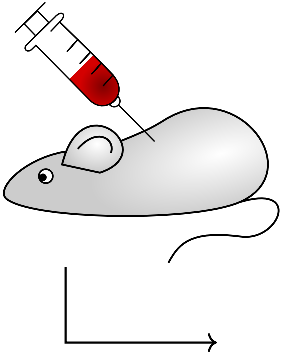


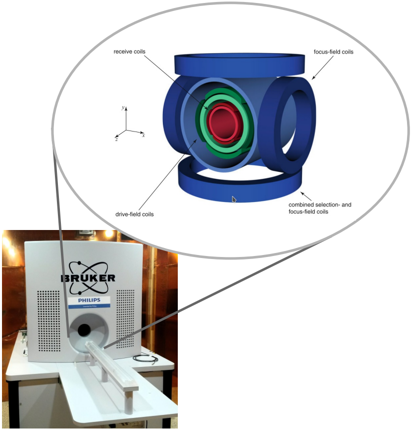
$$\mathcal{L}_{\alpha,\beta}(H) = \alpha \coth(\alpha\beta H) - \frac{1}{\beta H}$$

II) INJECT PARTICLES INTO A PATIENT



II) INJECT PARTICLES INTO A PATIENT

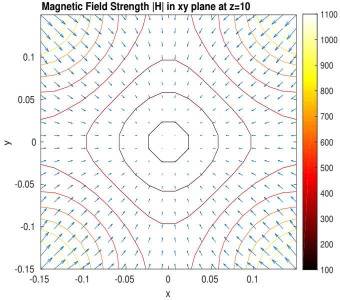




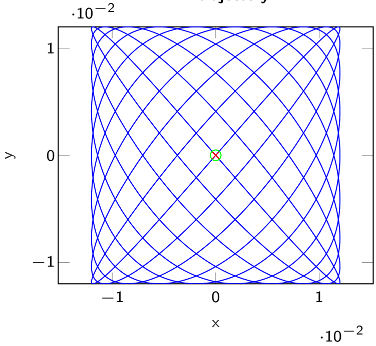
III) APPLY A TIME VARYING MAGNETIC FIELD

$$H(r,t) = \underbrace{\begin{pmatrix} G_x & 0 \\ 0 & G_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}}_{\text{Grad. Field}} + \underbrace{\begin{pmatrix} a_x \sin(2\pi f_x t + \varphi_x) \\ a_y \sin(2\pi f_y t + \varphi_y) \end{pmatrix}}_{\text{Drive Field}}$$

Gradient field



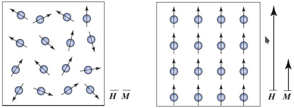
FFP trajectory



MEASURING A SIGNAL

1. FFP moves over some nanoparticles

(1)



MEASURING A SIGNAL

1. FFP moves over some nanoparticles
2. They react according to the Langevin function

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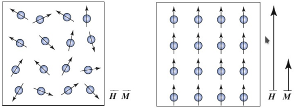
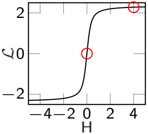


(2)



MEASURING A SIGNAL

1. FFP moves over some nanoparticles
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3. Magnetization changes

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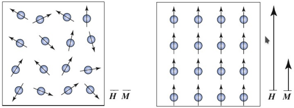
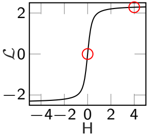


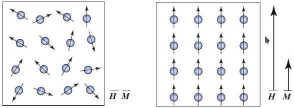
(2)



MEASURING A SIGNAL

1. FFP moves over some nanoparticles
2. They react according to the Langevin function
3. Magnetization changes
4. A voltage is induces in the receive coils → measurement

(1)



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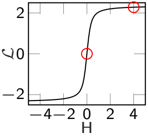
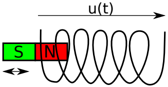


(4)



III) RECONSTRUCT THE PARTICLE CONCENTRATION

$$u_k = \overbrace{-\mu_0\, p\, a_k \int_\Omega \mathcal{F}\left(\frac{d\bar{m}}{dt}(r,t)\right)}^{\text{System Matrix}} \underbrace{c(r)}_{\text{Particle Concentration}} d^3 r$$

$$u_k = S_k\, c \qquad \text{Inverse Problem}$$

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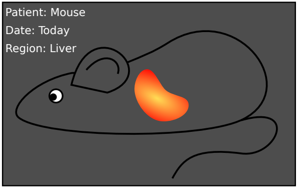
Solve for $c(r) \Rightarrow$



DYNAMIC INVERSE PROBLEM

Why do we have a dynamic inverse Problem?

- MPI is very fast $\Rightarrow$ Movies
- Multipatch: FOV is limited so that big volumes are stiched together out of smaller ones.
- Tracer concentration is not constant over time

$\Rightarrow$ How to regularize the inverse problem of MPI?

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THANK YOU FOR YOUR ATTENTION!

LITERATURE AND IMAGE RECOURCES

T. Knopp and T. Buzug.
Magnetic Particle Imaging: An Introduction to Imaging Principles and Scanner Instrumentation.
Springer Berlin Heidelberg, 2012.

OpenMPIData.

THANK YOU FOR YOUR ATTENTION!