# Kac-Moody instantons in space-time foam as an alternative solution to the black hole information paradox

Antonino Marcianò

Fudan University

based on work in collaboration with A.Addazi, P.Chen and Y.S. Wu

A. Addazi, P. Chen, A. Marciano & Y.S. Wu, arXiv:1707.00347

In preparation, with C. Fields

# The “vexata quaestio”

| | |
|---|---|
| Area increase and no hair | Wheeler et al. ‘71 |
| BH thermodynamics | Bekenstein ‘73 |
| BH radiation | Hawking ‘74 |
| Entanglement entropy | Sorkin ‘82 |
| Quantum hairs vs classical no hair | Veneziano ’86; Coleman, Preskill & Wilczek ‘92 |
| Holographic principle | ’t Hooft ’93; Susskind ‘95 |
| ER=EPR | Susskind & Maldacena ‘13 |
| BMS arguments | Hawking, Perry & Strominger ‘16 |

*Is the information lost during gravitational collapse in to BH?*

# BMS symmetries proposal

*Global symmetries of null asymptotic spacetimes & information encoding*

Infinite dimensional algebra of (hidden) symmetries in the soft infrared limit of scattering amplitudes

A. Strominger, P. Mitra, T. He ,V. Lysol et al. '14…

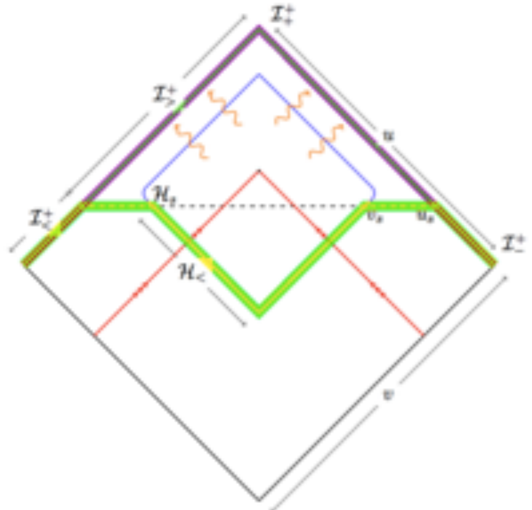
"Information is supposed to be stored not in the interior of the black hole, but on its event horizon"

S.W. Hawking '15

"Soft supertranslation hairs as soft gravitons or photons on the black hole horizon": complete information about their quantum state stored on a holographic plate at the future boundary of the horizon

S.W. Hawking, M.J. Perry & A. Strominger '16

$$Q^+_\varepsilon|0\rangle = \left(\frac{1}{e^2}\int_{\mathcal{I}^+} d\varepsilon \wedge *F\right)|0\rangle \neq 0 \qquad |M'\rangle = Q^{\mathcal{H}_<}_\varepsilon|M\rangle$$



# Reasons to reject the BMS argument

Not stable under radiative corrections: BMS is a classical tree-level symmetries that does not survive even 1-loop corrections

Not clear conservation of the angular momentum for asymptotic states

Canonical transformations decouple soft variables from hard dynamics: long-wavelength photons or gravitons undergo only trivial scattering: they simply pass through the interaction region

M. Mirbabayi and M. Porrati ’16; R. Bousso and M. Prorati ’17

Localized information is independent of fields outside a region: “soft hair play no role in encoding information”.

W. Donnelly and B. Giddings ’17

# New argument: Kac-Moody symmetries

From self-duality conditions to gauge instantons in 4D space-time, and link to Kac-Moody symmetries

Y.S. Wu et al. '82

On the space-time foam an infinity of different YM instantons with the same standard moduli interconnected by an infinite dimensional Kac-Moody algebra

Extend the same result to gravitational instantons

$S2 \times S2$ topologies as virtual black hole pairs fluctuations of the geometry

