TEICHMÜLLER SPACE, POLYGONAL BILLIARD, INTERVAL EXCHANGES
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Jayadev Athreya
Siegel-Veech transforms are in L^2
(joint work with Yitwah Cheung and Howard Masur).

Let H denote a connected component of a stratum of translation surfaces. We show that the Siegel-Veech transform of a bounded compactly supported function on $\mathbb{R}^2$ is in $L^2(H,\mu)$, where μ is the Masur-Veech measure on H, and give applications to bounding error terms for counting problems for saddle connections.

Corentin Boissy
Lengths spectrum of hyperelliptic components
(joint work with Erwan Lanneau).

We propose a general framework for studying pseudo-Anosov homeomorphisms on translation surfaces in hyperelliptic connected components. This new approach, among other consequences, allows us to compute the systole of these connected components, settling a question of Farb.

Dawei Chen
Principal boundary of strata of abelian differentials
(joint work with Qile Chen based on arXiv:1611.01591).

Eskin-Masur-Zorich described the principal boundary of strata of abelian differentials that parameterizes flat surfaces with a prescribed generic configuration of short parallel saddle connections. In this talk we describe the principal boundary algebraically over the Deligne-Mumford boundary of stable pointed curves. Along the way we deduce some interesting properties about meromorphic differentials on the Riemann sphere.

Diana Davis
Interval exchange transformations from tiling billiards

Tiling billiards is a dynamical system where beams of light refract through planar tilings. It turns out that, for a regular tiling of the plane by congruent triangles, the light trajectories can be described by interval exchange transformations. I will explain this surprising correspondence, give related results, and show computer simulations of the system.

Alex Eskin
Measure rigidity and orbit closures: a survey

I will review some old results in several areas of dynamics and suggest some new directions.

Sébastien Ferenczi
Rigidity for square tiled interval exchange transformations
(joint work with Pascal Hubert).

We look at interval exchanges defined as first return maps on the set of diagonals of a flow of direction α on a square-tiled surface: our main result is to show, by a combinatorial approach, that they are not rigid when the surface has at least one true singularity and α has bounded partial quotients.

Simion Filip
How to make a K3 surface with paper, scissors, and glue
(joint work in progress with P. Engel).

I will talk about some moduli spaces of K3 surfaces and the similarities they share with strata of translation surfaces. The constructions are based on an explicit way to describe and work with K3 surfaces.

Forni Giovanni
Limits of geodesic push-forwards of horocycle measures

We prove a couple of general conditional convergence results on ergodic averages for horocycle and geodesic subgroups of any continuous $\mathrm{SL}(2,\mathbb{R})$- action on a locally compact space. These results are motivated by theorems of Eskin, Mirzakhani and Mohammadi on the $\mathrm{SL}(2,\mathbb{R})$-action on the moduli space of Abelian differentials. By our argument we can derive from these theorems an improved version of the "weak convergence" of push-forwards of horocycle measures under the geodesic flow and a new short proof of a theorem of Chaika and Eskin on Birkhoff genericity in almost all directions for the Teichmüller geodesic flow.

Vaibhav Gadre
Random geodesics in moduli spaces
(some of this is joint work with J. Maher, and with J.Maher and G. Tiozzo).

Sullivan and Masur log-laws are a striking example of analogies between random Teichmüller geodesics and random hyperbolic geodesics on finite area hyperbolic surfaces with cusps. This talk extends this dictionary focusing on two notions of randomness. The first notion considers random Teichmüller geodesics for the holonomy measures (more generally $\mathrm{SL}(2,\mathbb{R})$-invariant measures) on moduli spaces on the one hand and random hyperbolic geodesics for the Liouville measure on the other. The second notion considers random geodesics for harmonic measures arising from random walks on mapping class groups on the one hand and non-uniform lattices in $\mathrm{SL}(2,\mathbb{R})$ on the other. Applications of this analysis of random geodesics include singularity of a large class of harmonic measures, and a trim-sum strong law generalising a strong law by Diamond-Vaaler for continued fractions.

Elise Goujard
Equidistribution of square-tiled surfaces of fixed combinatorial type
(some of this is joint work with V. Delecroix, P. Zograf, A. Zorich).

Dynamics in rational polygonal billiards is strongly related to dynamics and geometry in the moduli spaces of flat surfaces. In particular, evaluating the Masur-Veech volumes of such moduli spaces and computing Siegel-Veech constants give some quantitative results for the dynamics in the associated billiards. This evaluation can be done by counting square-tiled surfaces, with and without weight. I present several results about square-tiled surfaces of fixed combinatorial type related to this problem, and their relation to the question of the asymptotic of volumes, as the genus tends to infinity.

Ursula Hamenstädt
Simplicity of the Lyapunov spectrum revisited

We give an algebraic proof of the simplicity of the Lyapunov spectrum for the Teichmüller flow on strata of abelian differentials. This proof extends to the Kontsevich Zorich cocycle over strata of quadratic differentials and can also be used to study the algebraic degree of pseudo-Anosov stretch factors.

Chris Leininger
Surface bundles over Teichmüller curves
(joint work-in-progress with Dowdall, Durham, and Sisto).
I will discuss some work on the coarse geometry of the canonical surface bundle over a Teichmüller curve.

