The partially disjoint paths problem

Lex Schrijver

University of Amsterdam and CWI Amsterdam


Congratulations Tom and Jaime!!

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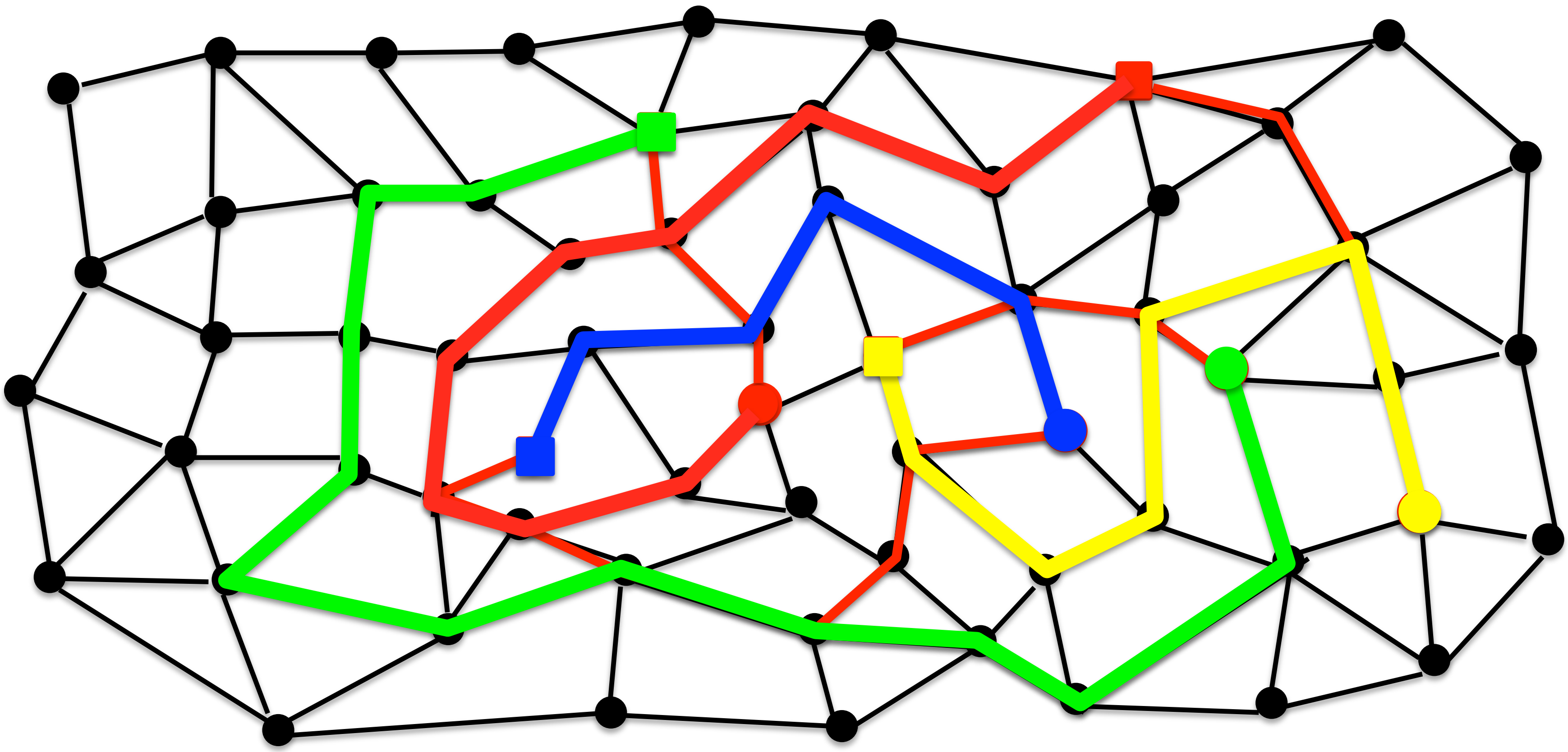
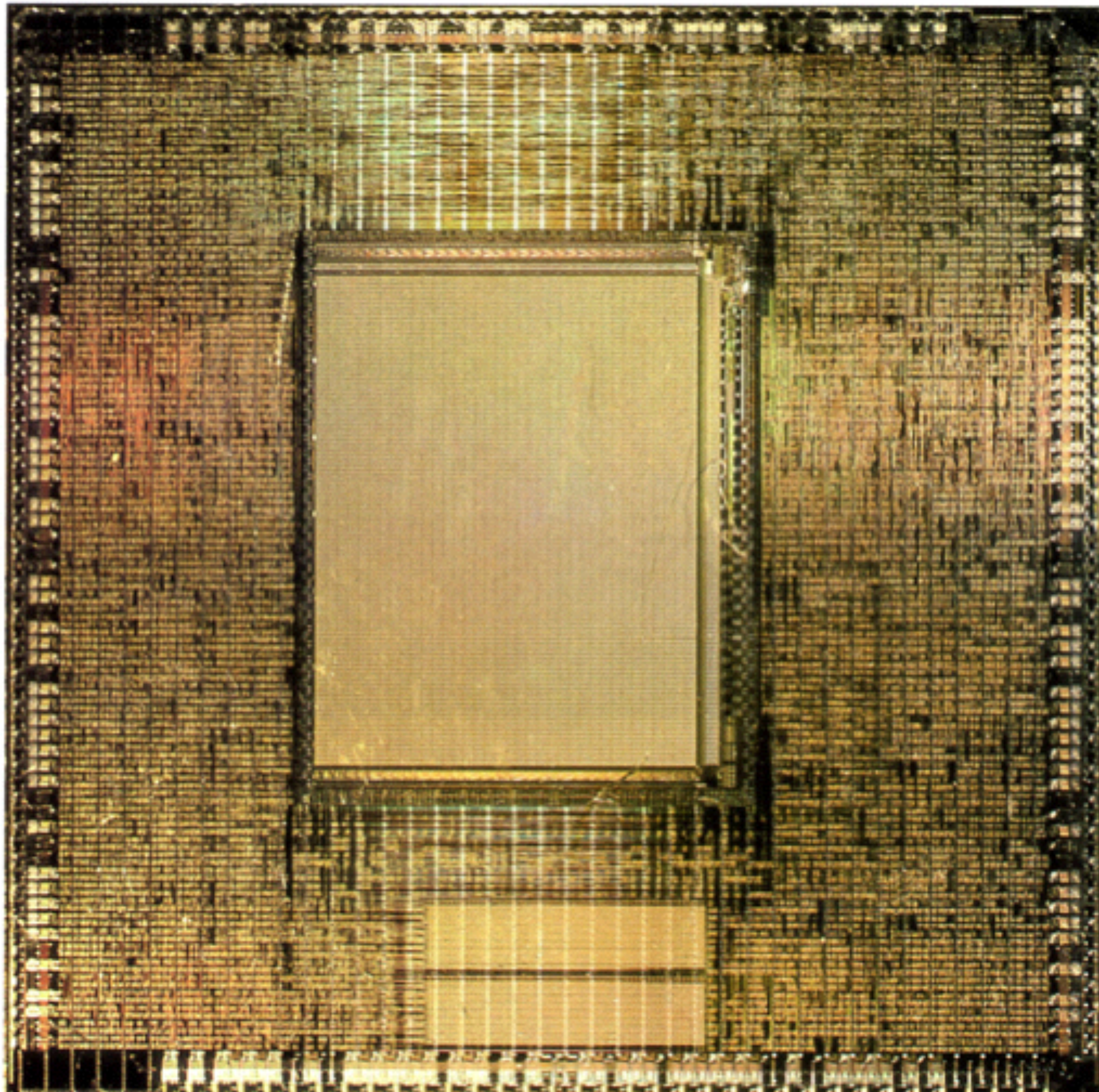