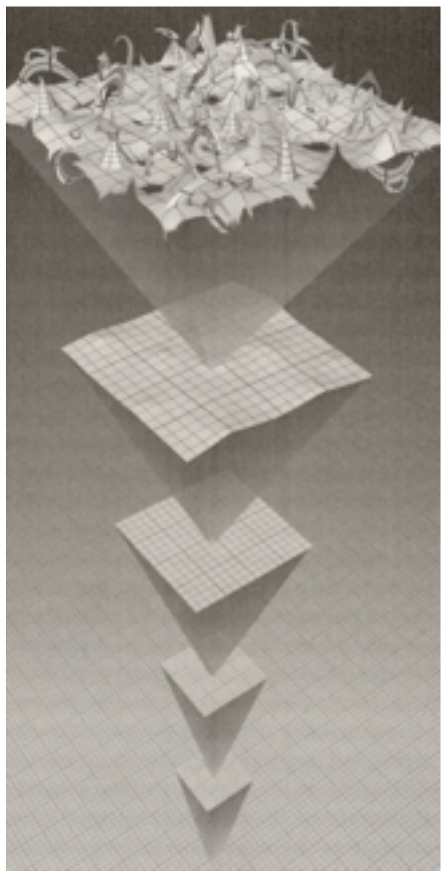
Instantonic solutions (infinite) are interconnected by the infinite dimensional Kac-Moody symmetry: they carry an infinite number of quantum hairs

Quantum hairs as excited moduli of instantonic modes on the foamy texture of space-time

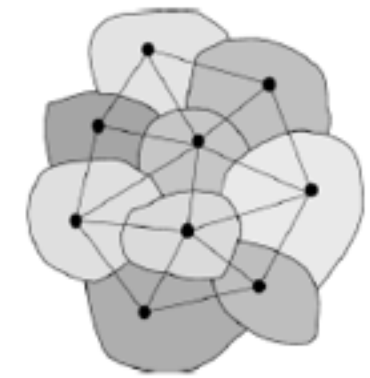
A. Addazi, P. Chen, A. Marciano & Y.S. Wu, arXiv:1707.00347

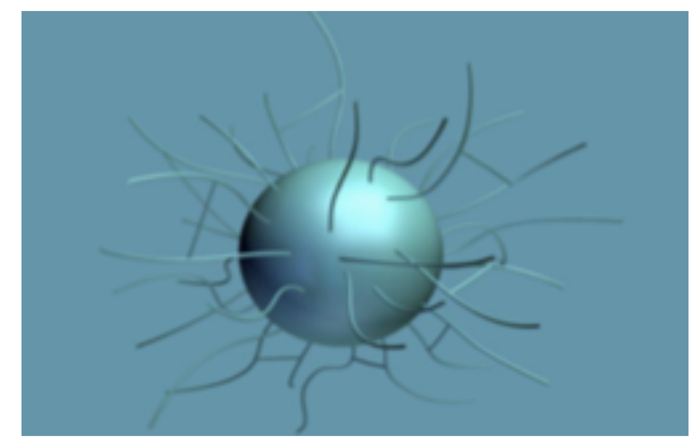
# From mesoscopic scales down to the Planck scale

Mesoscopic/semiclassical scales



Microscopic/Planckian scales





Instantonic solutions and moduli spaces

Kac-Moody symmetries

Holonomies and punctures

Kac-Moody symmetries shift holonomies

# Spacetime foam

Spacetime foam — Wheeler '54

Topological decomposition — Hawking '78

*Gravitational bubble: space-time foam can be topologically deconstructed into three building blocks:*

S2 ×S2, K3, CP2

Only S2xS2 is leading order in the (Euclidean) path integral

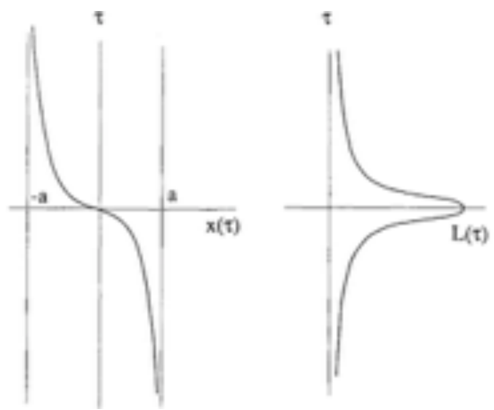
$$S_4 \longrightarrow S_4/\mathbb{Z}_2 \longrightarrow S_2 \times S_2 - \{p\}$$

The S2 ×S2 bubble may topologically correspond to the gravitational instanton with an Euclidean Nariai metric

$$ds^2 = \left(d\Omega^2_{(2)}(\psi,\chi) + d\Omega^2_{(2)}(\theta,\phi)\right)$$

# Instantons in non-perturbative QFT I

The WKB approach naturally leads to the concept of instantons in QM



$$Z(\beta) = \mathrm{Tr} e^{-\beta H} = \int_{x(0)}^{x(\beta)} \mathcal{D}x \; \exp - \frac{1}{g^2} \left( \int_0^\beta dt \, \frac{1}{2} \dot{x}^2 + V(x) \right)$$

