

# Higher Structures

CIRM, 21-25 January, 2019

## Schedule

	<b>Monday 21</b>	<b>Tuesday 22</b>	<b>Wednesday 23</b>	<b>Thursday 24</b>	<b>Friday 25</b>
<b>M1</b>	Willwacher	Ara	Wahl	Safronov	Robertson
<b>M2</b>	Iacono	Riehl	Idrissi	Dotsenko	Alekseev
<b>A1</b>	Scheimbauer	Bergner	<i>Free</i>	Robalo	<i>Free</i>
<b>A2</b>	Moerdijk	Horel	<i>Free</i>	Scherotzke	<i>Free</i>

**M1:** 9.30am-10.30am **M2:** 11am-12am **A1:** 4pm-5pm **A2:** 5.30pm-6.30pm

## Titles and Abstracts

**Anton ALEKSEEV** (Université de Genève)

TITLE. *The Goldman-Turaev Lie bialgebra and the Kashiwara-Vergne problem.*

ABSTRACT. The space spanned by free homotopy classes of curves on a compact oriented 2-dimensional manifold has a natural Lie bialgebra structure defined by intersections and self-intersections of curves. The corresponding Lie bracket was defined by Goldman and the Lie cobracket was defined by Turaev. The Goldman-Turaev (GT) Lie bialgebra carries a natural filtration and one may ask whether it is isomorphic to its associated graded. We show that this question is equivalent to a certain problem in the theory of free Lie algebras (the Kashiwara-Vergne problem) and explain that this problem always admits solutions. One of the important technical tools used in the proof is a novel characterization of conjugacy classes in the free Lie algebra in terms of cyclic words.

The talk is based on joint works with N. Kawazumi, Y. Kuno and F. Naef.

**Dimitri ARA** (Aix-Marseille Université)

TITLE. *A Quillen Theorem B for strict  $\infty$ -categories.*

ABSTRACT. Quillen's Theorem B is an important tool in the study of homotopy types of nerves of categories. It gives, under some assumptions, a categorical model for the homotopy fiber of the nerve of a functor. In this talk, I will explain how to generalize this theorem to strict  $\infty$ -categories. This will require the use of sesquifunctoriality properties of a comma construction for strict  $\infty$ -categories that we introduced with G. Maltiniotis. As an application, I will describe new models for certain Eilenberg Mac-Lance spaces.

**Julie BERGNER** (University of Virginia)

TITLE. *2-Segal spaces and algebraic K-theory.*

ABSTRACT. The structure of a 2-Segal space encodes data like that of a category, but for which composition need not always exist or be unique, yet is still associative. It was shown by Dyckerhoff and Kapranov, and independently by Gálvez-Carrillo, Kock, and Tonks, that applying Waldhausen's S-construction to an exact category results in a 2-Segal space. In joint work with Osorno, Ozornova, Rovelli, and Scheimbauer, we expand the input of this construction in such a way that the S-construction defines an equivalence of homotopy theories, as well as recovers other known generalizations. In work in progress with Zakharevich, we show that this general input has a close relationship with the CGW categories of Campbell and Zakharevich, which are also designed to be a very general context in which algebraic K-theory constructions can be made.

**Vladimir DOTSENKO** (Trinity College Dublin)

TITLE. *Monads and PBW theorems.*

ABSTRACT. I shall aim to reveal the "true" categorical meaning of the Poincaré-Birkhoff-Witt theorem and similar results for other left adjoint functors between categories of algebras over monads. As a by-product, I shall prove a PBW-type theorem for pre-Lie algebras, answering an old question of Loday. This is joint work with P. Tamaroff.

**Geoffroy HOREL** (Université Paris 13)

TITLE. *Galois group and space of knots.*

ABSTRACT. Goodwillie and Weiss introduced a method to study embedding space called manifold calculus. When specialized to the case of knots, this theory produces a tower of approximation that are related to finite type invariants of knots à la Vassiliev. From this tower, one can construct a spectral sequence, whose  $E_1$ -page consists of homotopy groups of spheres and which converges to an approximation of the space of knots. In recent work with Pedro Boavida de Brito, we construct an interesting action of the absolute Galois group of the rationals on this tower. This action implies some vanishing results on the differentials of this spectral sequence.

**Donatella IACONO** (Università degli Studi di Bari Aldo Moro)

TITLE. *DG-Lie algebras and deformation of pairs.*

ABSTRACT. For every smooth algebraic manifold  $X$  and every effective divisor  $D$  on  $X$ , we associate a DG-Lie algebroid such that the DG-Lie algebra of derived global sections controls the deformation theory of the pair  $(X, D)$ . Joint work in progress with Marco Manetti.

**Najib IDRISSE** (Université Paris Diderot)

TITLE. *Configuration spaces and operads.*

ABSTRACT. Configuration spaces consist of tuples of pairwise distinct points in a given space. Studying the homotopy type of configuration spaces of manifolds is a classical problem in algebraic topology. In this talk, I will explain how to use the theory of operads - more precisely, Kontsevich's proof of the formality of the little disks operads - to obtain results on the real homotopy type of configuration spaces of simply connected closed smooth manifolds. I will also talk about generalizations and applications: manifolds with boundary, framed configuration spaces, factorization homology, and work in progress on complements of submanifolds.

**Ieke MOERDIJK** (Universiteit Utrecht)

TITLE. *Dendroidal spaces and mapping spaces between little cubes operads.*

ABSTRACT. After introducing the category of dendroidal spaces, I will discuss a filtration on dendroidal spaces, and apply it to prove a connectivity result due to F. Goepl about the derived mapping space  $\mathrm{Map}(E(n), E(k))$ , where  $E(n)$  is the little  $n$ -cubes operad.

