A column generation approach for the Strong Network Orientation Problem

Alexandre X. Martins[1] Christophe Duhamel[2] Andréa C. Duhamel[1]

[1]Université de Technologie de Troyes, France

[2]Université Clermont Auvergne, France

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The problem

- Disruptions on urban networks
- Troyes: 8.000 disruptions each year
- Each disruption is handled by hand

The problem

Predictable

- Fairs and exhibitions
- Road maintenance

The problem

Unpredictable

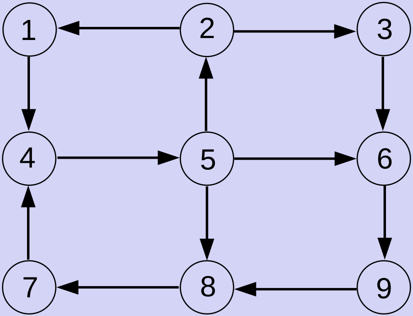
- Traffic accidents
- Natural accidents

The problem

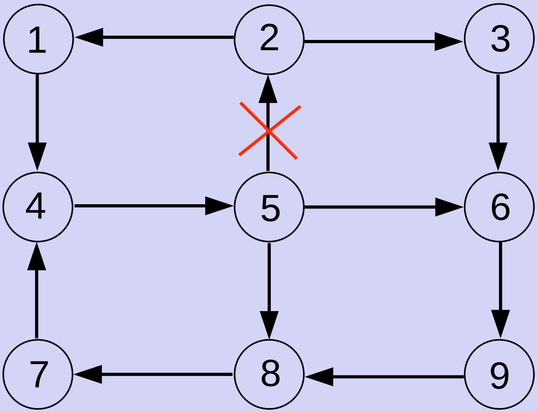
- Objective: set alternate paths to deviate the traffic flow
- Decision: reversing the flow direction (arc reversals) is allowed
- Constraints: the road network has to remain strongly connected
- Criteria
  - c_1 : Minimize the distance between the nodes
  - c_2 : Minimize the number of reversed arcs

Example

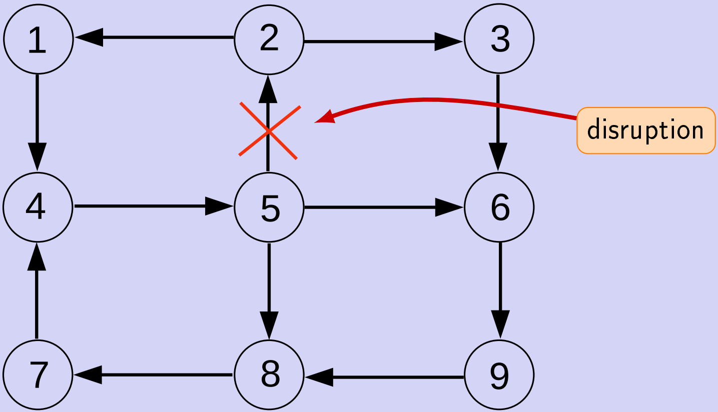
- Representation: directed connected graph
  - Nodes: crossroads, corners, roundabouts, etc
  - Arcs: roads, streets with initial orientation