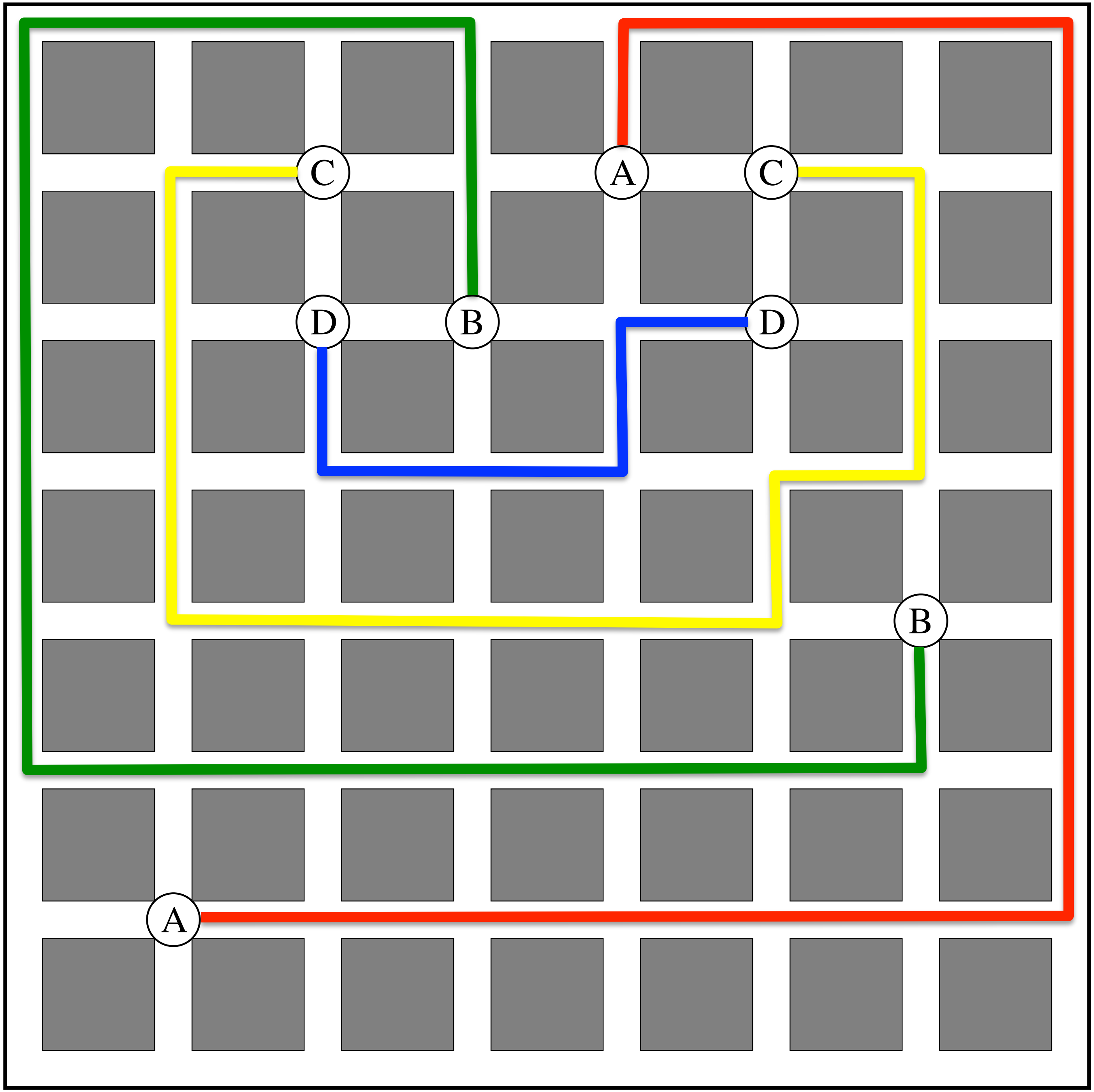
Connect sources with terminals arbitrarily: Easy

Connect given pairs of source and terminal: Hard

But for fixed number of pairs: Easy

(Robertson-Seymour 1995)

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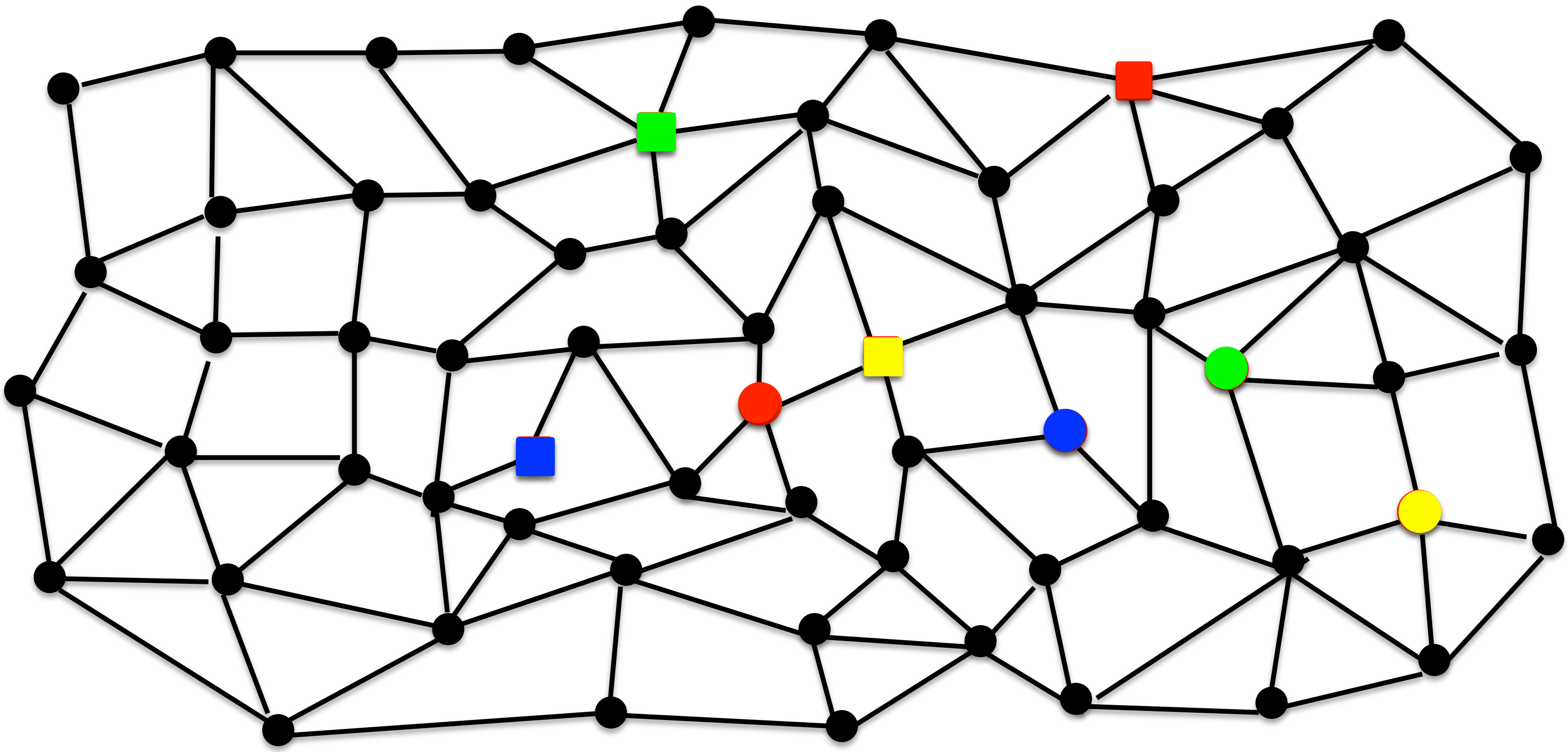
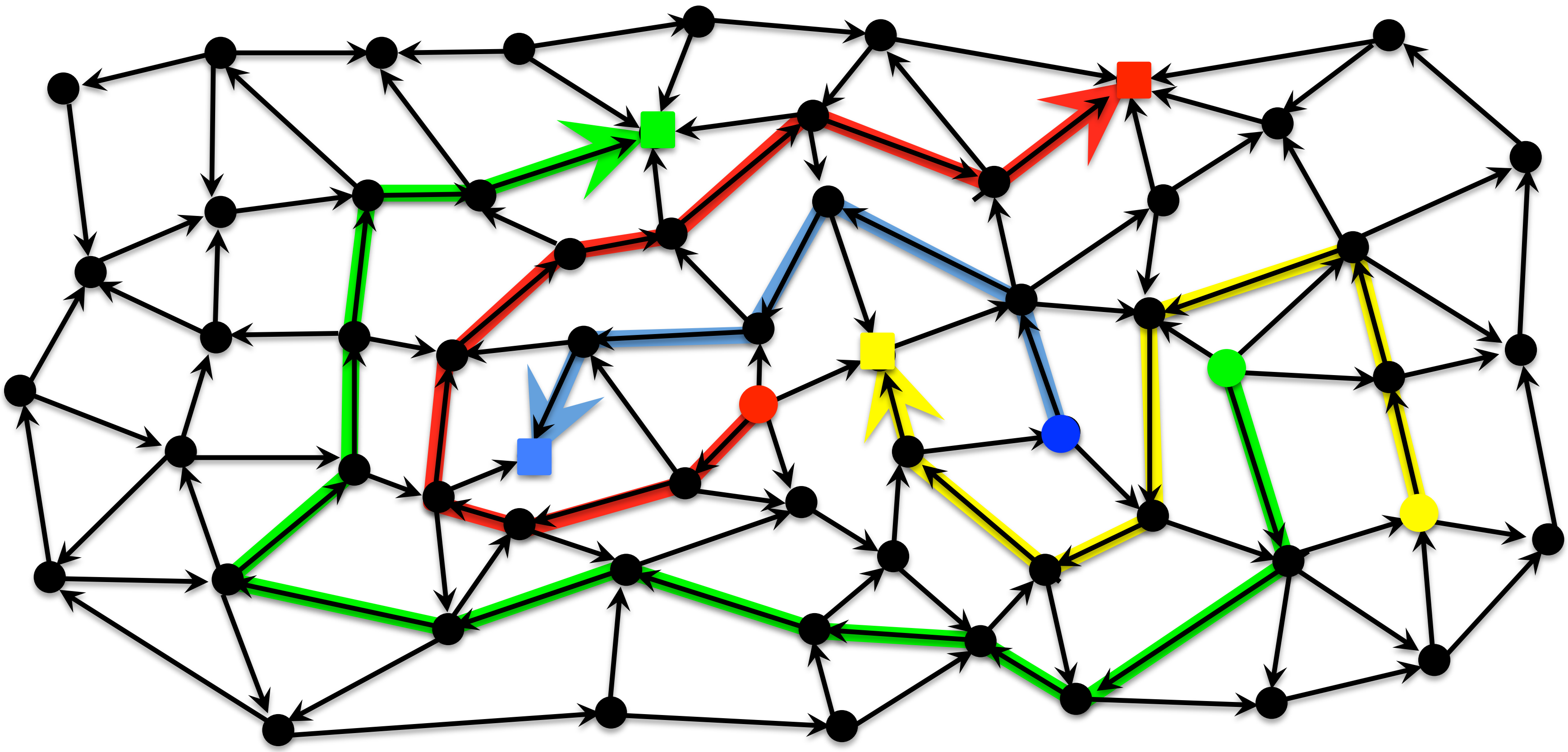