The tunneling amplitude is recovered in the dilute gas approximations, matching the WKB result

**Classical solutions of the Euclidean Yang-Mills equations**

$$\tilde{F}_{\mu\nu} = F_{\mu\nu} \qquad\qquad \tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$$

They have a finite action labelled by a natural number, and

$$S \geq S_{\mathrm{SD}} = \frac{8\pi^2 n}{g^2} \qquad \text{where} \qquad \pi_3(S_3) = \mathbb{Z}$$

# Instantons in non-perturbative QFT II

**Ex.** BPST solution for Euclidean SU(2): $A_\mu = \frac{1}{\imath g}\frac{x^2}{x^2+\lambda^2}(\partial_\mu \Omega)\Omega^{-1}$ $\qquad \Omega = \frac{x_4 \pm \imath \sigma_i x_i}{\sqrt{x^2}}$

**Particularly useful in QCD: theta-vacua solve the U(1) problem but the strong CP problem emerges**

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \theta \Delta n = \mathcal{L} + \frac{\theta}{16\pi^2}\mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Axial U(1) symmetry is not present in the particle spectrum of pi-mesons (no parity doubling):

$$\Delta Q_5 = \frac{N_f g^2}{2\pi^2}\int d^4x\, \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

The use of self-dual connections allow to find gravitational instantons

All the gravitational instantons are SU(2) Yang-Mills instantons

Ashtekar ’86, Samuel ’88, Oh, Park & Yang 2011

# Instantons on space-time foam I

A. Addazi, P. Chen, A. Marciano & Y.S. Wu, arXiv:1707.00347

**Three topological building blocks on space-time foam, but only S2 ×S2 dominant**

S2 ×S2 bubbles may topologically correspond to gravitational instantons with an Euclidean Nariai metric

$$ds^2 = \left( d\Omega^2_{(2)}(\psi, \chi) + d\Omega^2_{(2)}(\theta, \phi) \right)$$

Cast the metric in terms of complexified coordinates

$$z_1 = (x_1 + ix_2) \quad \bar{z}_1 = (x_1 - ix_2)$$
$$z_2 = (x_3 + ix_4) \quad \bar{z}_2 = (x_3 - ix_4)$$

$$ds^2 = \left( \frac{dz_1 d\bar{z}_1}{(1 + |z_1|^2)^2} + \frac{dz_2 d\bar{z}_2}{(1 + |z_2|^2)^2} \right)$$

equivalent to state that S2 × S2 ≃ CP1 × CP1, where CP1 manifolds have Fubini-Study metrics

# Instantons on space-time foam II

A. Addazi, P. Chen, A. Marciano & Y.S. Wu, arXiv:1707.00347

We consider a SU(N) YM theory with the self-duality constraint

$$F_{\alpha\beta} = \tilde{F}_{\alpha\beta} \longrightarrow F_{z_1 z_2} = F_{\bar{z}_1 \bar{z}_2} = F_{z_1 \bar{z}_1} + F_{z_2 \bar{z}_2} = 0$$

ADHM construction: gauge field components expressed in terms of (N + 2K) × N complex matrices Di , K standing for the instantonic topological charge and i = 1,2

$$A_{z_1} = \mathcal{D}^T_{(1)} \mathcal{D}_{(2),z_1} \quad A_{z_2} = \mathcal{D}^T_{(1)} \mathcal{D}_{(2),z_2}$$
$$A_{\bar{z}_1} = \bar{\mathcal{D}}^T_{(1)} \bar{\mathcal{D}}_{(2),\bar{z}_1} \quad A_{\bar{z}_2} = \bar{\mathcal{D}}^T_{(1)} \bar{\mathcal{D}}_{(2),\bar{z}_2}$$

Define a current J such that

$$\mathcal{J} = \mathcal{D}^T_{(2)} \bar{\mathcal{D}}_{(1)} \longrightarrow F_{\zeta,\bar{\zeta}} = -\overline{\mathcal{D}}_{(1)} (\mathcal{J}^{-1} \mathcal{J}_{,\zeta})_{,\bar{\zeta}} \overline{\mathcal{D}}_{(2)}, \quad (\zeta = z_1, z_2)$$

# Emergence of Kac-Moody symmetries

Y.S. Wu et al. '82

Because of the initial self-duality condition, the J-current respects a global infinite dimensional algebra

$$[\delta_\alpha^{(m)}, \delta_\beta^{(n)}]\mathcal{J} = \alpha^a \beta^b C_{ab}^c \delta_c^{(m+n)} \mathcal{J}$$