Example

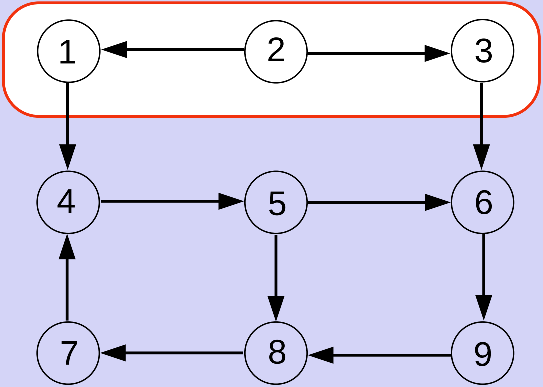


Example



Example

- Strong connectivity might be broken



Example

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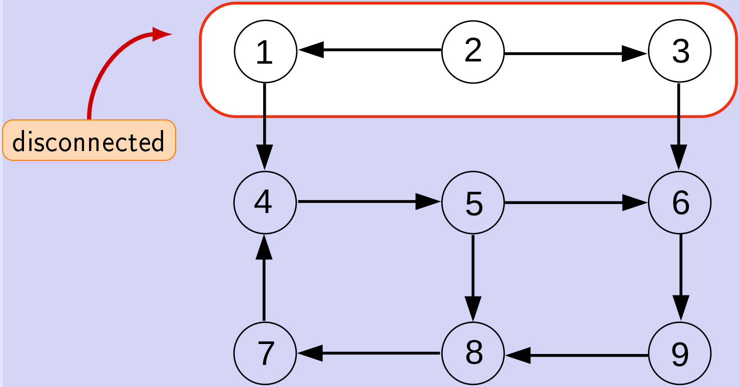
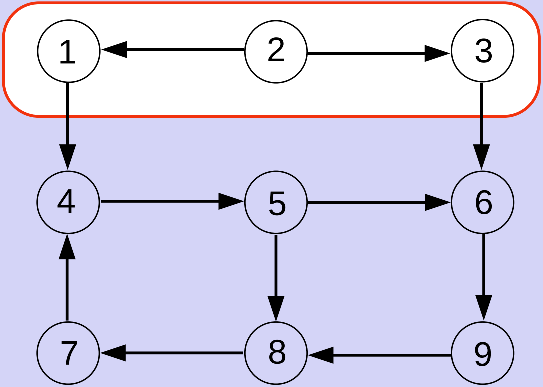


Example

- Strong connectivity might be broken

disconnected

4 SCC!



Bibliography

A theorem on graphs, with an application to a problem on traffic control {*Robbins, H.E. (1939)*}

- Conditions for strong connectivity

Distances in orientations of graphs {*Chvátal,V.and Thomassen,C. (1978)*}

- NP-Hard
- Equivalent to the problem of computing a minimum cost strong network orientation

Bibliography

Graph theory and its applications to problems of society {*Roberts, F.S. (1978)*}

- Orientation based on Depth First Search (DFS) with strong connection

Heuristics for urban road network design: lane layout and signal settings {*Cantarella, G. E., Pavone, G., Vitetta, A. (2006)*}

- Heuristics (HC, GA, SA,TS GA+TS, TS+PR)
- Real data of 3 Italian cities

Bibliography

A Meta-heuristic Algorithm for Solving the Road Network Design Problem in Regional Contexts {*Gallo, M., D'Ancierno, L., Montella, B. (2012)*}

- Scatter Search + ACO
- Optimization of investment for improving rural network
- Real data of Campania

Heuristics for setting directions in urban networks {*Santos; Duhamel; Prins (2013)*}

- Heuristics for the SNOP
- Real data of Clermont-Ferrand
- A simple method to create the initial solution (adapted from Roberts(78))

Bibliography

A bi-objective model to address disruptions on unidirectional road networks {*Huang; Santos; Duhamel (2016)*}

- bi-objective mathematical model
- Pareto-optimal: 25 nodes

Column Generation {*Desaulniers, G., Desrosiers, J., Solomon, M.M. (2005)*}

- Overview of the state of the art in integer programming column generation and its many applications

Formulations

- graph transformation
  - $A' \leftarrow A \cup \overline{A}$
  - complement with the reversed arcs

- Parameters
  - $G = (N, A')$, $n = |N|$
  - c_{ij}: unit flow cost for arc $(i,j) \in A'$
- Variables
  - $x_{ij} \in \{0,1\}, (i,j) \in A'$: arc use indicator
  - $f_{ij}^s \in \mathbb{R}$: flow from s on arc $(i,j) \in A'$
    - $f_{ij}^s \in [0, n-1]$