What about directed graphs?



What about <u>directed graphs</u>?

For <u>two</u> source/terminal pairs: Hard

But for <u>planar</u> graphs and <u>fixed</u> number of source/terminal pairs: Easy

(SIAM J. Computing 1994)

Method: based on the *free group* Γ generated by $g_1, \ldots, g_k$.

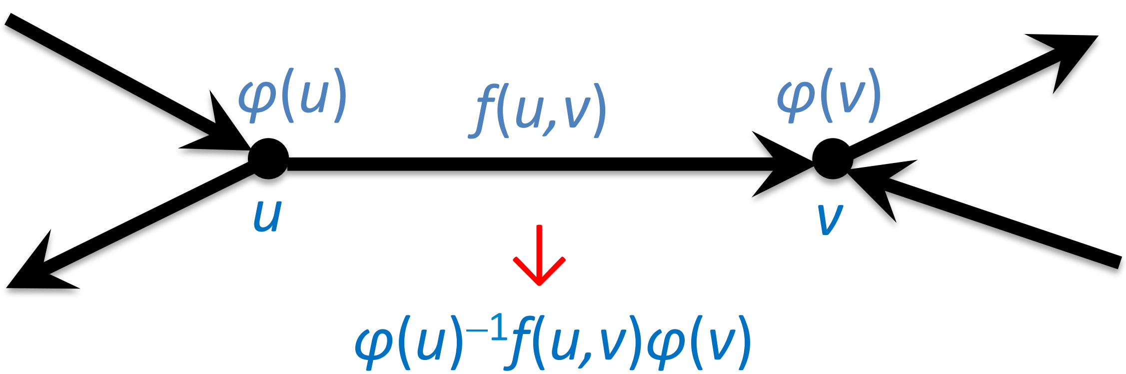
So: Γ = { words over the alphabet $g_1, g_1^{-1}, \ldots, g_k, g_k^{-1}$ without $g_i g_i^{-1}$ or $g_i^{-1} g_i$ }

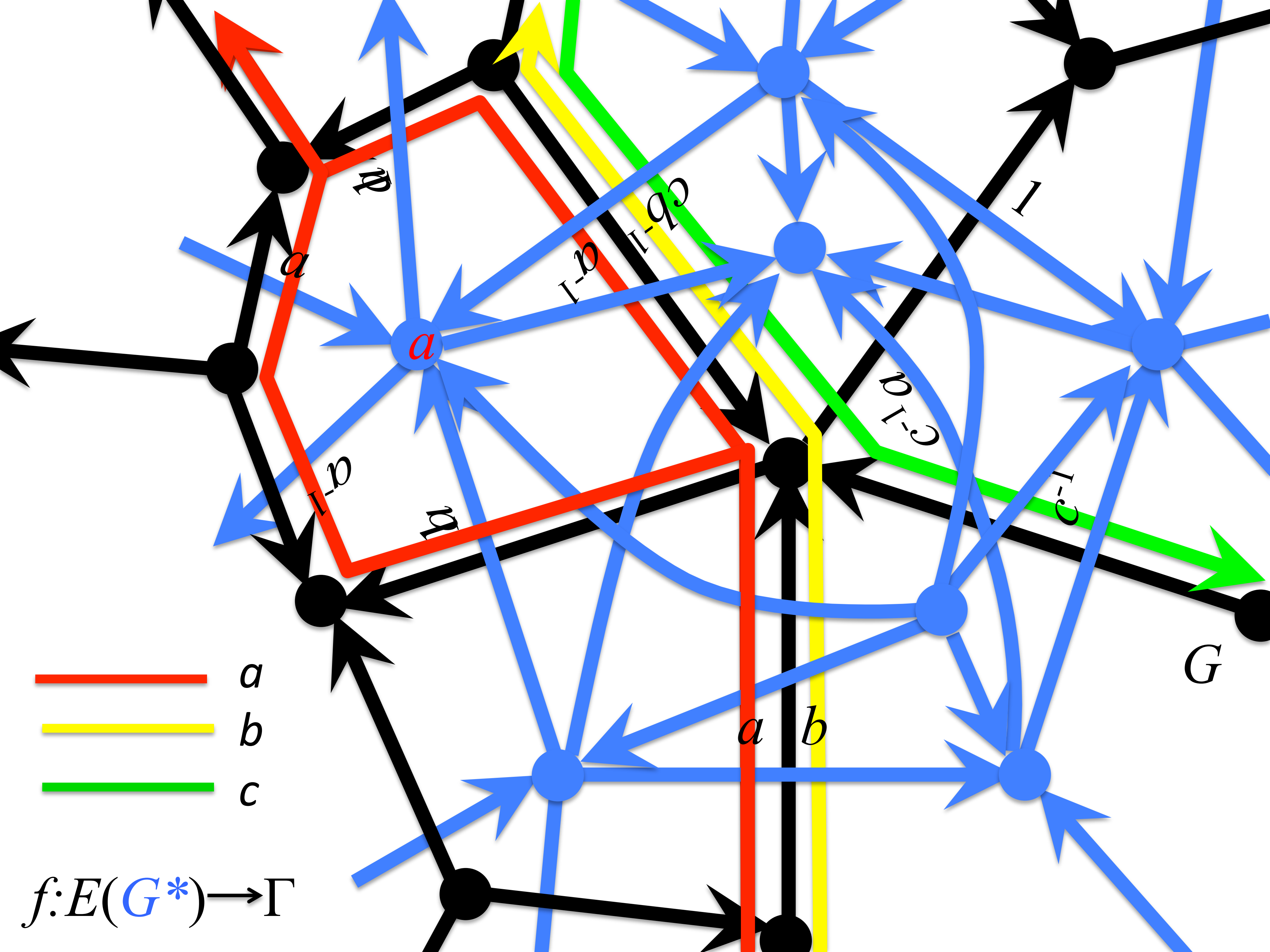
Theorem: *The following problem is solvable in polynomial time:*