J turns is the Kac-Moody current, shifted by the Kac-Moody group generators

$$\mathcal{Q}_a^m = -\int d^2 z_1 d^2 z_2 \mathbf{Tr}\left[\delta_a^{(m)} \mathcal{J} \frac{\delta}{\delta \mathcal{J}}\right]$$

which fulfill the Kac-Moody algebra

$$[\mathcal{Q}_a^{(m)}, \mathcal{Q}_b^{(n)}] = C_{ab}^c \mathcal{Q}_c^{(m+n)}$$

# Ashtekar self-dual variables & instantons

Samuel '88; Oh, Park Yang '11

Kac-Moody algebra also from the action of Einsteinian gravity

Use gravitational self-dual connection A, whose field strength $F = dA + A \wedge A$

$$S_{\mathrm{EH}} = \kappa \int \mathbf{Tr}\ \Sigma \wedge F - \frac{\lambda}{2} \mathbf{Tr}\ \Sigma \wedge \Sigma$$

Soldering 1-form $\gamma$ s.t. metric tensor with Lorentzian signature $g_{\mu\nu} = \mathrm{Tr}\gamma_\mu^\dagger \gamma_\nu$ ; Plebanski 2-form $\Sigma = \imath\gamma^\dagger \wedge \gamma$

Varying with respect to the phase-space variables, Cartan structure equation and $\gamma \wedge F = \lambda \gamma \wedge \Sigma$

Now use the ansatz $F = \lambda\Sigma$, and the fact that the Plebanski 2-form is self-dual

# Microscopic picture I

Gravitational instantonic solutions such as the Eguchi-Hanson metric, recast in term of self-dual SU(2) Ashtekar variables

Isolated horizons (IH) as non-expanding horizons that look like "apparent horizons in equilibrium", generally accounting for rotation and distortion, and with surface gravity is not evolving in time

*Link between the boundary of the Holst action on the IH and topological SU(2) Chern-Simons theory*

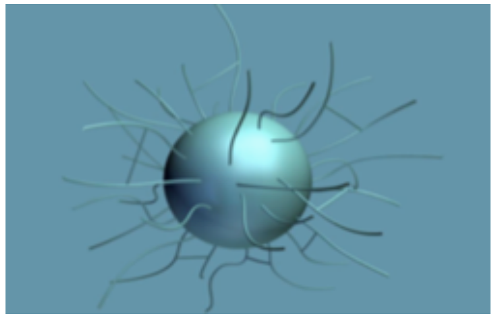
J. Engle, A. Perez and K. Noui '10

$$\gamma\kappa\Omega(\delta_1,\delta_2) = \int_\Delta \delta_1\Sigma_i \wedge \delta_2 A^i + \int_H \delta_1 e_i \wedge \delta_2 e^i \qquad \int_H \delta_1 e_i \wedge \delta_2 e^i = -\frac{a_H}{2(1-\gamma^2)\pi}\int_H \delta_1 A_i \wedge \delta_2 A^i$$

dimensional spatial sections H = Δ ∩ M of IH horizons Δ

$\downarrow$

*Chern-Simons that satisfies the e.o.m.* $\quad F^i_{ab}(A) = -\frac{2\pi}{a_H}\Sigma^i_{ab}$



$$\mathcal{S} = \mathcal{S}_{\rm CS} + \mathcal{S}_{\rm p} = \frac{k}{4\pi}\int_\Delta A^i \wedge dA_i + \frac{2}{3}A^i \wedge \epsilon_{ijk}A^j \wedge A^k + \Sigma^p_{p'=1}\lambda_{p'}\int_{c_{p'}} \mathbf{Tr}[\tau_3\,(\Lambda_p)^{-1}(d_A\Lambda_p)]$$

# Microscopic picture II

Regularize the action on spin-network states

$$\epsilon^{ab}\hat{\Sigma}^i_{ab}(x)|\Gamma, j_l, m_l\rangle = 2\kappa\gamma\delta(x,x_p)\tau^i_p\,|\Gamma, j_l, m_l\rangle \longrightarrow \hat{F}^i_{ab}(A) = -\frac{4\pi\kappa\gamma}{a_H}\sum_{p'=1}^{p}\delta(x,x_{p'})$$