Martin Möeller
Tropical curves and flat surfaces

We will provide a short introduction to tropical curves and explain why two questions in this context can be answered using flat surfaces.

Gabriele Mondello
Spherical metrics on conical surfaces
(joint work in progress with D. Panov).

A celebrated result by Poincaré states that a compact Riemann surface of positive genus has a conformal metric of constant curvature, unique up to rescaling. Clearly, the case of genus 0 is not so exciting: there is a unique complex structure and a unique metric of curvature 1 up to Möbius transformations. The problem becomes more interesting if we require such metrics to have conical singularities of prescribed angles at a finite subset of marked points. The case of negative and zero curvature was settled by McOwen and Troyanov: they established the existence and uniqueness of such a metric in each conformal class. The case of positive curvature is more delicate: existence and uniqueness results are known for small angles (Troyanov), whereas existence and non-uniqueness results are known in positive genus (Bartolucci-De Marchis-Malchiodi). Together with D.Panov we study the moduli space of such spherical metrics, first determining for which angle assignment such moduli space is not empty in genus 0 and then (in progress) we show that the number of such metrics in each conformal class is generically locally constant, provided the angles remain in the same "chamber".

Angel Pardo
Counting problem on wind-tree models

Wind-tree models are billiards in the plane endowed with $\mathbb{Z}^2$-periodically located identical connected symmetric right-angled obstacles. We will show asymptotic formulas for the counting problem of closed billiard trajectories (up to isotopy and $\mathbb{Z}^2$-translations) on wind-tree billiards, give the exact value of the associated Siegel-Veech constant for generic wind-tree billiards depending on the number of corners of the obstacle, exhibit a non-varying phenomenon in the classical case of rectangular obstacles and give a quantitative result for the error term in the case when the underlying surface is a Veech surface.

Luc Pirio
Moduli spaces of flat tori and elliptic hypergeometric functions
(joint work with S. Ghazouani).

In his 1993 paper "*Flat Surfaces*", Veech regards $\mathcal{M}_{g,n}$ as the moduli space of flat surfaces of genus g with n conical singularities whose associated cone angles $\theta_1,\dots,\theta_n$ are fixed. Using this identification, he constructs a natural foliation on $\mathcal{M}_{g,n}$ with complex leaves. Moreover, he shows that these latter carry an interesting geometric structure which depends only on the θ_i's.

In the genus 0 case, there is a unique leaf, namely the whole moduli space $\mathcal{M}_{0,n}$ whose geometric structure is complex hyperbolic if the θ_i's are all assumed to be strictly less than 2π. This case has previously been studied by Deligne and Mostow (in the realm of the theory of hypergeometric functions) and also by Thurston (in terms of flat surfaces).

My talk will be about some recent works carried out in collaboration with S. Ghazouani which allow us to give an explicit description of Veech's constructions in the genus 1 case.

Kasra Rafi
Unique ergodicity of geodesic flow in an infinite translation surface
(joint work with Anja Randecker).

The behaviour of infinite translation surfaces is, in many regards, very different from the finite case. For example, the geodesic flow is often not recurrent or is not even defined for infinite time in a generic direction. However, we show that if one focuses on a class of infinite translation surfaces that exclude the obvious counter-examples, one can adapted the proof of Kerckhoff, Masur, and Smillie and show that the geodesic flow is uniquely ergodic in almost every direction. We call this class of surface essentially finite.

Sasha Skripchenko
Novikov's problem: when interval exchange transformations are powerless
(joint work with I. Dynnikov and Pascal Hubert).

In 1982 S. P. Novikov posed a problem about asymptotic behavior of plane sections of 3-periodic surfaces. This problem can be easy reformulated in terms of the measured foliations on the compact surfaces. However, standard technique does not work here, and known results about interval exchange transformations are not applicable in a direct way. In my talk I will discuss how to fix this issue using some generalization of interval exchange transformations, called systems of isometries. We will describe the behavior of chaotic trajectories in Novikov's problem (joint work with Artur Avila and Pascal Hubert) in a very special situation first studied by Ivan Dynnikov. I will explain how to define a natural measure on the set of chaotic directions in Novikov's problem. This set is a fractal set of zero measure, thus this is a non trivial problem. Using some results on Lyapunov exponents and following some ideas from Zorich and Forni, I will describe the asymptotic behavior of a generic chaotic trajectory. We will also give some results on the ergodic properties of the associated foliations and explain how our object is related to famous family of IET introduced by Arnoux and Rauzy.

Alex Wright
Totally geodesic submanifolds of Teichmüller space and moduli space
(joint work with A. Eskin and S. Filip, with C. McMullen and R. Mukamel, and with Eskin, McMullen and Mukamel).

We consider "higher dimensional Teichmüller discs", by which we mean complex submanifolds of Teichmüller space that contain the Teichmüller disc joining any two of its points. We prove results in the higher dimensional setting that are opposite to the one dimensional behavior: every "higher dimensional Teichmüller disc" covers a "higher dimensional Teichmüller curve" and there are only finitely many "higher dimensional Teichmüller curves" in each moduli space. The proofs use recent results in Teichmüller dynamics, especially joint work with Eskin and Filip on the Kontsevich-Zorich cocycle. Joint work with McMullen and Mukamel as well as Eskin, McMullen and Mukamel shows that exotic examples of "higher dimensional Teichmüller discs" do exist.