Compact Model

Model

$$(\text{Compact}) \min \sum_{(i,j)\in A'} \sum_{s\in N} c_{ij} f_{ij}^s \quad (1)$$

st.:

$$x_{ij} + x_{ji} = 1 \qquad \forall\ (i,j) \in A \quad (2)$$

$$\sum_{(j,i)\in A'} f_{ji}^s - \sum_{(i,j)\in A'} f_{ij}^s = \begin{cases} n-1, & j = s \\ -1, & j \neq s \end{cases} \qquad \forall s \in N \quad (3)$$

$$f_{ij}^s - (n-1)x_{ij} \leq 0 \qquad \forall s \in N, (i,j) \in A' \quad (4)$$

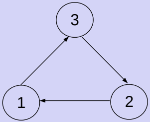
$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A' \quad (5)$$

$$f_{ij}^s \geq 0 \qquad \forall (i,j) \in A', s \in N \quad (6)$$

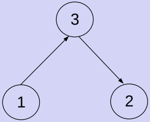
Column Generation

Decomposition

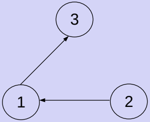
- idea: given a strongly connected graph, there is a least one spanning arborescence connecting any node to any other node



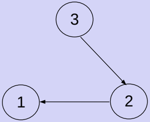
(a) Feasible solution



(b) source 1



(c) source 2



(d) source 3

- new model: for each node s, select a spanning arbo. rooted at s

Column Generation

- Parameters
  - T^s: set of all spanning arbo. rooted in s, $s \in N$
  - for a spanning arbo. $k \in T^s$
    - c_k^s : flow cost
    - a_{ij}^{sk}: if $(i,j) \in k$ or not
- Variables
  - $x_{ij} \in \{0,1\}, (i,j) \in A'$: arc use indicator
  - $y_k^s \in \{0,1\}$: if spanning arbo. k rooted at s is chosen or not

Column Generation

$$(\text{TreePath}) \min \sum_{s\in V} \sum_{k\in T^s} c_k^s y_k^s \qquad s.t. \qquad (7)$$

$$x_{ij} + x_{ji} = 1 \quad \forall\ (i,j) \in A \qquad (8)$$

$$\sum_{k\in T^s} y_k^s \geq 1 \quad \forall\ s \in N \qquad (9)$$

$$x_{ij} - \sum_{k\in T^s} a_{ij}^{sk} y_k^s \geq 0 \quad \forall\ (i,j) \in A', s \in N \qquad (10)$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in A' \qquad (11)$$

$$y_k^s \in \{0,1\} \quad \forall s \in N, k \in T^s \qquad (12)$$

Column Generation

- issue: exponential size for sets T^s
- column generation
  - linear relaxation
  - use subset $T'^s \subset T^s \rightarrow$ Restricted Master Problem (RMP)
  - improving columns y computed "on the fly"
  - inserted in T^s whenever their reduced cost $r(y) < 0$

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- dual variables
  - (9) $\sum_{k \in T^s} y_k^s \geq 1 \quad \forall \ s \in N \qquad \rightarrow \qquad \lambda^s \geq 0$
  - (10) $x_{ij} - \sum_{k \in T^s} a_{ij}^{sk} y_k^s \geq 0 \quad \forall \ (i,j) \in A', s \in N \qquad \rightarrow \qquad \mu_{ij}^s \geq 0$

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- reduced cost: $r(y_k^s) = c_k^s + \sum_{(i,j) \in A'} (a_{ij}^{sk} \mu_{ij}^s) - \lambda^s$

Column Generation

Subproblem

$$\mathrm{SP}(s)\ \min \sum_{(i,j)\in A'} (c_{ij} f_{ij}^s + \mu_{ij}^s x_{ij}) - \lambda^s \qquad s.t. \tag{13}$$

$$x_{ij} + x_{ji} \leq 1 \quad \forall\ (i,j) \in A \tag{14}$$

$$\sum_{j\in N} f_{ji}^s - \sum_{j\in N} f_{ij}^s = 1 \quad \forall\ i \in N \setminus \{s\} \tag{15}$$

$$\sum_{(s,j)\in A'} f_{sj}^s - \sum_{(j,s)\in A'} f_{js}^s = n-1 \tag{16}$$

$$f_{ij}^s - (n-1)x_{ij} \leq 0 \quad \forall\ (i,j) \in A' \tag{17}$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in A' \tag{18}$$

$$f_{ij}^s \geq 0 \quad \forall (i,j) \in A' \tag{19}$$

Column Generation

- All spanning arbo. can be picked
- Yet, some cannot be combined for a strongly connected network