given: a directed graph G and a function $f : E(G) \to \Gamma$,

find: a function $\varphi : V(G) \to \Gamma$ such that for each edge (u,v):

$\varphi(u)^{-1} f(u,v) \varphi(v)$ belongs to $\{ g_1, \ldots, g_k, 1\}$.




a
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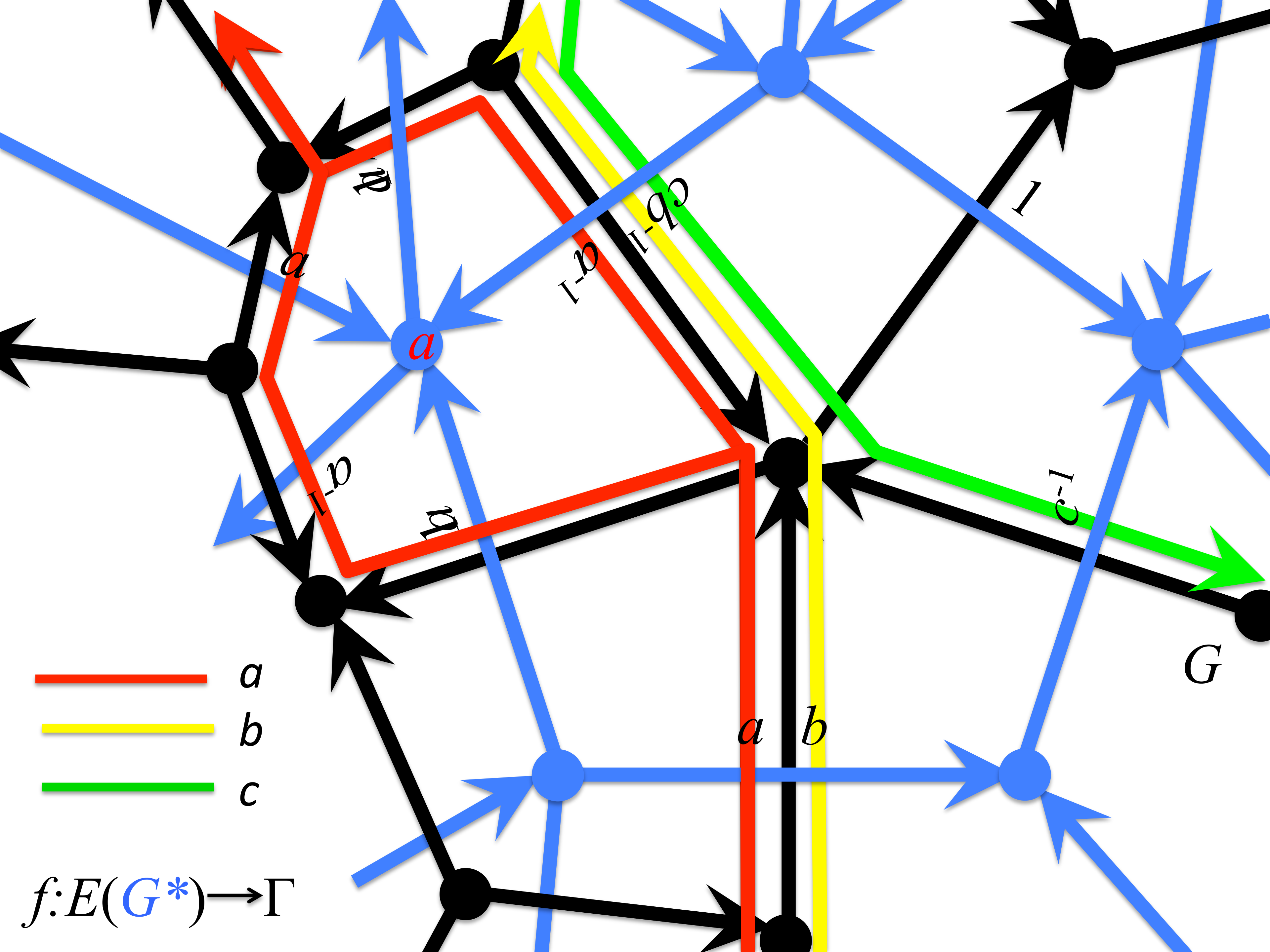
G
f:E(G*)→Γ



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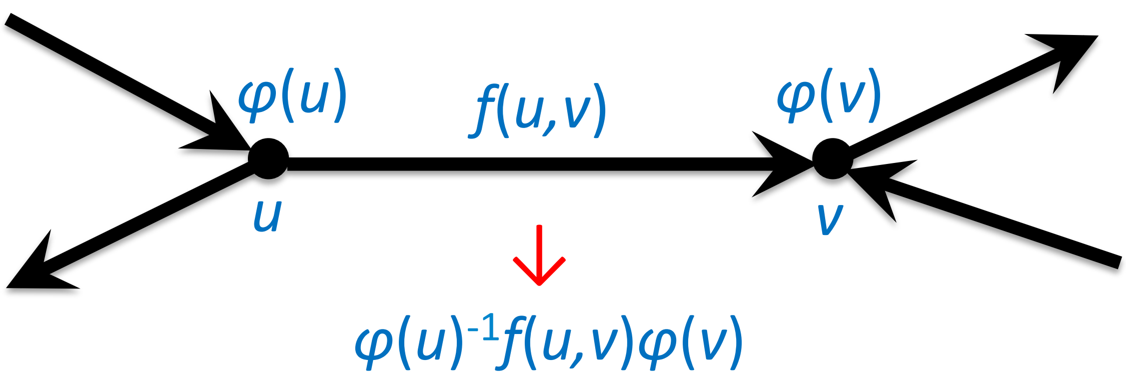

