Second order hyperbolic quasilinear PDEs and their applications in image problems

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Displacement Error[1] and Intensity Noise

♣ Displacement error:

$$\boxed{u^{\delta}(\mathbf{x}) = u(\mathbf{x} + \vec{\delta}(\mathbf{x})).}$$

♣ Additive intensity noise:

$$u^{\delta}(\mathbf{x}) = u(\mathbf{x}) + \delta(\mathbf{x}).$$

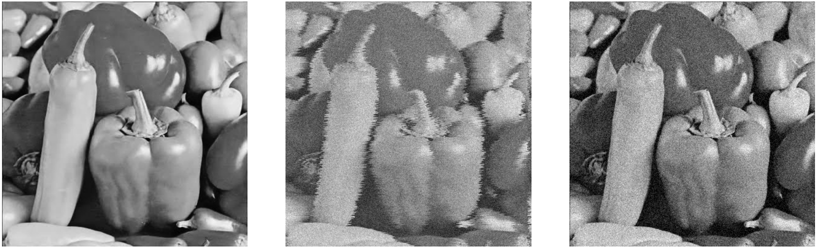

Figure: Displacement error and additive noise error

[1] G. Dong and O. Scherzer. Nonlinear flows for displacement correction and applications in tomography. In F. Lauze et. al, editors, *proceeding SSVM 2017*, volume 10302 of *LNCS*, pages 283–294. Springer, 2017.

Total variation flow and mean curvature flow of level sets

- First order TV flow[2]

$$\dot{u} = \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) \text{ on } \mathbb{R}^m \times (0,\infty),$$
$$u(0) = u^\delta \text{ on } \mathbb{R}^m.$$

- First order level sets MC flow [3]

$$\dot{u} = |\nabla u| \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) \text{ on } \mathbb{R}^m \times (0,\infty),$$
$$u(0) = u^\delta \text{ on } \mathbb{R}^m.$$

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[2] F. Andreu, C. Ballester , and V. Caselles. The Dirichlet problem for the total variation flow. *Journal of Functional Analysis*, 180:347–403, 2001.

[3] L. Evans and J. Spruck. Motion of level sets by mean curvature. I. *Journal of Differential Geometry*, 33:635–681, 1991.

Second order damping flows[4]

- Second order damping TV flow

$$\ddot{u} + \eta\dot{u} = \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) \text{ on } \mathbb{R}^m \times (0,\infty),$$
$$u(0) = u^\delta, \quad \dot{u}(0) = v_0 \quad \text{on } \mathbb{R}^m.$$

- Second order damping level sets MC flow

$$\ddot{u} + \eta\dot{u} = |\nabla u| \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right) \text{ on } \mathbb{R}^m \times (0,\infty),$$
$$u(0) = u^\delta, \quad \dot{u}(0) = v_0 \quad \text{on } \mathbb{R}^m.$$

[4] G. Dong, M. Hintermüller and Y. Zhang, Second order quasilinear hyperbolic PDEs and their applications in image problems, research report, 2018.

Numerical results

Figure: Denoising using the algorithms by TVFs and level sets MCFs. Middle: TV flows; Right: level sets MC flows (Above: first order; Below: second order.)

Figure: Dejittering: the results of algorithms from damped second order TVF (middle) and damped second order level sets MCF (right).

State and open problems

- Well-poseness of first order TV flow in $\mathbb{R}^n$ (Andreu, Ballester, Caselles and Mazón) and first order MC flows in $\mathbb{R}^n$ (Chen, Giga and Goto; Evans and Spruck); of second order TV flow in $\mathbb{R}^2$ (Dong, Hintermüller and Zhang);
- Asymptotic of the solution for the first order TV flow and the first order MC flows in $\mathbb{R}^n$ (Caselles, Evans, et. al);
- Well-posedness of the second order level sets MC flow;
- Asymptotic analysis for the second order TV flow and the second order MC flow.

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Thank you for your attention!

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