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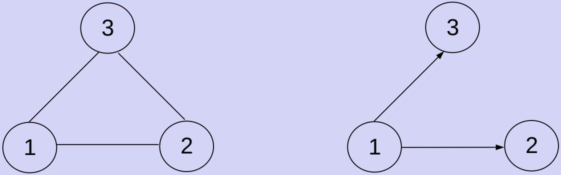
Example



(a) Instance (b) source 1

Column Generation

Improved Column Generation

- idea: only consider spanning arbo. that can belong to (a least) one strongly connected orientation
- force a flow $g_{ij}^s \geq 0$ from all nodes to node s wrt. arc orientations
- no change in the RMP since we restrict to a subset of T^s

Column Generation

Improved Column Generation

- idea: only consider spanning arbo. that can belong to (a least) one strongly connected orientation
- force a flow $g_{ij}^s \geq 0$ from all nodes to node s wrt. arc orientations
- no change in the RMP since we restrict to a subset of T^s

$$\sum_{j \in N} g_{ij}^s - \sum_{j \in N} g_{ji}^s = 1 \quad \forall \; i \in N \setminus \{s\} \tag{20}$$

$$\sum_{(j,s) \in A'} g_{js}^s - \sum_{(s,j) \in A'} g_{sj}^s = n - 1 \tag{21}$$

$$g_{ij}^s - (n-1) x_{ij} \leq 0 \quad \forall \; (i,j) \in A' \tag{22}$$

$$g_{ij}^s \geq 0 \quad \forall (i,j) \in A' \tag{23}$$

Column Generation

Initial Columns

- if $T'^s = \emptyset \quad \forall s \in N$, then the RMP if unfeasible

$$\sum_{k \in T^s} y_k^s \geq 1 \quad \forall s \in N$$

Column Generation

Initial Columns

- if $T'^{s} = \emptyset \quad \forall s \in N$, then the RMP if unfeasible

$$\sum_{k \in T^s} y_k^s \geq 1 \quad \forall s \in N$$

- Add a slack variable $z^s \geq 0 \quad \forall s \in N$

$$\min \sum_{s \in N} \sum_{k \in T^s} c_k^s y_k^s + M \sum_{s \in N} z^s$$

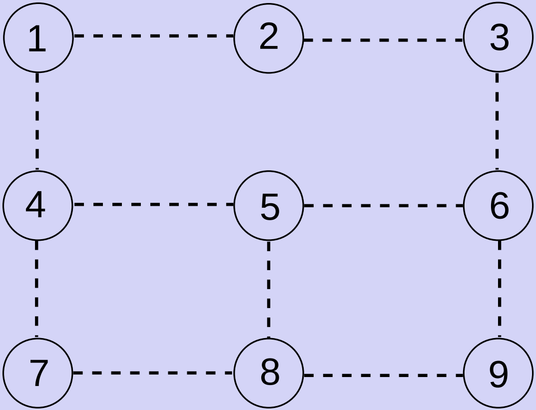
$$\sum_{k \in T^s} y_k^s + z^s \geq 1 \quad \forall s \in N$$

$$z^s \geq 0 \quad \forall s \in N$$

Column Generation

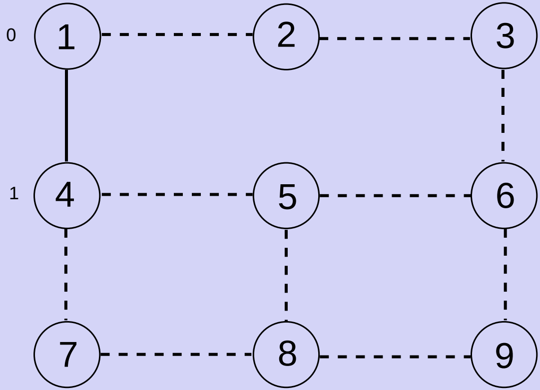
Computing initial columns

- Depth First Search (DFS) algorithm, adapted from (Roberts 78)