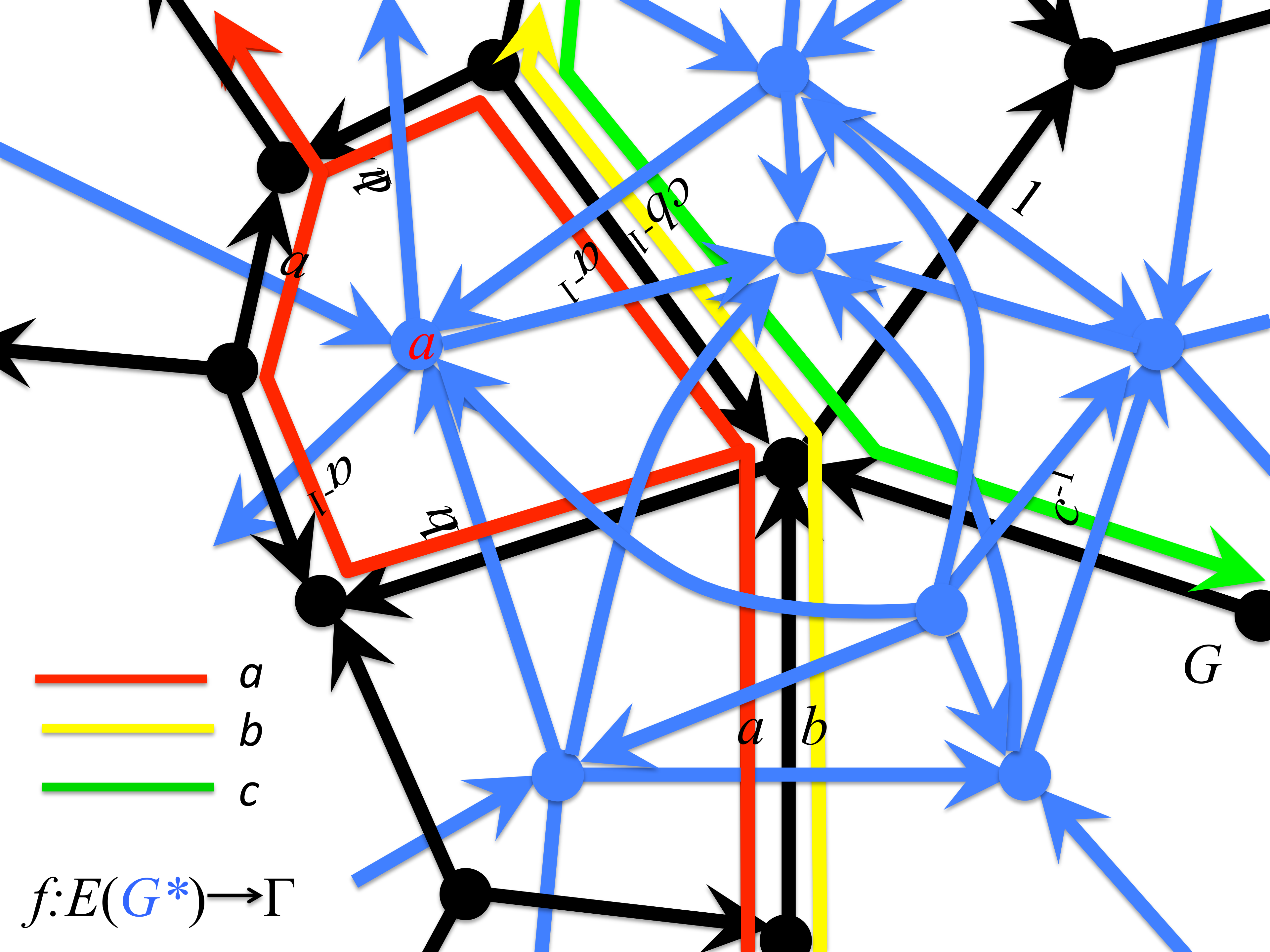

a
a
a
b
G
a
b
c
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The *k* disjoint paths problem:

given: a directed graph G and vertices $r_1,s_1,\ldots,r_k,s_k$ of G,

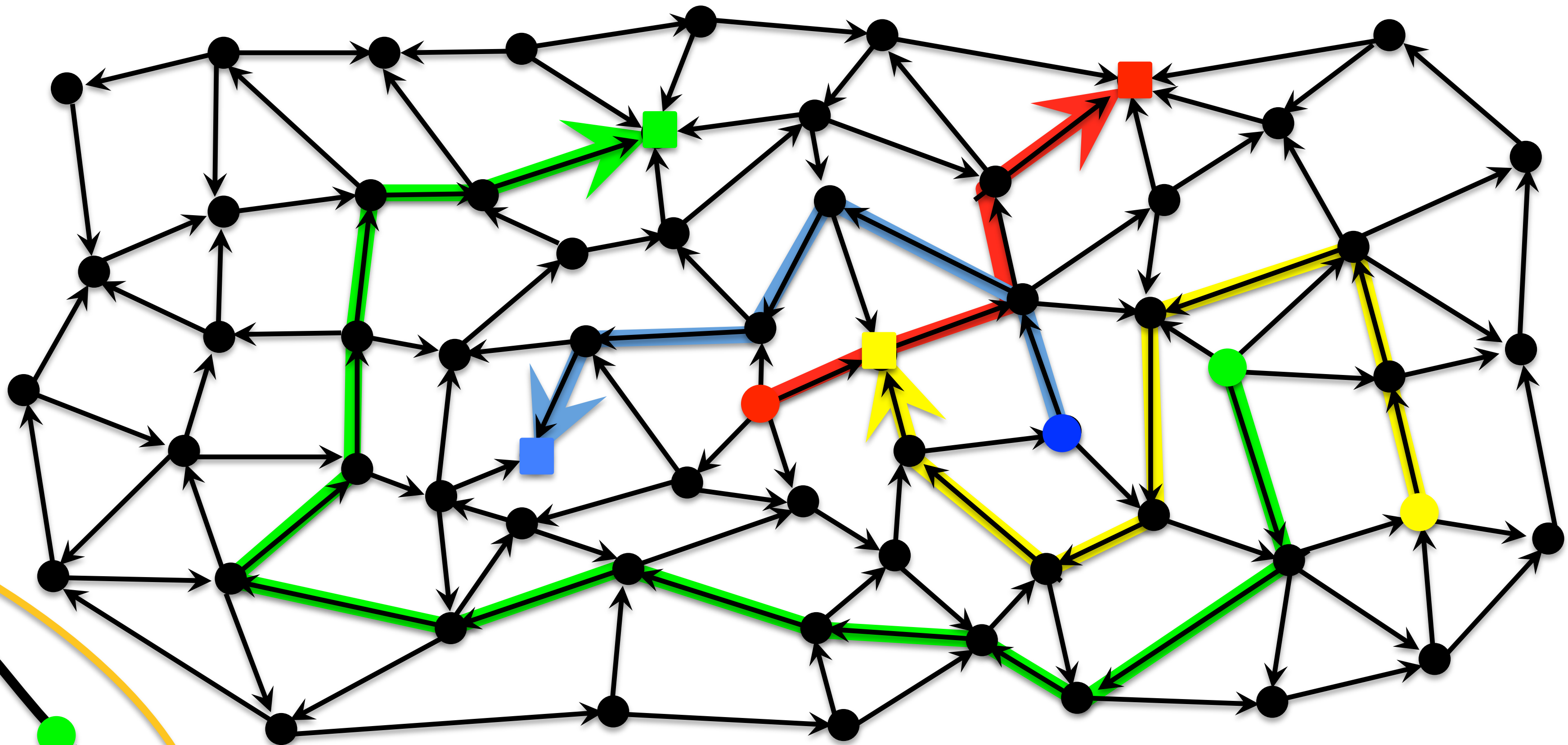
find: pairwise disjoint paths $P_1,\ldots,P_k$ where P_i runs from r_i to s_i.

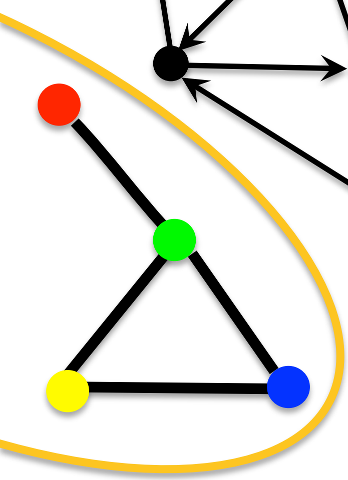
Call $P_1,\ldots,P_k$ a *pre-solution* if

- they are undirected paths (i.e., they may go against the direction of the edges)
- they do not cross one another.