*Quantization of SU(2) CS with real Ashtekar-Barbero connection encodes reps of the quantum group SU$_q$(2)*

$$\downarrow$$

$$g_k(p,d_l) = \frac{2}{2+k}\sum_{d=1}^{k+1}\sin^2\left(\frac{\pi d}{k+2}\right)\prod_{l=1}^{p}\frac{\sin\left(\frac{\pi}{k+2}dd_l\right)}{\sin\left(\frac{\pi}{k+2}d\right)}$$

*Finite volume of H*

*Self-dual Ashtekar connection recovered by an analytical continuation*

J. Ben Achour, A. Mouchet and K. Noui '15

$$\gamma = \pm\imath \longrightarrow k = \mp\imath\lambda, \quad j_l = \frac{1}{2}(\imath s_l - 1)$$
$$\lambda, s_l \in \mathbb{R}^+$$

$$\longrightarrow$$

$$a_H(j_l) = 8\pi l_p^2\gamma\sum_l\sqrt{j_l(j_l+1)} \quad \rightarrow$$
$$\rightarrow \quad a_H(s_l) = 4\pi l_p^2\sum_l\sqrt{s_l^2+1}$$

# From loops to bubbles

Minimum area gap: $a_H = 4\pi\, l_P^2\, \lambda$

$\downarrow$

Finite dimension of the kinematical Hilbert space (reps bounded by the level of the Chern-Simons theory λ).
In the semiclassical limit this the entropy asymptotically approaches the measure of H

*Gravitational instantons: supported at semiclassical level on S3 manifolds (Wick-rotated to S2 × R); at the quantum level defined on a finite set of worldlines that discretize the domain into a piece-wise linear manifold*

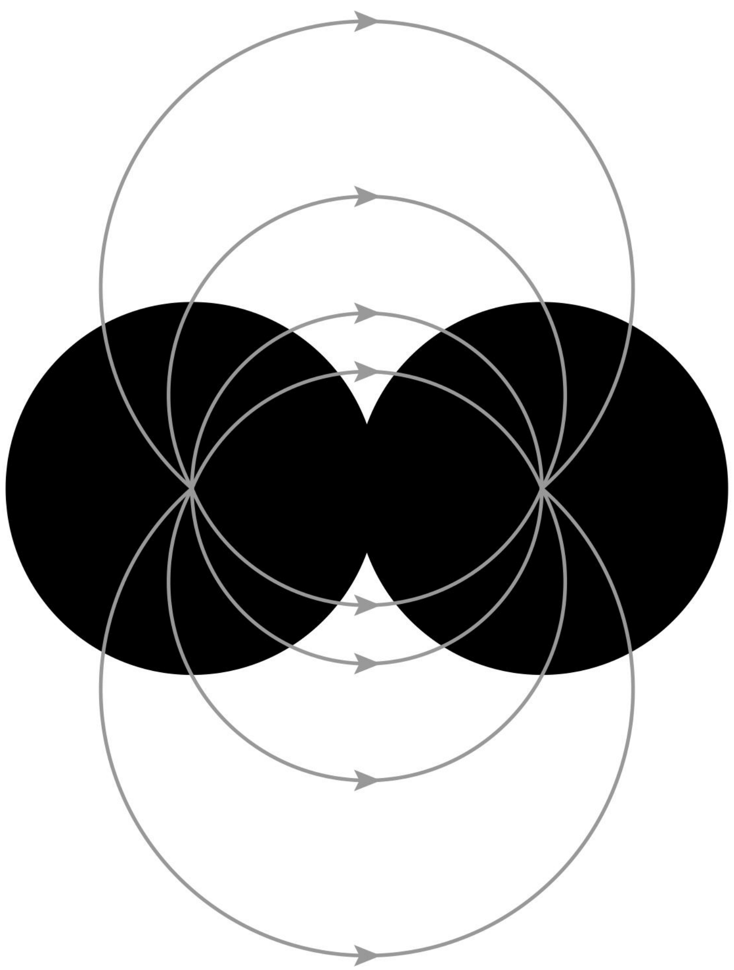
*Quantum hairs on space-time foam: finiteness of the Hilbert space (and of the measure of H) suggests to consider only finite amount of instantons, with related Kac-Moody charges*