**Emily RIEHL** (Johns Hopkins University)

TITLE. *The complicial sets model of higher  $\infty$ -categories.*

ABSTRACT. While it's undeniably sexy to work with infinite-dimensional categories "model-independently," we contend there is a categorical imperative to familiarize oneself with at least one concrete model in order to check that proposed model-independent constructions interpret correctly. With this aim in mind, we recount the  $n$ -complicial sets model of  $(\infty, n)$ -categories for  $0 \leq n \leq \infty$ , the combinatorics of which are quite similar to its low-dimensional special cases: quasi-categories ( $n = 1$ ) and Kan complexes ( $n = 0$ ). We conclude by reporting on an encounter with 2-complicial sets in the wild, where a suitably-defined fibration of 2-complicial sets enables the *comprehension construction* introduced in joint work with Verity. Special cases of the comprehension construction can be used to "straighten" a co/cartesian fibration of  $(\infty, 1)$ -categories into a homotopy coherent functor, exhibit a quasi-categorical version of the "unstraightening" construction, and define an internal model of the Yoneda embedding for  $(\infty, 1)$ -categories.

**Marco ROBALO** (Sorbonne Université)

TITLE.

ABSTRACT.

**Marcy ROBERTSON** (University of Melbourne)

TITLE. *Tangles, modular operads, and a group like  $GT$ .*

ABSTRACT. Tangles are embeddings  $\coprod S^1 \cup \coprod [0, 1] \rightarrow \mathbb{R}^3$ . If instead we embed our circles and intervals into a surface, we have the notion of  $v$ -tangles or  $w$ -tangles. Building on work of Bar-Natan and Dansco, we give modular operads that captures the Reidemeister theory of  $v$  and  $w$  tangles. We explain how the group of homotopy automorphisms of (a weakened version of) the modular operad for  $w$ -tangles acts on the space of solutions to the Kashiwara-Vergne problem and will give some updates on ongoing work describing a conjectured relationship between this group and the Grothendieck-Teichmüller group. This talk includes pieces of joint work with Z. Dansco, I. Halacheva, P. Hackney and D. Yau.

**Pavel SAFRONOV** (Universität Zürich)

TITLE. *Noncommutative Poisson geometry and the Kashiwara—Vergne problem.*

ABSTRACT. In this talk, I will explain an interpretation of the Kashiwara—Vergne problem (a property of the Baker—Campbell—Hausdorff series) in terms of noncommutative geometry. Namely, one can reformulate it as a formality statement for the Calabi—Yau algebra of cochains on a Riemann surface equipped with a trivialization of the Euler class. In the talk, I will also describe noncommutative versions of familiar concepts such as shifted Poisson structures and their unimodular versions. This is a report on work in progress with Florian Naef.

**Claudia SCHEIMBAUER** (Norwegian University of Science and Technology)

TITLE. *A complete model for the cobordism category.*

ABSTRACT. The model of complete Segal spaces as a model for  $(\infty, 1)$ -categories and their higher analogs are very useful for constructing cobordism categories, as was famously illustrated in Lurie's approach to the Cobordism Hypothesis. A drawback is that the first natural definitions only give Segal spaces, which, for high dimensions, are not complete. This is a feature following from the s-cobordism theorem. In this talk, I will explain a very simple model of cobordisms, which is a completion of the usual one, and in particular indeed is complete. This is joint work with Ulrike Tillmann.

**Sarah SCHEROTZKE** (Universität Münster)

TITLE. *The Chern character and categorification.*

ABSTRACT. The Chern character is a central construction which appears in topology, representation theory and algebraic geometry. In algebraic topology it is for instance used to probe algebraic K-theory which is notoriously hard to compute, in representation theory it takes the form of classical character theory. Recently, Toen and Vezzosi suggested a construction, using derived algebraic geometry, which allows to unify the various Chern characters. We will categorify this Chern character. In the categorified picture algebraic K-theory is replaced by the category of non-commutative motives. It turns out that the categorified Chern character has many interesting applications. For instance we show that the DeRham realisation functor is of non-commutative origin.

**Nathalie WAHL** (Københavns Universitet)

TITLE. *Products, coproducts, and "higher structures" in string topology.*

ABSTRACT. String topology is the study of a certain type of structure on the homology of free loop spaces. I'll describe the string topology product and coproduct as well as associated "higher structures".

**Thomas WILLWACHER** (ETH Zürich)

TITLE. *Deformation quantization and the homology of knot spaces.*

ABSTRACT. I will describe some links between deformation quantization and the homology of knot spaces. More concretely, it is known by work of Sinha that the spaces of long knots in  $\mathbb{R}^n$  (modulo immersions) with  $n \geq 4$ , carry the structure of a double loop space. Hence the (rational) homology of those spaces carries a Gerstenhaber algebra structure. This  $E_2$  structure has been identified (independently) by Songhafouo Tsopmene and Moriya as the one on the Hochschild complex of the  $E_n$  operad. On the other hand, the rational homology has been computed by Lambrechts–Turchin–Volic in terms of hairy graph cohomology. We will see that the resulting Gerstenhaber structure on hairy graphs may be identified using methods of deformation quantization. In particular, the hairy graphs are akin to multivector fields, and the Hochschild complex of  $E_n$  is akin to the multidifferential operators in Kontsevich's Formality Theorem in Deformation Quantization.