Algorithm:

1. Enumerate `sufficiently many' pre-solutions.
2. For each such pre-solution, check if it is homotopic to a solution.

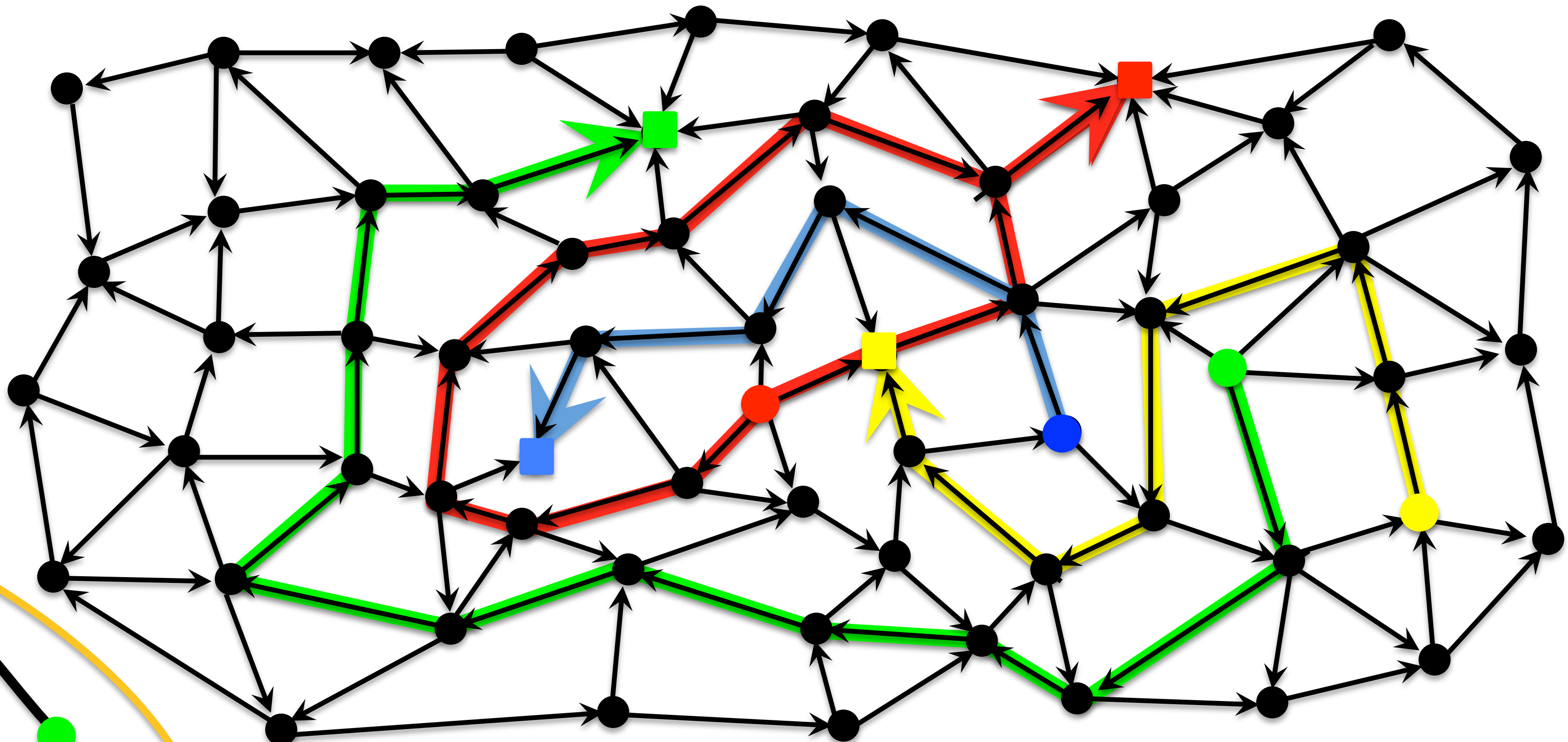


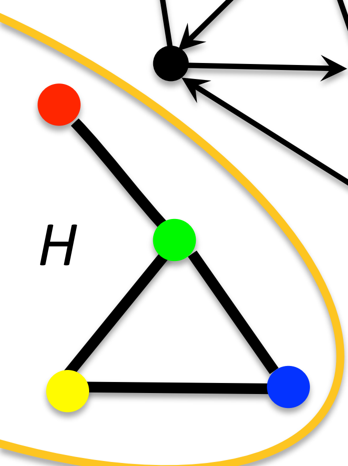


Also easy for directed planar graphs and fixed # of source/terminal pairs:

if only certain pairs of paths need to be disjoint

(Method: flow over the free partially commutative group)



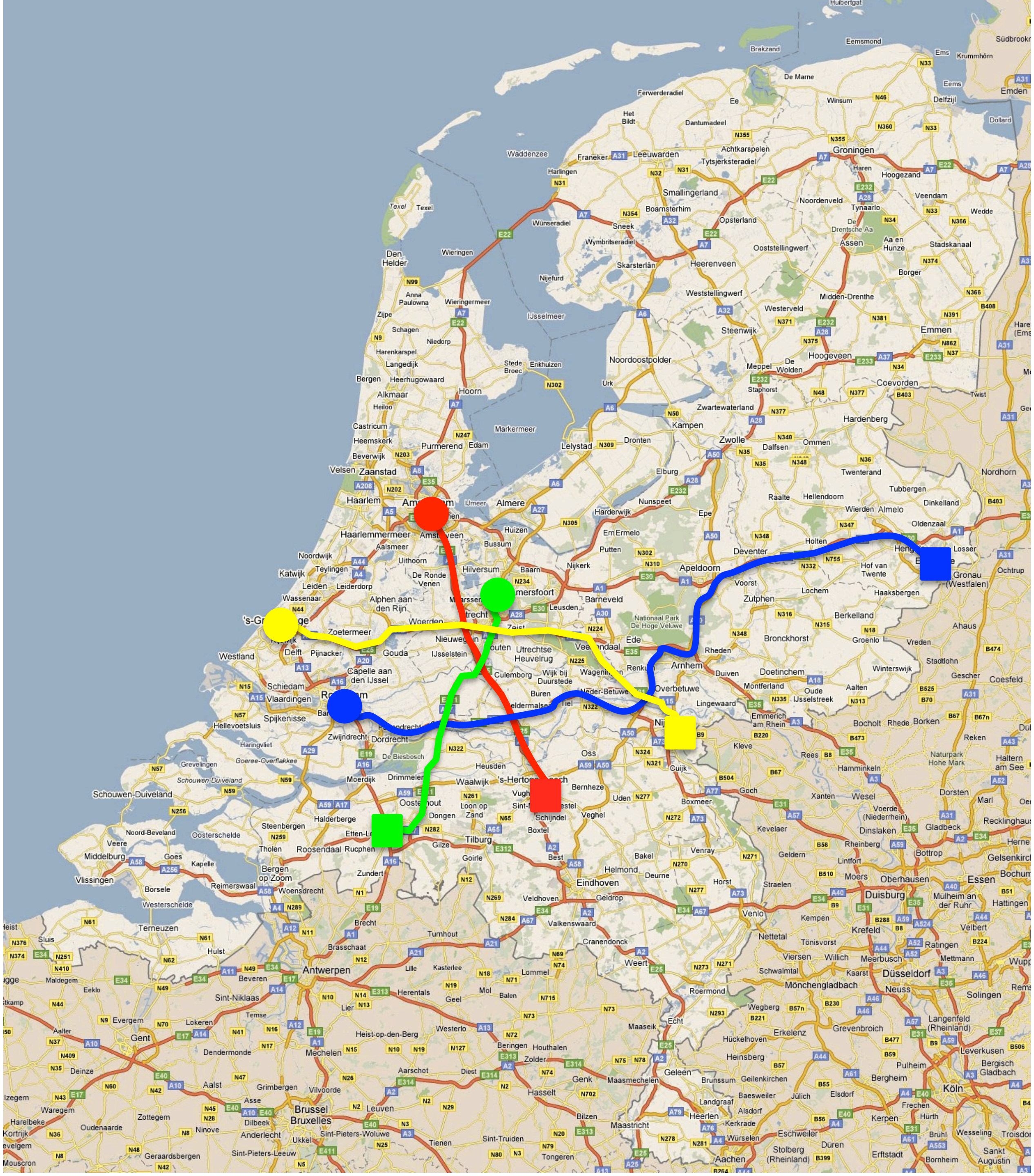


partially ← P_i and P_j disjoint if i and j are connected in H

The disjoint paths problem

is solvable in polynomial time for directed planar graphs

<u>for any fixed number of terminals</u>.