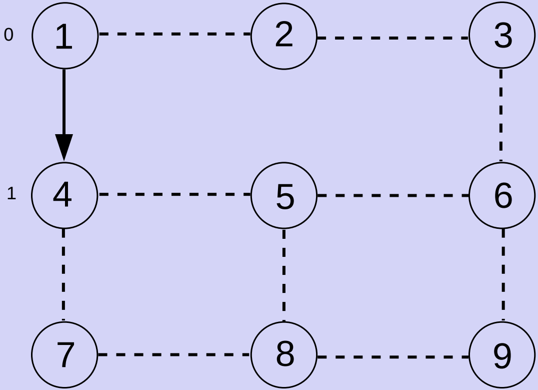
Column Generation

- DFS scan



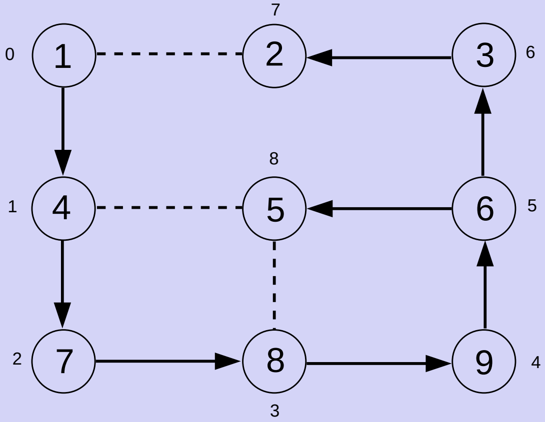
Column Generation

- The arcs are oriented the way they are traversed during the DFS



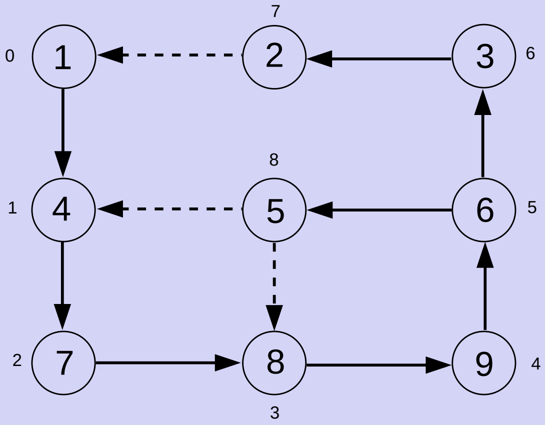
Column Generation

- DFS solution



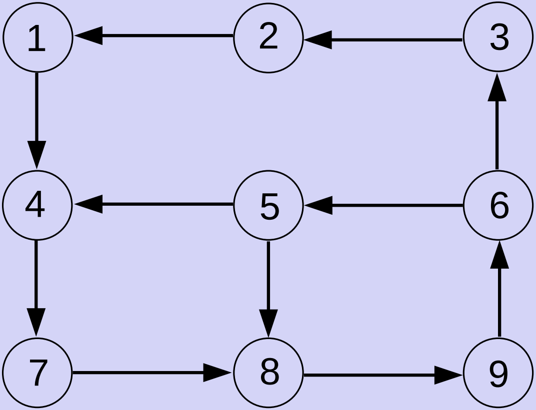
Column Generation

- Remaining edges oriented from the highest rank node to the lowest rank node



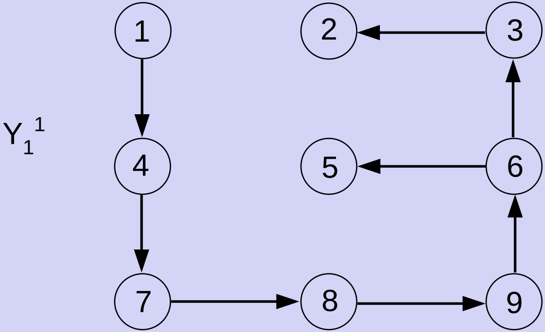
Column Generation

- Proven strongly connected orientation



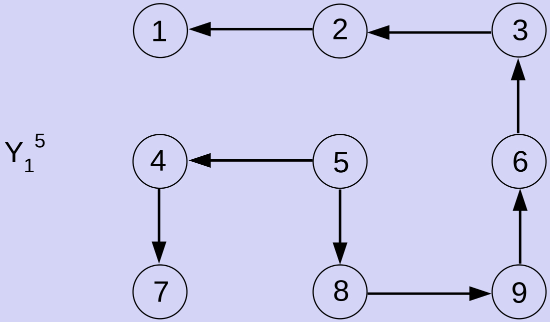
Column Generation

- Decomposition for source node 1



Column Generation

- Decomposition for source node 5



Column Generation

procedure *ColumnGeneration*(G)

do

Solve RMP(T'^s)

Get the duals λ and μ

for each $i \in N$

$r(y_i) \leftarrow$ Solve SubProblem(i, λ, μ)

if $r(y_i) < 0$

then $T'^s \leftarrow T'^s \oplus \{y_i\}$

while New columns added

end

Results

- C++ with CPLEX 12.6
- Instances from 'A bi-objective model to address disruptions on unidirectional road networks' {*Huang; Santos; Duhamel (2016)*}
- Three exact methods
  - CG: $T'^s = \{\emptyset\}$, basic column generation
  - DFS+CG: initial columns from DFS, basic column generation
  - DFS+ICG: initial columns from DFS, improved column generation

Results

Impact on the linear relaxation

Instances		Compact model	Column Generation	