$$\mathrm{Kac\!-\!Moody:}\;\; U_\gamma \to U_\gamma + \delta U_\gamma$$

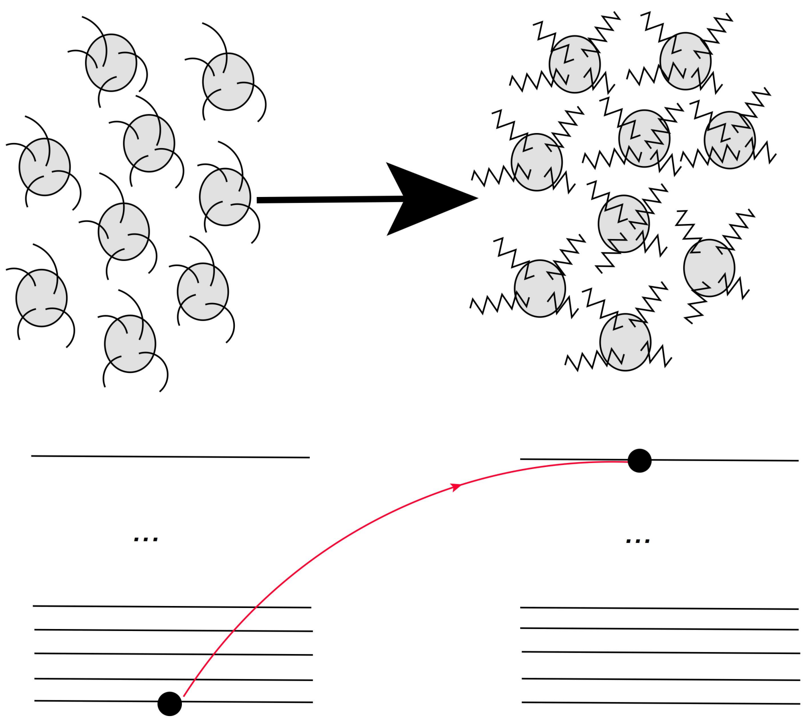
*The Kac-Moody algebra connects an infinite number of gravitational loops associated to the same punctures*.

# Information encoding

Microscopic



Mesoscopic




*Kac-Moody algebra connects an infinite number of holonomies, associated to the same punctures. Wilson lines are not instantons, but Kac-Moody symmetry remerges as symmetry of holonomies.*

*The black hole change of state corresponds to a transformation of the instantonic hairs. This changes the energy state of the system and the information processed through mass gap of energy levels*

# Dynamically breakdown of Kac-Moody

A. Addazi, P. Chen, A. Marciano & Y.S. Wu, arXiv:1707.00347

Action of interacting instanton and anti-instanton of centers $x_0$ and $y_0$

$$S + S' + S_{\rm int}(x_0, y_0)$$

$$S = \int_{S_2 \times S_2 - B} d\zeta^2 d\bar{\zeta}^2 \left[ \frac{1}{4} F\tilde{F} + \frac{1}{8}(F - \tilde{F})^2 \right]$$

The action has no saddle point now for instantonic configurations, but instead is minimized when

$$(F - \tilde{F}) = -L_{int}$$

Thus we can easily conclude that the Kac-Moody algebra is dynamically broken by interactions among instantons

# Information encoding

A. Addazi, P. Chen, A. Marciano & Y.S. Wu, arXiv:1707.00347

Instantonic moduli are associated to zero modes. These are Nambu-Goldstone bosons of the symmetry spontaneously broken by instantonic solutions.

They gain dynamically a mass, becoming pseudo-Nambu-Goldstone bosons: generation of a mass gap.

Mass gap roughly controlled by the confinement scale of the YM theory. At the first level of the Kac-Moody ascendent scale ($M = 1$), we expect an energy level $E_1 \sim \Lambda$, while at the M-th level an energy level $E_M \sim M\Lambda$…

M different instantons connected by Kac-Moody transformations have an energy difference $E_M - E_{M-1} \sim \Lambda$.

# Conclusions

Infinite number of different Yang-Mills and gravitational instantons (with the same standard moduli) in space-time foam are connected by the Kac-Moody symmetry

Bubbles that are topologically BH-WH pairs are the building blocks of the semiclassical limit, and to each generic element of the decomposition will correspond a family of gravitational instantons

Irreducible representations of punctures on H label homotopy classes of instantons that interpolate between BH and WH 2-spheres constituting the $S2 \times S2$ space-time bubbles

Infalling collapsing matter into the black hole excites ground state instantons of determined size and center into instantons with the same size and center but different (quantum reduced) Kac-Moody charges

Differently than in the BMS picture, in our picture information is no more supposed to be stored at the event horizon, but everywhere around the (would-be) singularity: holographic principle applies only locally!

Kac-Moody charges are stored in the virtual BH pairs: vaguely reminiscent of the ER=EPR conjecture

谢谢

Merci!

Thank you!

Grazie!