Waddenzee
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Markermeer
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Groningen
Emden
Franeker
Harlingen
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Assen
Emmen
Hoogeveen
Meppel
Steenwijk
Zwolle
Kampen
Lelystad
Dronten
Almere
Alkmaar
Hoorn
Purmerend
Zaanstad
Haarlem
Haarlemmermeer
Hilversum
Amersfoort
Apeldoorn
Deventer
Zutphen
Arnhem
Ede
Barneveld
Leiden
Katwijk
Zoetermeer
Gouda
Delft
Westland
Schiedam
Vlaardingen
Spijkenisse
Dordrecht
Zwijndrecht
Culemborg
Tiel
Oss
Nijmegen
Cuijk
Heusden
Waalwijk
Tilburg
Breda
Oosterhout
Roosendaal
Bergen op Zoom
Eindhoven
Helmond
Venlo
Roermond
Weert
Vlissingen
Middelburg
Schouwen-Duiveland
Terneuzen
Antwerpen
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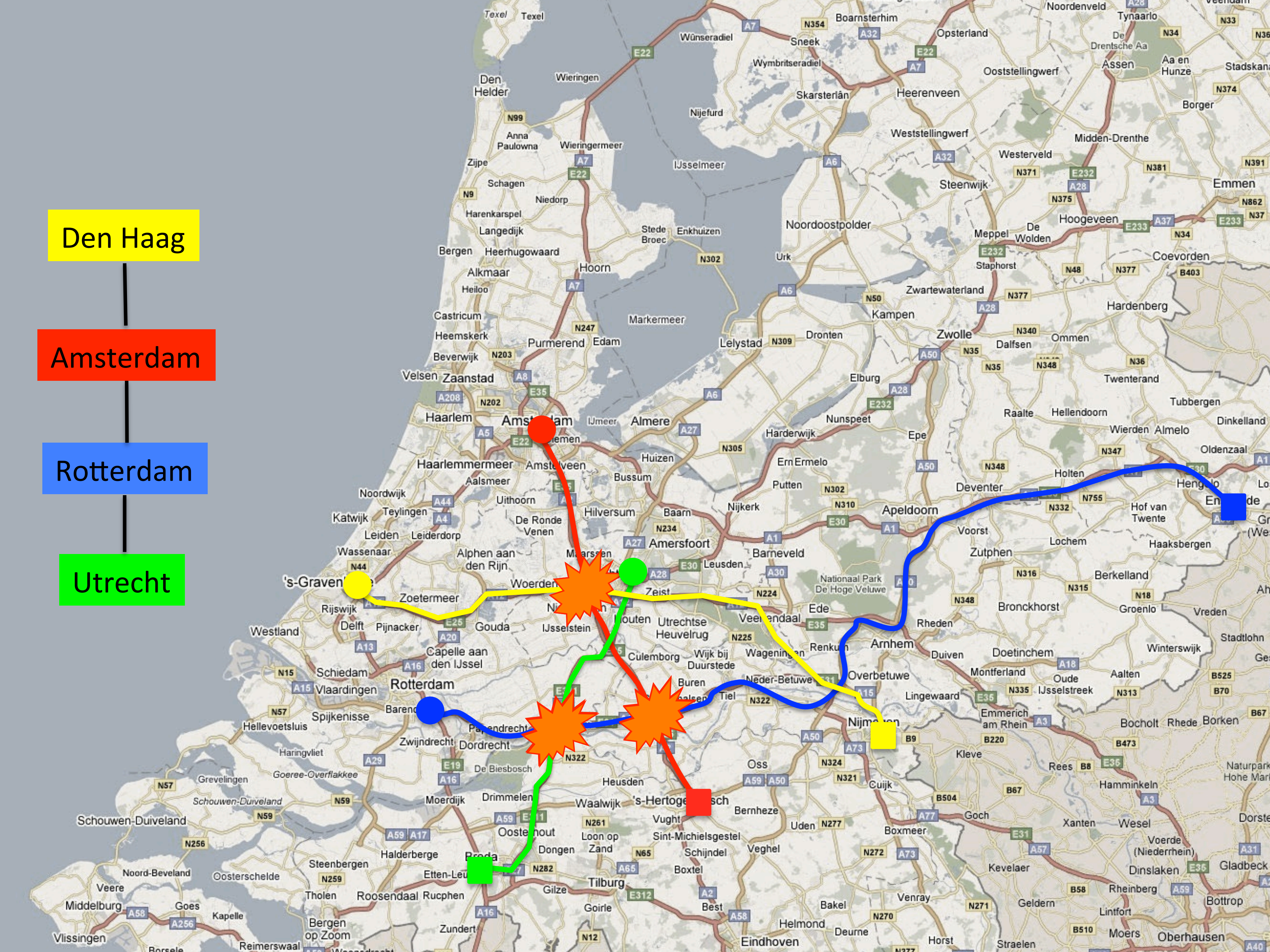
Mechelen
Brussel Bruxelles
Leuven
Hasselt
Genk
Maastricht
Aachen
Köln
Düsseldorf
Duisburg
Essen
Mönchengladbach
Krefeld
Bocholt
Borken
Enschede
Hengelo
Almelo
Oldenzaal
Gronau (Westfalen)
Nordhorn
Winterswijk
Doetinchem


Den Haag
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Versions:

G undirected / directed

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Theorem: The k partially disjoint paths problem for directed planar graphs is polynomial-time solvable, for each fixed k.

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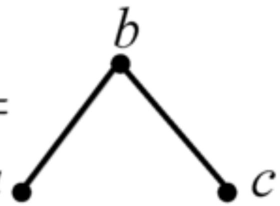
Graph groups

Graph groups

$H =$ 

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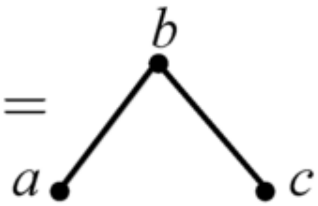
$H =$ 

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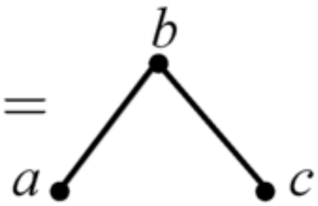
$H =$ 

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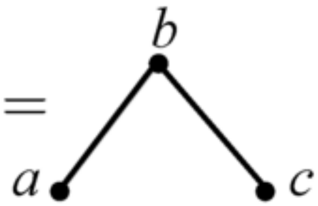
$H =$ 

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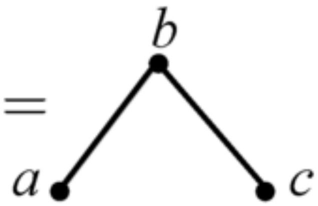
$H =$ 

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$aba^{-1}c$

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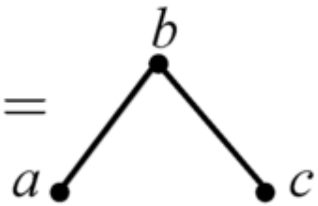
$H =$ 

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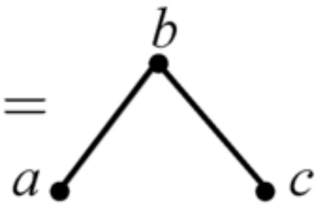
$H =$ 

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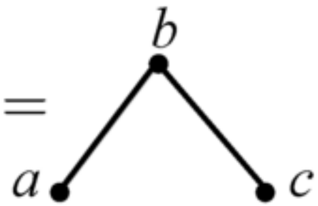
$H =$ 

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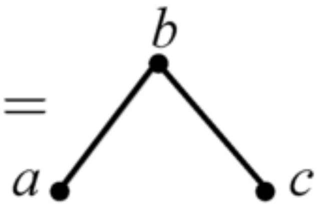
$H =$ 

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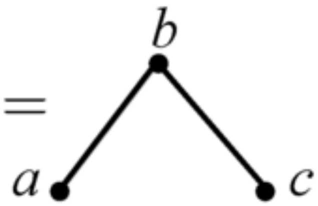
$H =$ 