size	$\|B\|$	LR	LR	OPT
4x4	1	786	**789**	912
	2	836	**841**	968
	4	**1216**	**1216**	1392
	6	**1088**	**1088**	1304
5x5	1	2292	**2297**	2638
	2	2404	**2409**	2748
	4	2636	**2645**	3032
	6	2672	**2674**	3116

Results

Comparison of the three methods

Instances		CG		DFS+CG		DFS+ICG		Init. cols.
size	$\|B\|$	CPU(s.)	cols.	CPU(s.)	cols.	CPU(s.)	cols.	
4x4	1	22.58	725	27.89	840	**18.13**	**667**	256
	2	21.89	673	21.46	823	**16.53**	**609**	
	4	7.24	**306**	10.18	568	**6.08**	476	
	6	5.06	**252**	8.38	462	**2.92**	341	
5x5	1	381.41	3667	327.31	4023	**323.19**	**2679**	625
	2	1064.84	5992	277.58	3641	**224.94**	**2231**	
	4	1137.00	6369	667.16	6080	**188.34**	**2043**	
	6	867.52	6565	575.75	6076	**140.53**	**1703**	

Results

Impact of the subproblem

Instances		DFS+ICG		
size	$\|B\|$	CPU Master(s.)	CPU Sub(s.)	% Sub
4x4	1	0.41	17.72	97.3
	2	0.29	16.24	98.2
	4	0.14	5.94	97.7
	6	0.09	2.83	96.9
5x5	1	12.20	310.99	96.2
	2	7.84	217.10	96.5
	4	6.66	181.68	96.5
	6	4.66	135.88	96.7

Conclusions

- The CG strategy is able to solve the SNOP up to 25 nodes
- The method improves the lower bounds for almost all instances
- The modified subproblem
  - forbids infeasible columns
  - improves the time to get the solutions in all instances

Future works

- Heuristics to solve the subproblem
- Other heuristics to create initial columns
- Branching strategies

Thank you all!!
Merci tout le monde!!