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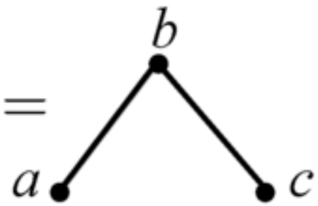
$H =$ 

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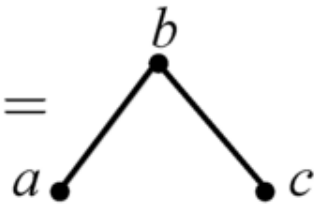
$H =$ 

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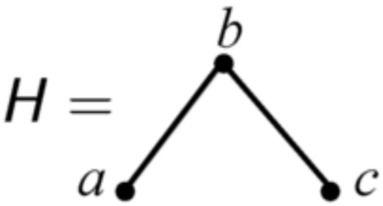

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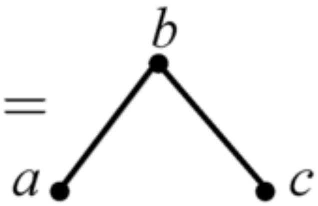
$H =$ 

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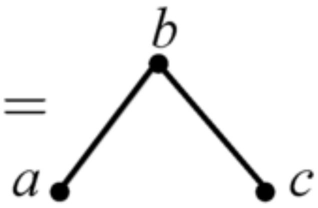
$H =$ 

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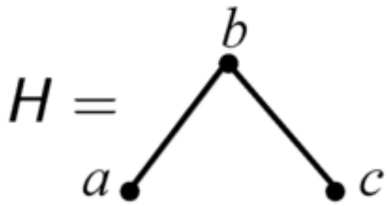
Graph groups



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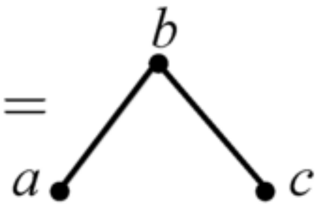
$H =$ 

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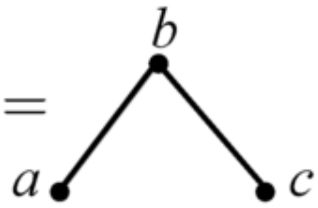
$H =$ 

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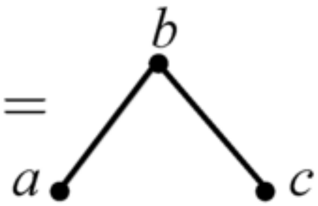
$H =$ 

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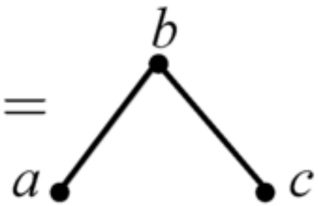
$H =$ 

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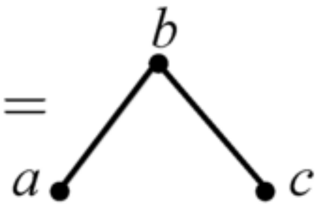
$H =$ 

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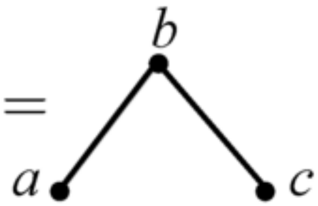
$H =$ 

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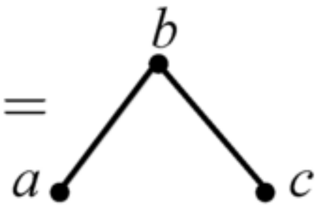
$H =$ 

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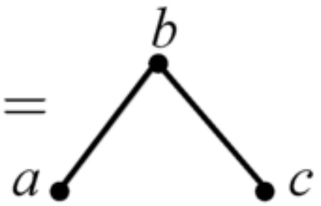
$H =$ 

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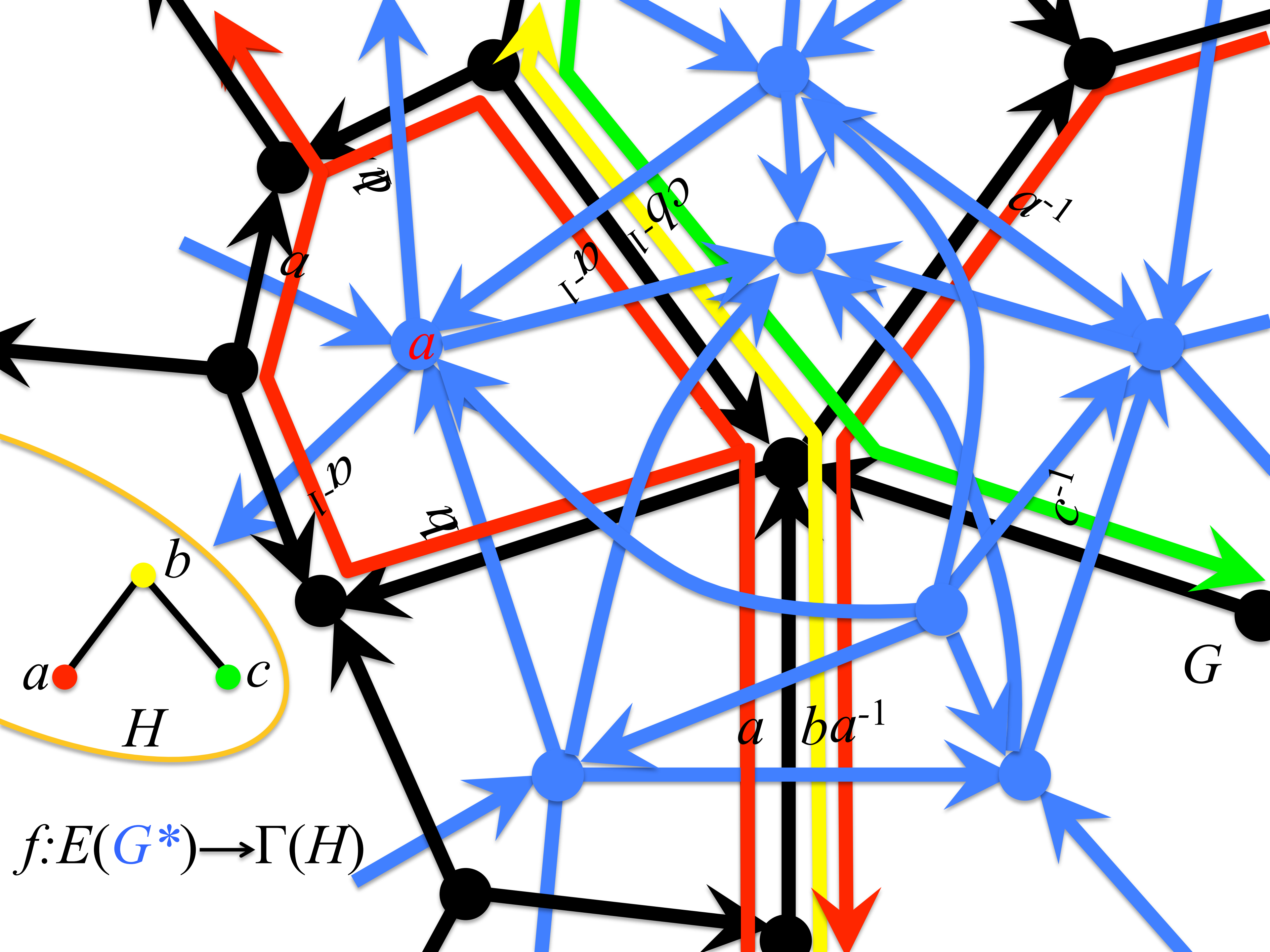
a
b
c
H
G
a
ba-1
a-1
cb-1
c-1
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Algorithm

Algorithm

The k partially disjoint paths problem:

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given: a graph G, vertices $r_1, s_1, \ldots, r_k, s_k$ of G,
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Algorithm:

1. Enumerate 'sufficiently many' pre-solutions
2. For each such pre-solution, check if it is homotopic to a solution.

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THANK YOU !